Relative mirror velocity estimation

LIGO-T060170-00-R

M. Varvella* for Virgo LAL and LIGO Caltech

October 16, 2006

Abstract

The raw data of the 40m interferometer have been used to estimate the relative velocity of the arm mirrors with two reconstruction methods. The obtained estimated velocity has been used to simulate in e2e framework the ringdown of the interferometer. The real data, the theoretical curve and the simulated data of the transmitted power and of the Pound-Drever signal have been compared.

1 Introduction

The optical parameters of the 40m interferometer [1, 2] can be investigated using the ringdown of the transmitted power and the Pound-Drever signal.

During lock acquisitions the mirrors frequently pass through resonances of the cavity. As one of the mirrors approaches a resonant position, the light in the cavity builds up. Immediately after the mirror passes a resonance position, a field transient in the form of damped oscillations occurs. Those oscillations are also called ringdown.

According to the time of the day and of the night those oscillations can change and with them also the mirror velocity. The quieter time for the interferometer is during the night when there is less human activity and the seismic motion is reduced. The following study has been done using 40m raw data obtained around midnight in August 11th, 2005 during lock acquisition attempts with the central part of the interferometer already locked. Using those data it has been possible to estimate the relative mirror velocity of the arms mirrors (XARM and YARM); the data have been fitted with two reconstruction methods [3, 4, 5] used for a similar purpose independently.

Once the relative mirror velocity has been obtained by fitting the real data its value can be used to simulate the ringdown of the interferometer: the simulation in the time domain of the 40m interferometer has been done in e2e

*Laboratoire de l'Accelerateur Lineaire - LAL, Orsay, France

framework [6]; the transmitted power of XARM and YARM have been performed. It has been possible to compare the real data and the simulated data of the transmitted power for the two arms. In the same way, the Pound-Drever signal can be compared adding to the comparison the theoretical curve obtained with a formula that includes the oscillations.

2 The relative mirror velocity

Assuming that the central part of the 40m interferometer already locked, let's try to acquire the lock of the complete 40m configuration adding the arms (XARM and YARM): during these attempts the mirrors pass through resonances of the cavity. The reconstruction of the relative mirror velocity is based on the assumption that the mirrors are moving slowly through Fabry-Perot fringes with no feedback control (not affected by the feedback). The data taken during lock acquisition attempts of the 40m have been fitted with two methods [3, 4] and the relative mirror velocities have been estimated both for the XARM and the YARM mirrors.

The parameters used for the analysis are listed in table 2:

L (m)	$\lambda(nm)$	T_{ITM}	L_{ITM} (ppm)	T_{ETM}	L_{ETM} (ppm)
38.55	1064	0.005	100	10^{-5}	100

2.1 First kind of data fitting

The first one [3] has been used in 2000 to reconstruct the mirror velocity of the 30m mode-cleaner installed in Orsay at LAL. The procedure is described in the following.

Let's consider the transmitted power and its time derivative. The positions of the minima and maxima, with the exception of the main peak, are almost independent of the *finesse* value. The derivative zeros depend only on the relative mirror velocity. Let the position of the curve's derivative zeros, t_n, be labeled by the index n.

The n-th zero of the derivative is a quadratic function of the zero crossing time t_n. The curves of the measured transmitted power are used to fit the expression $n = p_1 + p_2\ t + p_3\ t^2$, where p_1, p_2 and p_3 are fitting parameters. Empirically the coefficient p_3 can be written as $p_3 = cv/(\lambda L)$ where L is the arm cavity length, v is the relative mirror velocity, L is the length of the arm and λ the laser wavelength.

The parabolic fit has been done using the MATLAB function "polyfit" and the errors on the index n, corresponding to the n-th derivative zero, have been evaluated using the MATLAB function "polyval" on the results of the parabolic fit **??**. Knowing the coefficient p_3 from the parabolic fit of the transmitted power it is possible to obtain an estimation of the relative mirror velocity.

Figure (1) shows, at the top, the measured DC transmitted power of the XARM as function of time for the full 40m interferometer and at the bottom, the index n, corresponding to the n-th derivative zeros, as a function of time. The comparison of the parabolic fit and the corresponding n-th derivative zeros (red stars) are shown with the error bars on the y coordinate. The value of $p_3 = 2.3 \times 10^6$. The reconstructed mirror velocity value is 0.35 ± 0.01 $\mu m/s$.

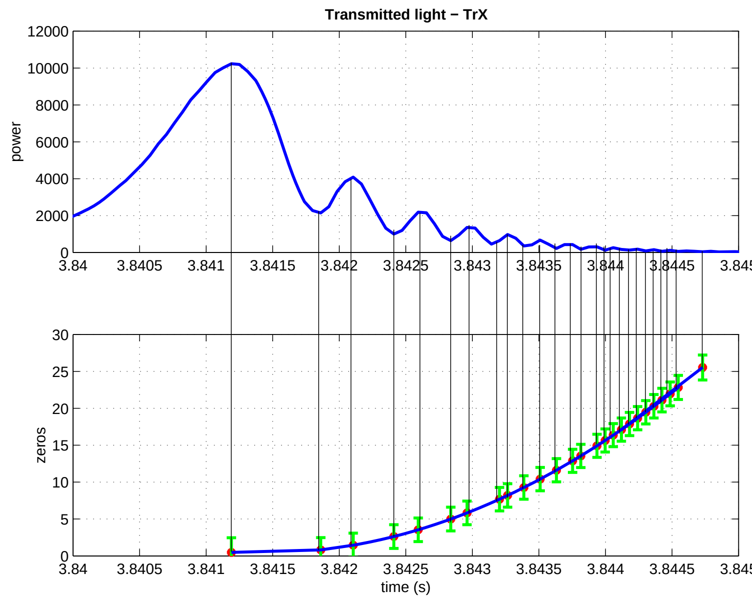

Figure 1: *Fit results for the XARM relative mirror velocity reconstruction. Top figure: The measured DC transmitted power of the XARM as function of time taken on August 11th, 2005, during the night around midnight. Bottom figure: the index n, corresponding to the n-th derivative zero, as a function of time. Th result of the parabolic fit applied to the zeros of the transmitted power is shown. The solid line is the predicted fit and the red stars are the experimental points with the error bars on the y coordinate. The reconstructed mirror velocity value is* 0.35 ± 0.01 $\mu m/s$. *The value of* $p_3 = 2.3 \times 10^6$.

The estimation has been repeated using the YARM data. The value of $p_3 = 1.95 \times 10^6$. The reconstructed mirror velocity value is 0.26 ± 0.005 $\mu m/s$ for the YARM.

2.2 Second kind of data fitting

The second method [4, 5] for fitting the ringdown data has been used to reconstruct the relative mirror velocity for Initial LIGO. The analysis method was developed in 1997 when the 40m one arm FP measurement was done. Referring to [4, 5], the approach is based on the linear shift of the frequency of the Pound-Drever signal or the transmission signal. The mirror velocity can be found by studying either the peaks or the zero-crossing of the transmitted signal.

The formula for the transient is

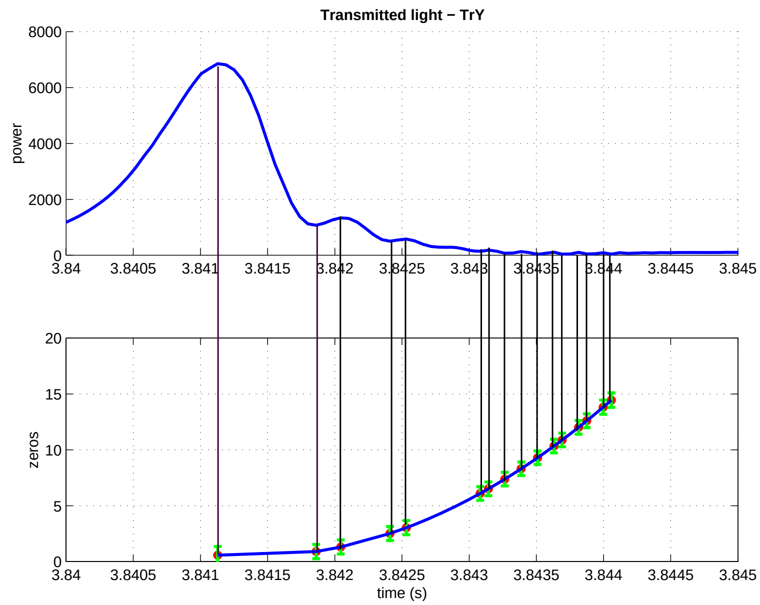

Figure 2: *Fit results for the YARM relative mirror velocity reconstruction. Top and bottom figure are the same as Figure 1 for YARM. The reconstructed mirror velocity value is* 0.26 ± 0.005 $\mu m/s$ *for YARM. The value of* $p_3 = 1.95 \times 10^6$.

$$E(t) = \frac{t_a A}{1 - r_a r_b \; exp(-2i\frac{2\pi v}{\lambda}t)} + D_0 \; exp\Big(-\frac{t}{\tau} - i\frac{\pi c v}{\lambda L}t^2\Big) \qquad (1)$$

where t_a is the transmissivity of one mirror, r_a and r_b are the reflectivities, A and D_0 are amplitudes, λ is the wave length of the laser, v the velocity, τ the time constant and L is the arm cavity length. The second term of equation 1 includes the Doppler induced oscillations of the stored field as one sweeps across the resonance. The corresponding formula for the transmitted power can be easily obtained. The formula of the error signal with the oscillations is the following [4, 5]

$$V_D(t) = -A \; |D_0| \; exp(-\frac{t - t_0}{\tau}) + sin(\gamma + \delta - \frac{\pi c v}{\lambda L} \; (t - t_0)^2) \qquad (2)$$

where t_0 is the time when the mirror passes a center of the resonance and $\delta = arg \; D_0$. Let's call the times for the zero-crossing t_n with n integer (they are the equivalent of t_{zero}). The values for t_n are derived by the equation (2) and they are given by

$$\frac{\pi c v}{\lambda \tau_0}(t_n - t_0)^2 = \pi n + \gamma + \delta; \qquad (3)$$

these values depend on the demodulation frequency, while the difference is independent. Here is the difference

$$\frac{kv}{2\tau_0}((t_{n+2} - t_0)^2 - (t_n - t_0)^2) = 2\pi. \tag{4}$$

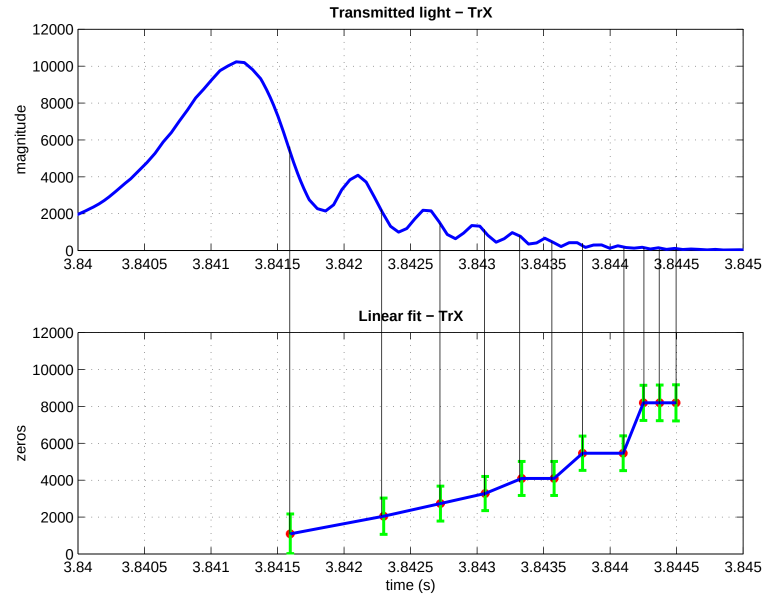

Figure 3: *Fit results for the XARM relative mirror velocity reconstruction. Top figure: The measured DC transmitted power of the XARM as function of time when the full 40m interferometer is considered. Bottom figure: Result of the linear fit applied to the spacing peaks and mid-points of the transmitted power. The solid line is the predicted fit and the red stars are the experimental points with the error bars on the y coordinate. The reconstructed mirror velocity value is* $0.39 \pm 0.04\ \mu m/s$.

For the reconstruction of the relative mirror velocity, spacing peaks and mid-points are considered instead of all the zeros of the transmitted power as done in the previous procedure. Let's define he spacings between the zero crossings as $\Delta t_n = t_{n+2} - t_n$, and the positions of the midpoints as $\bar{t_n} = t_n + t_{n+1}/2$. The averages frequencies of the oscillations are defined as $\bar{\nu_n} = \frac{1}{2\ \Delta t_n}$ and they satisfy the equation

$$\bar{\nu_n} = \frac{v}{\lambda T}(\bar{t_n} - t_0). \tag{5}$$

A linear fit can be applied to the data $v(t) = at + b$. The slope is related to the mirror velocity as $v = \lambda T a$ and the intercept to the time when the mirror passes through the center of the resonance, $t = -b/a$.

Figure (3) shows the results of the linear fit for the XARM: at the top the measured DC transmitted power as function of time is shown (with the central part of the 40m interferometer already locked) and at the bottom the linear fit done on the spacing peaks and the mid-points of the transmitted power is shown. The reconstructed mirror velocity value is $0.39 \pm 0.04\ \mu m/s$.

In the same way, Figure (4) shows the results of the linear fit for the YARM. The reconstructed mirror velocity values are $0.39 \pm 0.04\ \mu m/s$ for the XARM and $0.27 \pm 0.04\ \mu m/s$ for the YARM.

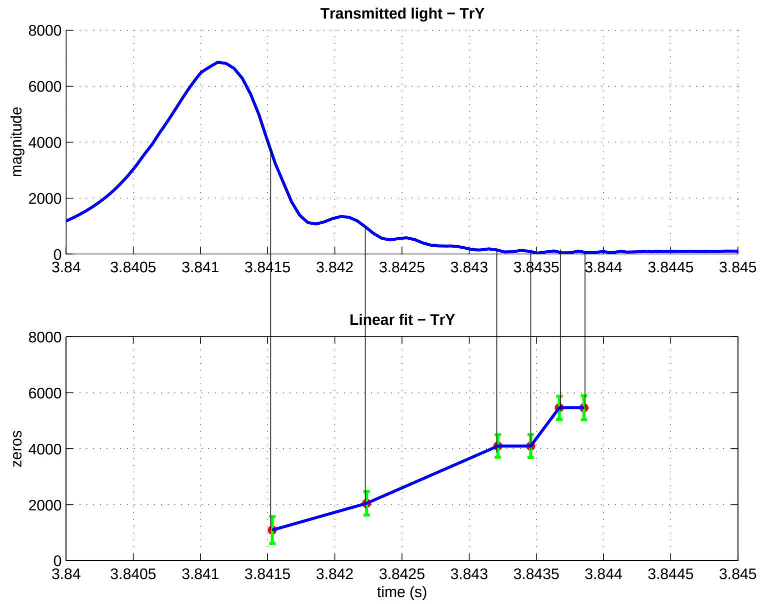

Figure 4: *Fit results for the YARM relative mirror velocity reconstruction. Top and bottom figure are the same as Figure 3 for YARM. The reconstructed mirror velocity value is* $0.27 \pm 0.04\ \mu m/s$ *for YARM.*

The linear fit has been done using the MATLAB function "polyfit" and the errors on the index n, corresponding to the n-th derivative zero, have been evaluated using the MATLAB function "polyval" on the results of the linear fit.

3 Measurement and simulation

The mirror velocity value obtained with the fit can be used to perform simulation in e2e framework [6].

It is possible to compare the error signal (adiabatic + with oscillations) with the 40m raw data, the theoretical curve (adiabatic + oscillations) and the e2e simulation.

The formula for the transient, equation (1), can be used for extracting the cavity parameters from the signal. It is convenient to remove the adiabatic component from the signal described already with the formula (2).

The measured error signal and the theoretical prediction based on equation (2) are shown in Figure (5): the agreement for the oscillations looks quite good apart from the initial region where the approximated formula is not valid.

The measured transmitted power for both the arms can be also simulated in e2e [6] framework, using a time domain model of the 40m interferometer : during the simulation the central part of the interferometer were blocked and the two arms were freely swinging under the effect of the induced velocity.

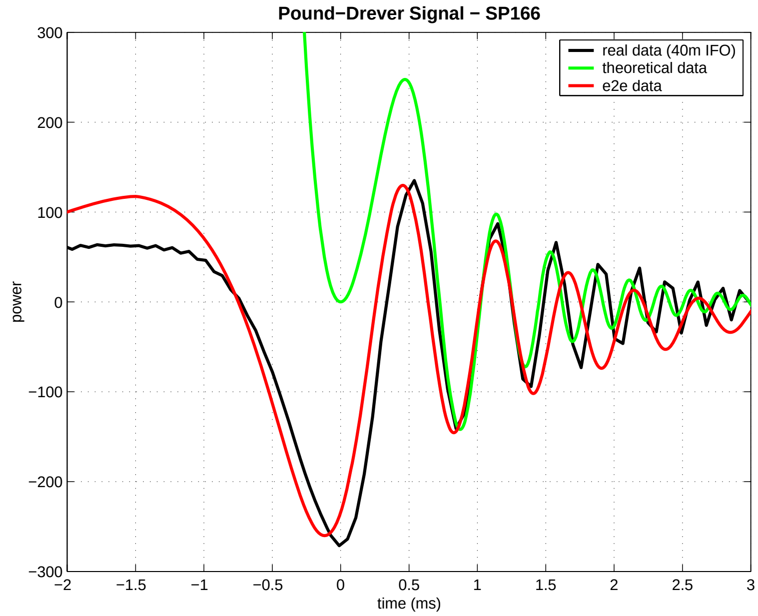

Figure 5: *Measured error signal: the figure shows the comparison among the real data (black), the theoretical prediction (green) and the e2e simulation (red).*

A comparison between measured and e2e simulated transient signals is shown in Figure (6); the relative mirror velocities values estimated with the Virgo LAL method [3] have been used for the e2e simulation. The estimated relative mirror velocities are 0.35 ± 0.01 $\mu m/s$ for XARM (top figure) and 0.26 ± 0.01 $\mu m/s$ for YARM (bottom figure). From Figure 6 one can notice that the simulated data start around t=-2s; it seems that before that time the transmitted curve becomes zero: this behavior has not been investigated.

4 Conclusions

The relative mirror velocity for the arms mirrors of the 40m interferometer, XARM and YARM, has been estimated comparing two kind of fitting procedures to analyze the raw data: the first procedure fits directly the times of the minima and maxima of the transmitted signal using a parabolic fit while the second one fits the spacings and mid-points of the transmitted signal using a linear fit. The obtained values agree within the errors. A simulation in e2e framework of the transmitted signal for both the arms has been performed using the value obtained with the parabolic fit: the agreement with raw data can be seen in Figure (6).

The reconstructed mirror velocity values are 0.35 ± 0.01 $\mu m/s$ for the XARM and 0.26 ± 0.01 $\mu m/s$ for the YARM.

A comparison of the error signal with oscillations of the theoretical curve, the raw data and the e2e simulation has also been done.

As it is clear from Figure 6 and Figure 5, the agreement with the theory, simulation and real data is not perfect. This can be explained at least with

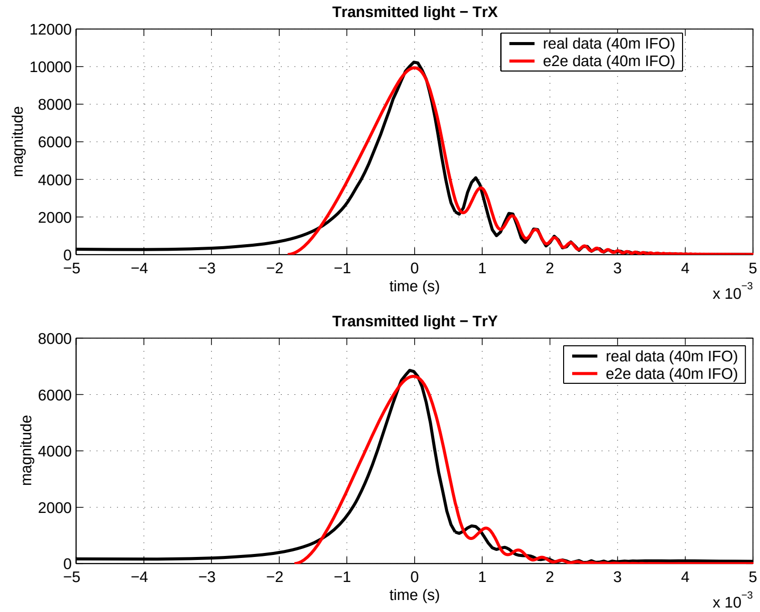

Figure 6: *Comparison between transient signals obtained in real data (black) and e2e simulation (red) using the relative mirror velocities values estimated with the Virgo LAL method [3]. The estimated relative mirror velocities are* 0.35 ± 0.01 $\mu m/s$ *for XARM (top figure) and* 0.26 ± 0.01 $\mu m/s$ *for YARM (bottom figure).*

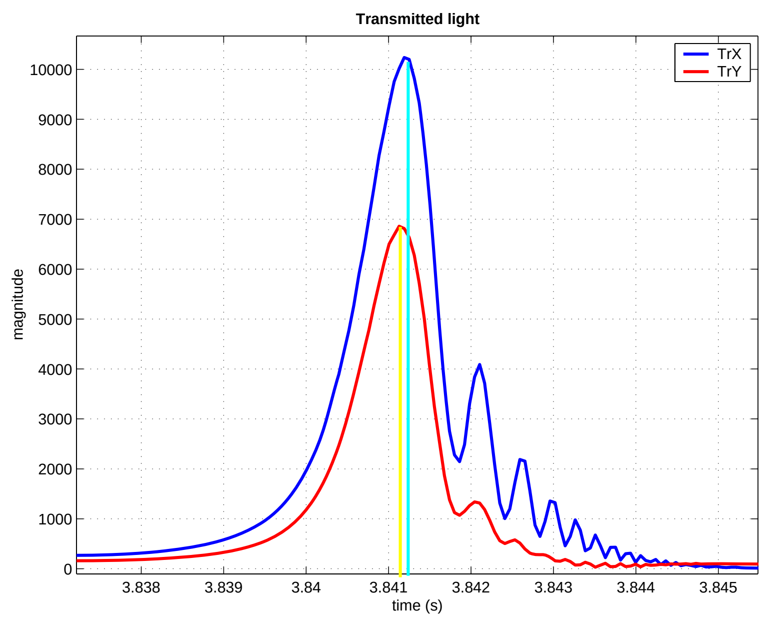

Figure 7: *Comparison between transient signals in real data: XARM (blue) and YARM (red) transmitted lights. The two straight lines shows that there is a delay between the resonance peaks, but the overlap remains and introduce an interference between the arms. The delay between the resonance peaks is less than* 0.1 ms*.*

four reasons:

1. the two arms don't behave as simple Fabry-Perot cavities. There is an interference between the arms due to the influence of the central part of the interferometer. The central part of the interferometer was locked when the transmitted power and the error signal have been acquired, so there was a coupling between the arms. The overlap between the two arms crossing the resonance can be seen in Figure (7): the delay between the resonance peaks is less than 0.1 ms.

2. the relative mirror velocity used for the simulation could be wrong. Even if the order of magnitude seems to be correct looking at the comparison in Figure (6) and Figure (5) probably the value approximation is not so accurate to reproduce exactly the same shape of the oscillations.

3. the model used for the theoretical curve could be wrong; a more accurate model is needed to describe a situation of interference between the two arms. In addition the approximated formula used to describe the theoretical curve is not valid for the description of the resonance point and this causes a mismatching of the initial points.

4. the shape of the ringdown of the transmitted signal can change according to the time of the measurement: this is because the interferometer is noisier during daytime and quieter during the night. Consequently, the oscillations correspond to different velocities. There are no direct measurements which have monitored this behavior: it could be interesting to check it and estimate the existing difference (in order of magnitude) all along the day. Under these conditions the simulation could match just in a very particular case.

Further investigations should be necessary to check the reason of the imperfect agreement. The simulation in e2e framework [6] can be used to investigate the interference between the two arms and also to understand the correct value of the mirror velocities.

There could be other reasons for the mismatching of the theoretical, simulated and real curves apart from the three possibilities already discussed. This could be argument of further studies about the optical characterization of the interferometer.

References

[1] 40m Proposal.
http://www.ligo.caltech.edu/~ajw/40m_cdr/40m_cdr.pdf

[2] 40m website.
http://www.ligo.caltech.edu/~ajw/40m_upgrade.html

[3] L.Matone et al., Phys. Lett. A271, 314-318 (2000),
http://www.lal.in2p3.fr/recherche/virgo/Publication/PLA_271_303.pdf

[4] M. Rakhmanov, Applied Optics, Vol.40, No.12, 20 April 2001

[5] H. Yamamoto, "Fringe structure of LHO 2k FP, a modeler view" LIGO-G990130-01, December 1999

[6] http://www.ligo.caltech.edu/~e2e