LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

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Acoustic Coupling Through Cabling for the HAM-SAS System

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1 Introduction

The review committee, for the proposed HAM-SAS experiment/prototype at LASTI, expressed a concern (M060004-00) that electrical cabling might short circuit the isolation by providing a path for ground vibration to excite the table. This potential for an acoustic short is more pronounced for a "soft" (mechanically compliant) system like the Seismic Attenuation System (SAS) for the LIGO HAM vacuum chamber than for a "stiff" system. In the following analysis it is shown that the ground noise excited vibration of the anticipated cabling should not compromise the isolation performance of the HAM-SAS system.

2 Reference Documents

M060004-00	Review Report for HAM-SAS LASTI experiment prototype proposal
T060021-01	Response to Findings from the HAM-SAS LASTI Experiment/Prototype Review
T060020-00	HAM-SAS System Dynamics Model
T960065-03	(Initial LIGO) Seismic Isolation Design Requirements Document
E990303-03	(Advanced LIGO) Seismic Isolation Subsystem Design Requirements
E060008-00	Tentative HAM SAS internal cabling list and connections scheme

3 Mass-String, Frequency-independent Model

Consider a string terminated with a mechanical impedance[1], as indicated in Figure 1.

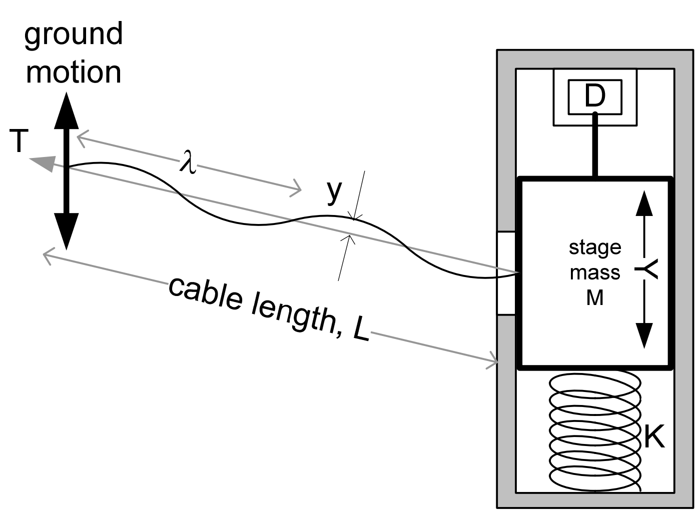

Figure 1: String Terminated with an Impedance
Stage (mass M, stiffness K, damping D) excited by a cable (string length L with tension T)

If a transverse sinusoidal force is applied to the input end of the string, a wave will travel to the termination at the mass and reflect. The ratio of the motion at the mass, Y, to the input motion, y, is equal to 1 plus the reflection coefficient:

[1] See for example, G. Swenson, Principles of Modern Acoustics, Boston Technical Pub., 1965, p.33-49.

$$\frac{Y}{y} = 1 + R = 1 + \frac{Z_0 - Z_T}{Z_0 + Z_T} = \frac{2Z_0}{Z_0 + Z_T}$$

where Z_0 is the input impedance and Z_T is the termination impedance.

$$Z_0 = \left(\frac{f}{\dot{y}}\right) = \frac{T}{v} = \mu v$$

where T is the tension, $v = \sqrt{T/\mu}$ is the wave speed, and μ is the string lineal density.

$$Z_T = D + j\left(\omega M - \frac{K}{\omega}\right)$$

where D is the stage (viscous) damping, M is the stage mass, K is the effective stage stiffness and ω is the angular frequency. If the stage impedance is approximated by inertia only, then $Z_T = j\omega M$ and if $Z_0 << Z_T$, then

$$\left|\frac{Y}{y}\right| \approx \frac{2\mu v}{\omega M} = \frac{\mu\lambda}{\pi M}$$

where the relationship between the wave velocity, frequency and wavelength, $\frac{\omega}{v} = \frac{2\pi}{\lambda}$, has been used. This is the expression given in the initial LIGO seismic isolation requirements document[2]. For the initial LIGO seismic isolation design requirements an upper bound on the wavelength, λ = L, the cable length, was used. Then assuming an amplification of the cable motion at resonance of Q and imposing a requirement that the cable induced motion on the stage should not compromise the required attenuation of the stage, T_s:

$$\left|\frac{YQ}{y}\right| = \frac{mQ}{\pi M} \le T_s$$

or

$$m \le \frac{\pi M T_s}{Q}$$

where m=μλ is the mass of a single cable. To account for n cables, we conservatively assume that the effects of each cable add in phase, so

$$m \le \frac{\pi M T_s}{Qn}$$

A better approximation for the wavelength is based on the modal mass for the first cable resonance. The modal mass, m_m, normalized by the beam mass, m, is defined as follows[3]:

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[2] F. Raab, (Initial LIGO) Seismic Isolation Design Requirements Document, T960065-03, appendix F.

[3] D. Karnopp, D. Margolis, R. Rosenberg, System Dynamics: Modeling and Simulation of Mechatronic Systems, 3rd edition, John Wiley & Sons, cr 2000, p. 378, eqn. 10.103.

$$\frac{m_m}{m} = \int_0^1 Y_n^2\left(\frac{x}{L}\right) d\frac{x}{L}$$

where x is the distance along the beam axis, L is the beam length and $Y_n\left(\frac{x}{L}\right)$ is the nth mode shape. The mode shape for a beam which is simply supported (pinned) at both ends[4] is:

$$Y_n\left(\frac{x}{L}\right) = \sin\left(\frac{n\pi x}{L}\right)$$

With this mode shape in the above definition for modal mass one obtains a normalized modal mass of ½ for the first beam bending mode. Similarly the normalized modal mass for the 1st longitudinal mode of a bar[5] is ½ . Consequently, the limit on cable mass is:

$$m \le \frac{2\pi M T_s}{Qn} \text{ or } \frac{Qmn}{2\pi M} \le T_s$$

The parameter values for the HAM-SAS system are as follows:

M ~ 520 kg (for the spring box[6])

Q ≈ 1 (for the cable – a guess)

n = 20 (an estimate[7] of the required cabling)

μ = 0.120 kg/m measured density[8]

L < 0.6 m

m = μL = 0.072 kg

T_s=0.0005 (required isolation factor[9] at 10 Hz)

$$\frac{Qm\sqrt{n}}{2\pi M} = 0.005 \approx T_s$$

This approximate analysis seems to indicate that the cabling might cause an acoustic short which compromises the required isolation. However this analysis does not indicate the frequency at which the amplitude of cable-induced, stage motion exceeds the maximum isolation factor. In the next section a frequency dependent analysis shows that the isolation requirements are not exceeded.

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[4] R. Blevins, Formulas for Natural Frequency and Mode Shape, Krieger Publishing Co., cr 1979, p. 108, Table 8.1, case 5.

[5] D. Karnopp, et. al., op. cit., p.369.

[6] D. Coyne, HAM-SAS System Dynamics Model, T060020-00

[7] R. DeSalvo, D. Coyne, Response to Findings from the HAM-SAS LASTI Experiment/Prototype Review, T060021-01.

[8] value measured for a similar 25-conductor cable, not the in-vacuum cable that will be used for HAM-SAS

[9] R. DeSalvo, D. Coyne, op. cit.

4 Mass-Beam, Frequency-dependent Model

Consider a mass attached to the ground by a beam[10], as indicated in Figure 2.

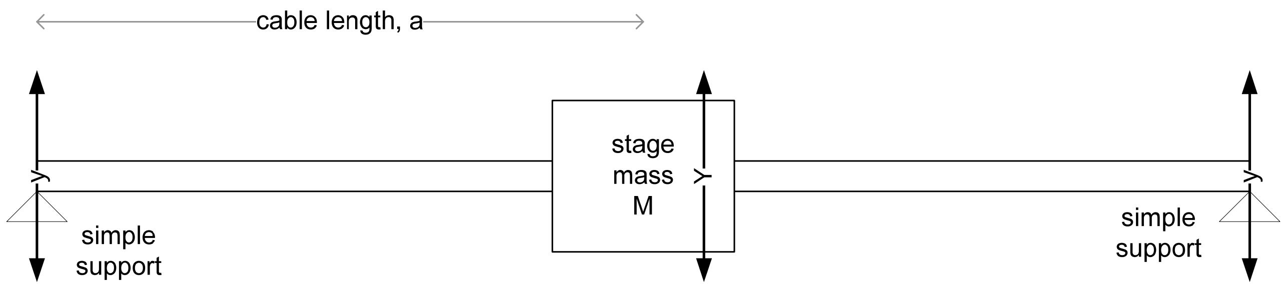

Figure 2: Mass-Beam Model

Stage (mass M) excited by a cable (beam of length 2a)

The transmissibility across the mass-loaded beam (from supported ends of the beam to the mass) is:

$$T_m = \left| \frac{\cosh(n^* a) + \cos(n^* a)}{2\cosh(n^* a)\cos(n^* a) + \gamma(n^* a)(\sinh(n^* a)\cos(n^* a) - \cosh(n^* a)\sin(n^* a))} \right|$$

where the beam length is 2a and

$$n^* = \left(\frac{\omega^2 \rho}{E r^2 (1 + j\delta)} \right)^{1/4}$$

ω = frequency

ρ = 1730 kg/m^3, beam density

E = cable (beam) effective elastic modulus

r = 5.25 mm , cable (beam) radius of gyration

δ = 1/Q =0.2 is the beam damping factor

$\gamma = \dfrac{M}{m\sqrt{n/2}} = 1124$ is the effective mass ratio

The effective elastic modulus for the beam has been calculated from the measured[11] static stiffness of a 1 ft. length of the intended in-vacuum cable, k = 1 N/m:

$$E = \frac{kL^3}{3I} = \frac{4kL^3}{15\pi r^4} = 4.9\ MPa$$

The value of (n^*a) determines the frequencies of the system based on the beam density, ρ, and not directly from the mass ratio, γ. The mass ratio, γ, has been adjusted to account for the multiple

[10] J. Snowdon, Vibration and Shock in Damped Mechanical Systems, John Wiley & Sons Inc., cr 1968, section 7.5, p. 230.

[11] R. DeSalvo, D. Coyne, op. cit.

cables, assuming that all n cables are not in phase, but instead add in quadrature. (The factor of ½ in the effective mass ratio accounts for the fact that the basic mass-loaded beam model includes 2 cables (beams) supporting the stage mass.)

The calculated transmission is plotted with the required attenuation in Figure 3. The first resonance is simply the mass-spring resonance ($f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{M}} = 7\ mHz$). The lightest stage (spring box) mass has been assumed (M=520 kg). However at this frequency the optics table and payload are not decoupled and so the actual mass participating in the first resonance is much higher (M=1426 kg) and the first resonance will be lower (f_1 = 4 mHz). The spring box and optics table are decoupled at about 1.2 Hz, so use of the spring box mass as the effective stage mass for calculating the transmission at cable self-resonances is more appropriate than the combined mass of the spring box and optics table. The first cable self-resonance (beam bending mode) is ~2 Hz. At this frequency, the ground motion transmission through the cabling to the stage would equal the required attenuation if the cable has a Q of 5 (which seems high; blue curve in Figure 3). If the cable has a (more likely) Q of 1, then the ground noise transmission due to cabling is a factor of ~3.5 lower than required. Probably adequate, but not with a high safety margin.

As a follow up to this analysis:

1) The actual cabling should be measured for the mass per unit length, rather than relying upon “similar” cabling.
2) Consider doubling the cable length (to ~1.2 m) to shift the cable resonances to lower frequency (see Figure 4).

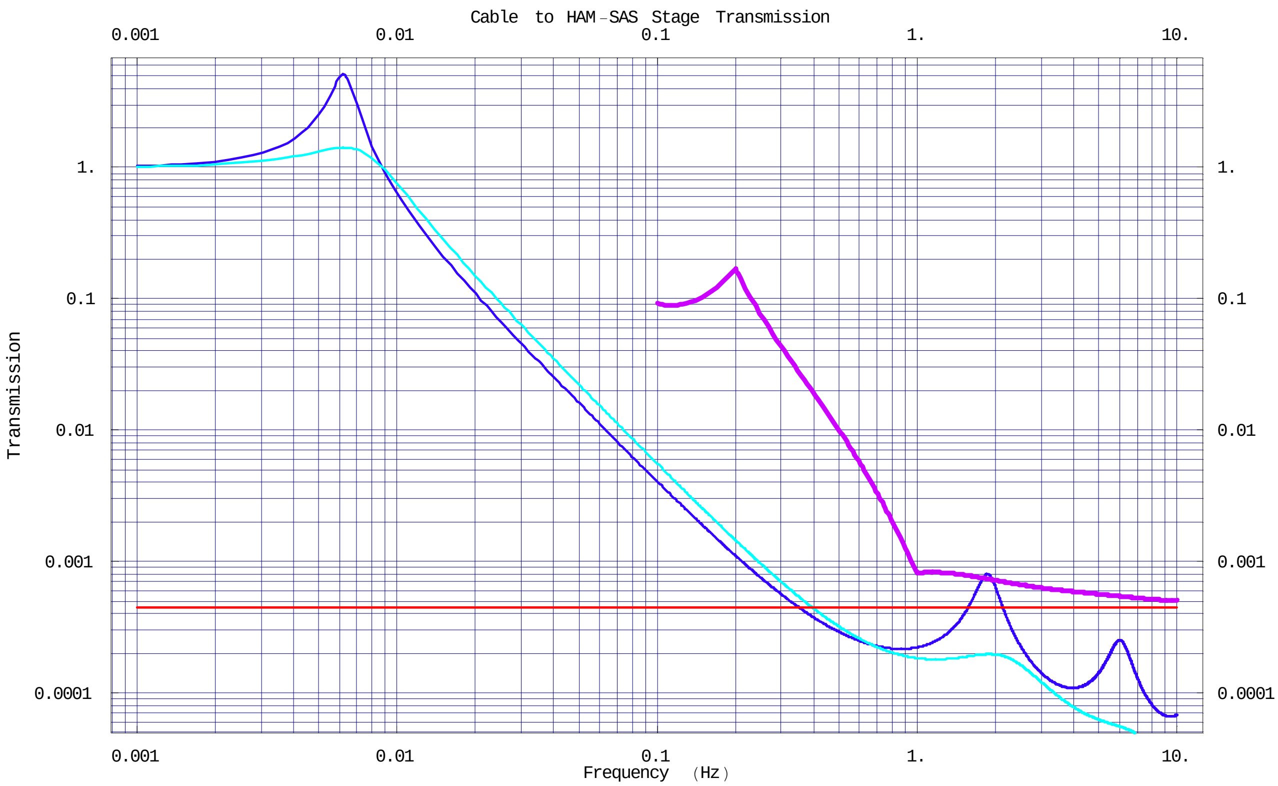

Figure 3: Mass-Beam Transmissibility (nominal length cables)

With 20 cables of nominal length (0.6 m (2 ft.) cables)
Magenta Curve: Required Attenuation (from E990303-03, only defined > 0.1 Hz with ground motion spectrum from T060021-01)
Red Curve: Frequency-independent model with Q=1
Blue Curve: Frequency-dependent model with Q=1
Cyan Curve: Frequency-dependent model with Q=5

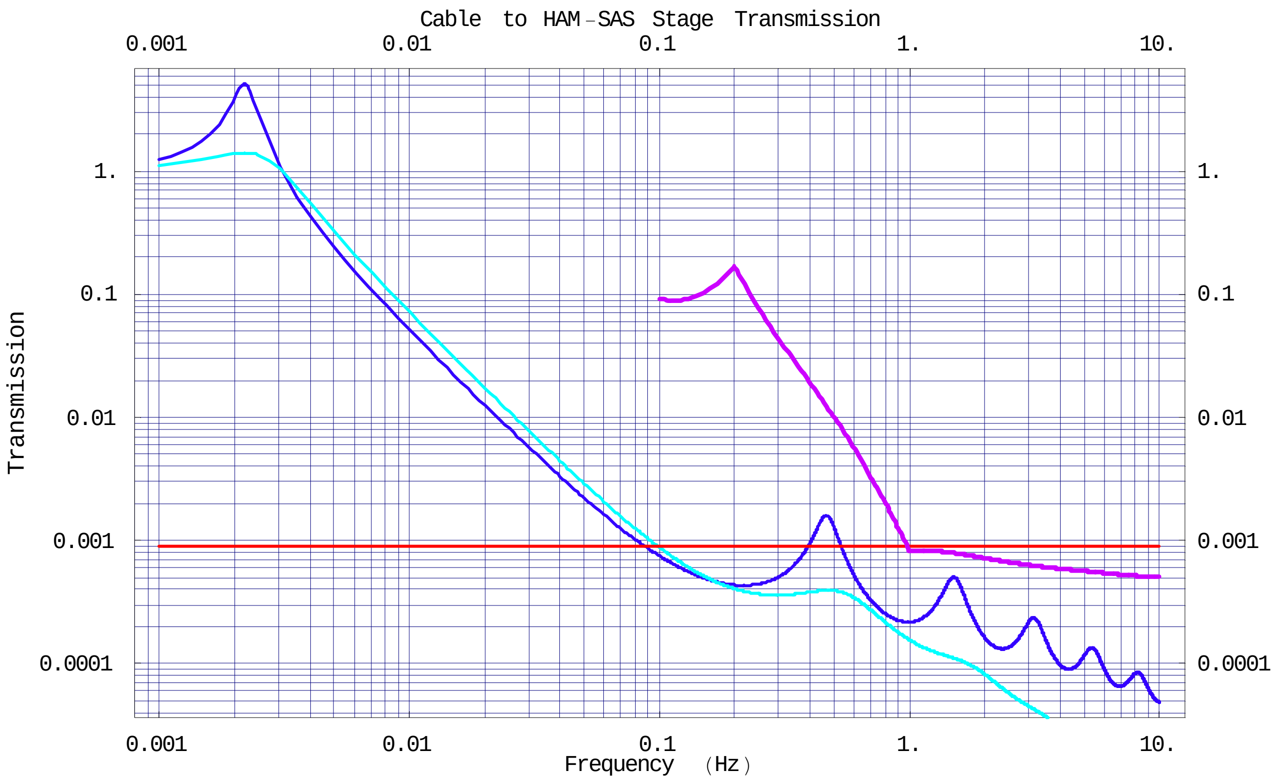

Figure 4: Mass-Beam Transmissibility (long cables)

With 20 cables of long length (1.2 m (4 ft.) cables)
Magenta Curve: Required Attenuation (from E990303-03, only defined > 0.1 Hz with ground motion spectrum from T060021-01)
Red Curve: Frequency-independent model with Q=1
Blue Curve: Frequency-dependent model with Q=1
Cyan Curve: Frequency-dependent model with Q=5