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Orientation of Quadrant Diode for Wave Front Sensing

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Abstract

In current LIGO I interferometers, quadrant diodes for wave front sensors (WFS) are oriented in such a way that the lines (i.e. gaps) between the segments forms “X” instead of “+” pattern. The segment 1 and 3 are used for YAW degree of freedom, while the segment 2 and 4 are used for PIT. Though at a first glance it looks as if there are no fundamental difference between “X” and “+” configuration, “X” is more prone to the crosstalk between YAW and PIT. There are other things, like DC centering annoyances for human operators. All in all, there’s no reason to prefer “X” to “+”, and there are some reasons to hate “X”. We should not use “X” configuration for future LIGO interferometers.

1 Wave Front Sensing, quadrant diodes, “X” and “+” section pattern

There are two known techniques to use radio frequency sidebands to detect the misalignment of the optical resonators. One [1] is now commonly called Wave Front Sensing [2] and the other is called Anderson technique [3]. There are some minor technical differences between these two, but they are more or less the same from hardware perspective: The sensing hardware detects the spatial interference pattern between fundamental and the first order off-axis mode of Hermite-Gaussian modal set. Although this document discusses the LIGO system specifically, the same argument can be applied to Anderson technique as well.

Let’s assume that the laser beam is propagating along z-axis, and the misalignment is in horizontal (YAW) direction. Apart from the sinusoidal oscillation caused by the phase modulation scheme that is needed for technical reason, the interference pattern would be proportional to the product of the Gaussian distribution and the first order Hermite polynomial:

$$I(x, y) \propto H_1(\sqrt{2}x/w) \exp\left[-\frac{x^2+y^2}{w^2}\right] \tag{1}$$

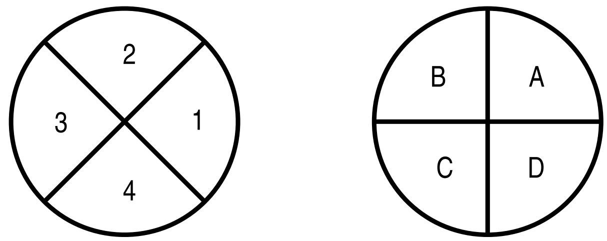

Figure 1: Quadrant diode patterns: On the left is the "X" section pattern for current LIGO WFS. On the right is an alternative "+" section pattern.

where x and y are horizontal and vertical axis respectively, and w is the beam radius at the measurement point .

Since such a pattern is averaged out and becomes zero using a spatially uniform receiver, usually a quadrant diode is used to intentionally introduce non-uniformity. In LIGO I interferometers, the diode is oriented in such a way that the gaps between the diode segments form an "X" pattern [2] (Fig.1, left). For YAW signal we subtract the signal of segment 3 from that of 1, while 2 and 4 are used for PIT:

$$E_{\times\mathrm{p}} = I_2 - I_4 \qquad (2)$$

$$E_{\times\mathrm{y}} = I_1 - I_3 \qquad (3)$$

where I_n $(n = 1, 2, 3, 4)$ is the surface integral of the interference pattern over n-th segment.[1]

This is of course not the only possibility. On the right of Fig. 1 is the "+" wave fron sensor pattern used by other interferometers, e.g. TAMA300, and LIGO optical levers (although optical levers are not wave front sensors). In this configuration, one subtract the left half of the signal from the right half to calculate the YAW, and the bottom from the top for PIT:

$$E_{+\mathrm{p}} = I_\mathrm{A} + I_\mathrm{B} - I_\mathrm{C} - I_\mathrm{D} \qquad (4)$$

$$E_{+\mathrm{y}} = I_\mathrm{A} - I_\mathrm{B} - I_\mathrm{C} + I_\mathrm{D}. \qquad (5)$$

[1]Here we pretend as if the gap between the diode segments are infinitely thin "lines". Though the actual gap size is important for signal/noise analysis, it is beyond the scope of this document. For simplicity I'll focus on things that can be discussed without introducing device specific features like gap size.

At a first glance, it looks as if the two are almost equivalent. After all, it's a simple matter of coordinate rotation. For example, if you want to measure the misalignment in the direction of $x = y$ (i.e. the line parting segment 1 and 2 in LIGO WFS), you want to add PIT and YAW signal. For misalignment in the direction of $x = -y$, you subtract YAW from PIT. Using "+" pattern, that gives the following:

$$E_{+\mathrm{p}} + E_{+\mathrm{y}} \propto I_{\mathrm{A}} - I_{\mathrm{C}} \tag{6}$$

$$E_{+\mathrm{p}} - E_{+\mathrm{y}} \propto I_{\mathrm{B}} - I_{\mathrm{D}}. \tag{7}$$

Comparing the above equations with Eqs.4 and 5, it is very tempting to conclude that there are no inherent difference between "X" and "+" configuration. Indeed that is almost true under ideal situation. However, off-centering of the beam would make things worse for "X".

2 YAW-PIT WFS crosstalk via Off-centering

"X" configuration is mathematically identical to "+" when, and only when, the beam is perfectly centered on the diode. Off-center the beam, and the differences become apparent. Among these differences, the most evil is probably the crosstalk between YAW and PIT. In LIGO-type isolation system, the PIT and YAW noise are never the same (it seems that typically PIT is larger than YAW). Under a finite crosstalk, the error signal of quieter degree of freedom (typically YAW) is "polluted" by breeding of the noisier degree of freedom (PIT). Feed it back to any of the optics, and they would be misaligned because of the false error signal. Let us calculate the crosstalk via off-centering for "X" and "+".

Since the interference pattern caused by misalignment is a linear function of misalignment, we can assume without losing generality that the misalignment is only in YAW direction. Under this assumption, let's suppose that the center of the interference pattern is offset from the center of the diode. Apart from apparent decrease in the optical gain, this offset can

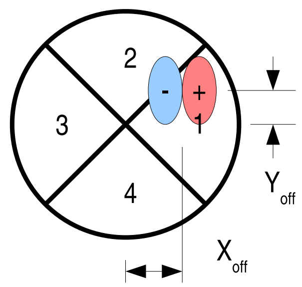

Figure 2: When using “X” configuration, the off-centering of the beam easily couples to the misalignment pattern to confuse WFS. In this figure, there is a finite signal in segment 2, which would produce a false PIT signal.

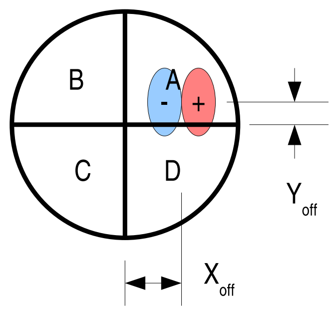

Figure 3: With “+” pattern, no matter where you place the beam, WFS wouldn’t confuse YAW with PIT (as far as the entire beam is on the diode). In this figure, the integral over top half becomes zero, and the same holds true for the bottom, therefore PIT signal always becomes zero.

couple to the geometry of the WFS diode and generate YAW-to-PIT crosstalk.

In Fig.2, the YAW-misalignment interference pattern represented by pink and blue ellipses is not centered on the diode. The segment 2 has small minus value while segment 4 is almost zero. Even though the misalignment itself is strictly in YAW, WFS is confused and “thinks” that the field has also some misalignment in PIT. Since the beam position itself is dependent on the alignment in this real world, this kind of coupling can complicate the behavior of the WFS servo.

On the other hand, if you use “+” pattern instead of “X”, the separation is maintained even if there is off-centering (Fig.3). No matter where you place the beam, PIT signal always becomes zero because the integral over the top half alone and also over the bottom half alone are both zero.

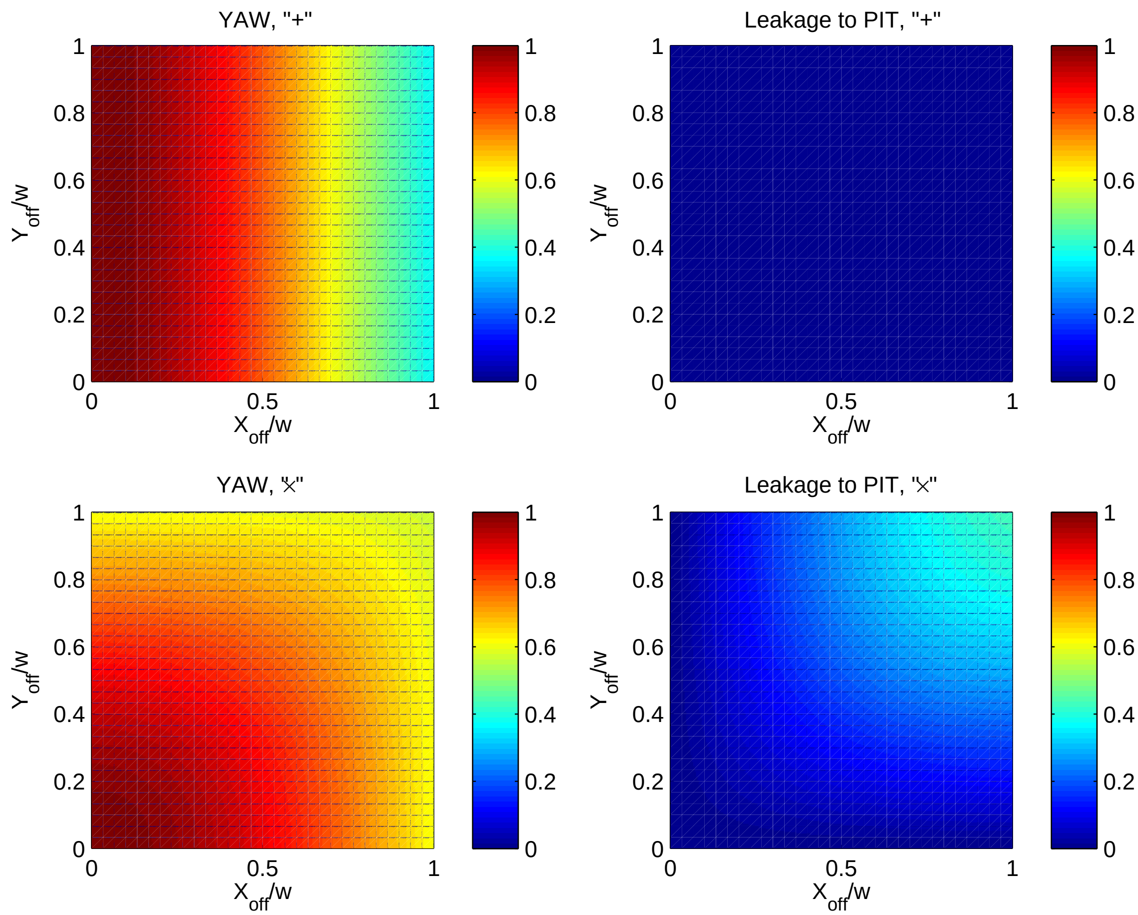

Figure 4: YAW and PIT signal for pure YAW misalignment when the beam is not centered. The YAW signal amplitude (top and bottom left) can be considered as the optical gain, while PIT (top and bottom right) represents the crosstalk. The signal amplitude is normalized by the value without off-centering (i.e. $X_{\rm off} = Y_{\rm off} = 0$), as the “X” section pattern and “+” section pattern give different signal amplitude. The “+” section pattern gives no crosstalk while it was not negligible when using “X” section.

2.1 A Simple Calculation

Figure 4 shows the numerical calculation for YAW and PIT signal obtained by “X” and “+” pattern for a pure YAW misalignment under the existence of off-centering. The interference pattern shown in Figs. 2 and 3 were calculated, integrating over each of the quadrants. Off-centering parameters $X_{\rm off}$ and $Y_{\rm off}$ are normalized by the beam radius w. The signal amplitude is normalized by the value without off-centering (i.e. the one with $X_{\rm off} = Y_{\rm off} = 0$), as the ”X” and ”+” pattern give different signal amplitude.

For “+” pattern, one can see that the YAW optical gain is not affected by PIT (i.e. Y)

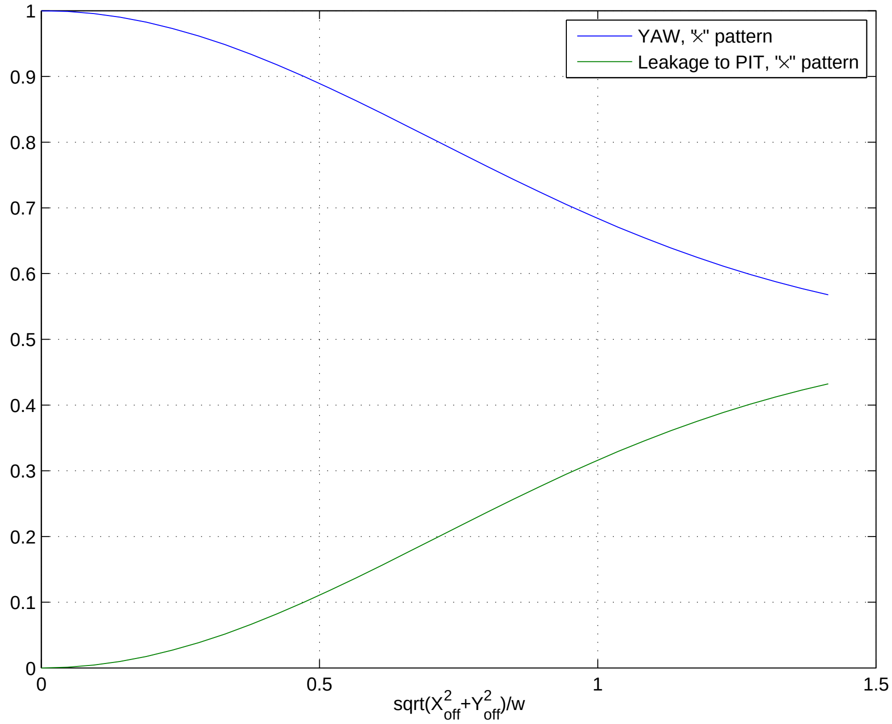

Figure 5: The cross section of the bottom two plots of Fig.4 on $X_{\text{off}} = Y_{\text{off}}$ line where the crosstalk becomes largest.

displacement (Fig.4, top left), and that the crosstalk is zero (note the scale of the top right plot).

On the other hand, for "X" pattern, the optical gain (Fig.4, bottom left) is symmetric for X and Y off-centering. The displacement of the beam in any direction results in the reduction of both YAW and PIT optical gain. However, more important is the fact that there is a fake PIT signal caused by the off-centering (bottom right).

For a given amplitude of displacement, the crosstalk from YAW to PIT in "X" becomes the largest when the beam displacement is in the direction of $x = \pm y$. Figure 5 shows the YAW and PIT signal for "X" pattern on $X_{\text{off}} = Y_{\text{off}}$. According to this plot, when the displacement is quarter of the beam diameter, the cross coupling is roughly 10%. Because of the symmetry, the crosstalk from PIT to YAW of the same amount should appear.

Although this number doesn't look catastrophic, it can become an issue when the external disturbances such as seismic excitation is driving the optics, or when the IFO is recovering from lock loss.

3 Other Things to Consider

Several things that are more or less obvious are studied. Readers can skip this section.

3.1 DC Centering Crosstalk (aka Operator Annoyance Factor)

We can use the DC signal from WFS quadrants to center the beam on WFS. Indeed, in all of LIGO I interferometers, such centering mechanism have been in use for all of the WFS diodes, and for some diodes there are automatic servo system to do that. Are there any fundamental difference for this aspect between "X" and "+"? Figure 6 shows the vector mapping of the DC centering error signal obtained by "+" (blue) and "X" (red) configuration. Ideally the vectors should be parallel to the radial line, and the length of the vectors should be proportional to the radial distance from the center.

As you see, neither configuration is ideal. Both of the configuration are good near the center, x axis, y axis, and $x = \pm y$ lines. However, as they go away further from these lines, "X" configuration tends to become either horizontal or vertical, while "+" tends to point to ± 45 degree directions. Mathematically speaking they are quite similar. If you rotate "+" plot by 45 degrees you get something very similar to "X" plot.

However, one thing that should be noted is the fact that we generally use "PIT" and "YAW" direction for control. In this context, we can say that in "+" configuration, when the error signal for either YAW or PIT is very small, the actual error in that degree of freedom is indeed very small. This is not at all true for "X".

Suppose that the beam is positioned at, say, (3,2/3) in Fig.6. Suppose that you manually center the beam using DC error signal. When you use "X" configuration, you think that the error is almost pure YAW, and try to move YAW. As you move closer to $x = 0$, YAW error signal becomes smaller, but the bad news is that the PIT error grows. Even though the movement is purely in YAW, it seems to you as if the beam is moving further away from x

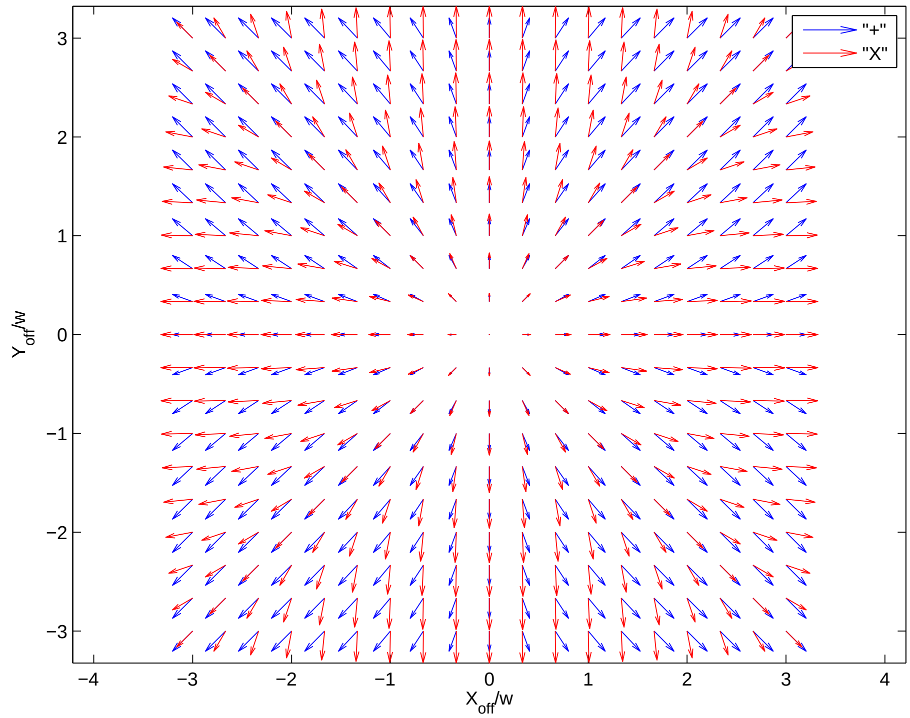

Figure 6: DC centering error signal for "+" (blue) and "X" (red) configuration.

axis (Fig.7). Not only is this annoying to operators, this can be problematic to large signal responce of the DC centering servo, as it becomes impossible to properly diagonalize the system for large signals.

On the other hand, using "+" configuration, as you approach y axis, only the YAW error becomes smaller while PIT error (which was large from the start, unlike "X") doesn't change. Since YAW error is independent of PIT position and vice versa, everything looks reasonable.

3.2 LSC to ASC Crosstalk via Off-centering

LSC signal would couple to off-centering and appear in WFS signal (Fig.8). If LSC gains are infinite, this is not the problem, but in reality there is always some LSC-ASC leak, which we can observe daily in LIGO interferometers by looking at low frequency LSC calibration lines (e.g. 48Hz in H1).

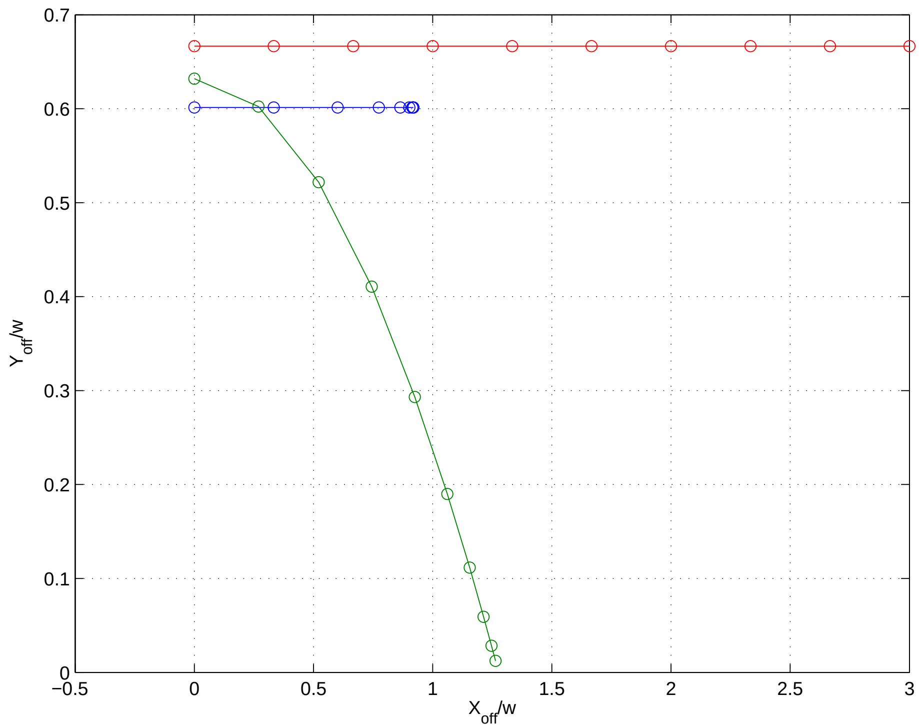

Figure 7: The error signal of "+" (blue) and "X" (green) configuration when the beam moved horizontally from $(3, 2/3)$ to $(3, 0)$ (red). In "+", any horizontal motion always looks horizontal because PIT and YAW are independent. In "X", the horizontal motion appears as if it were some weird trajectory.

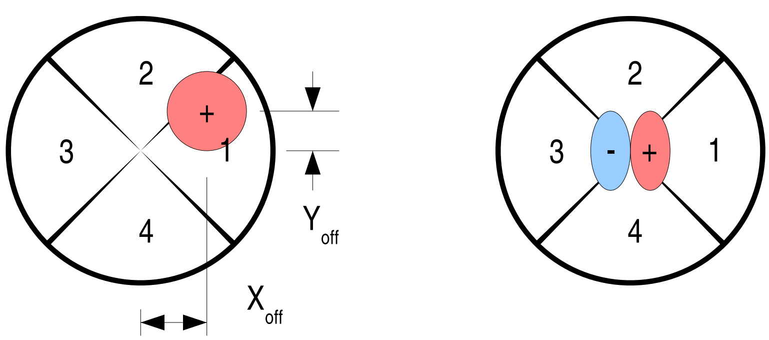

Figure 8: Off-centering of the beam couples to LSC signal, which has a TEM00 pattern, and shows up in WFS signal (left). Normalizing the amplitude of this signal by the WFS signal for TEM10 pattern (right), one can evaluate the LSC to ASC crosstalk.

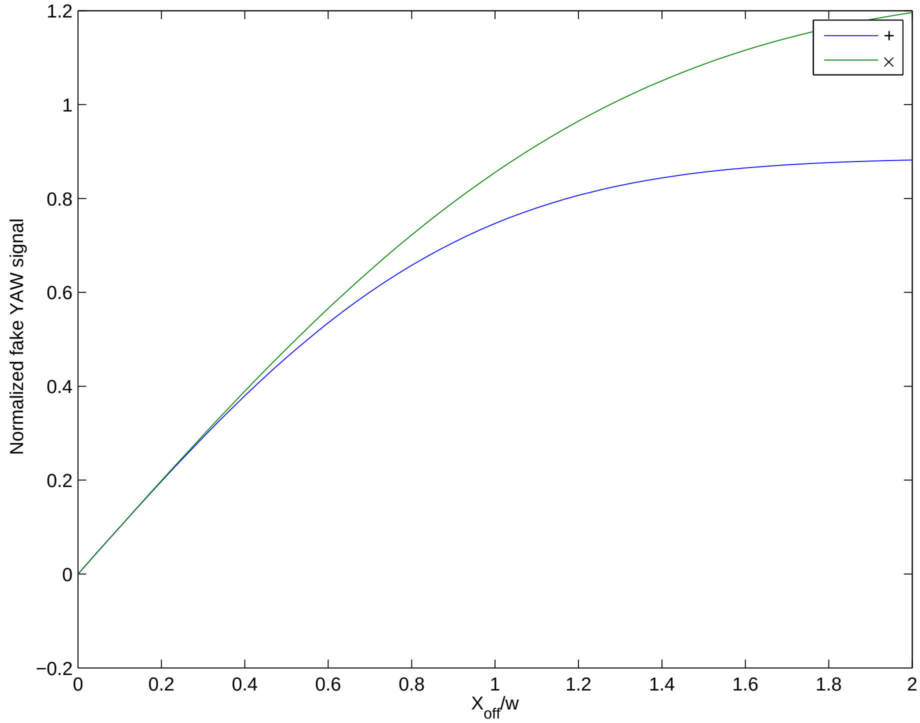

Figure 9: Fake YAW signal caused by the off-centering of LSC signal. The YAW signal was first calculated for TEM00 pattern with a certain field amplitude for various beam positions, and then normalized by the signal generated by a centered YAW (i.e. Hermite-Gaussian 01 and 00 product) field.

As a measure of this crosstalk, we can calculate the WFS signals (Eqs.2, 3, 6 and 7) generated by an offset TEM00-TEM00 pattern, and normalize them by the TEM00-TEM01 pattern positioned at the center of the diode (Fig.9). As you expect, the crosstalk is zero at the center for both of the configurations, and coupling is somewhat smaller for "+" pattern.

3.3 Optical Gain Reduction by Off-centering

As we saw in Fig.4, there is coupling of the PIT beam position to YAW optical gain for "X" configuration. Because of the symmetry, the optical gain for YAW and optical gain for PIT have exactly same dependence on the off-centering in "X". This means that the PIT and YAW optical gain are reduced at the same time by the same amount by beam displacement in any direction. The YAW optical gain for "+" configuration on the other hand is dependent only on YAW off-centering, and PIT only on PIT. However, the reduction of YAW optical

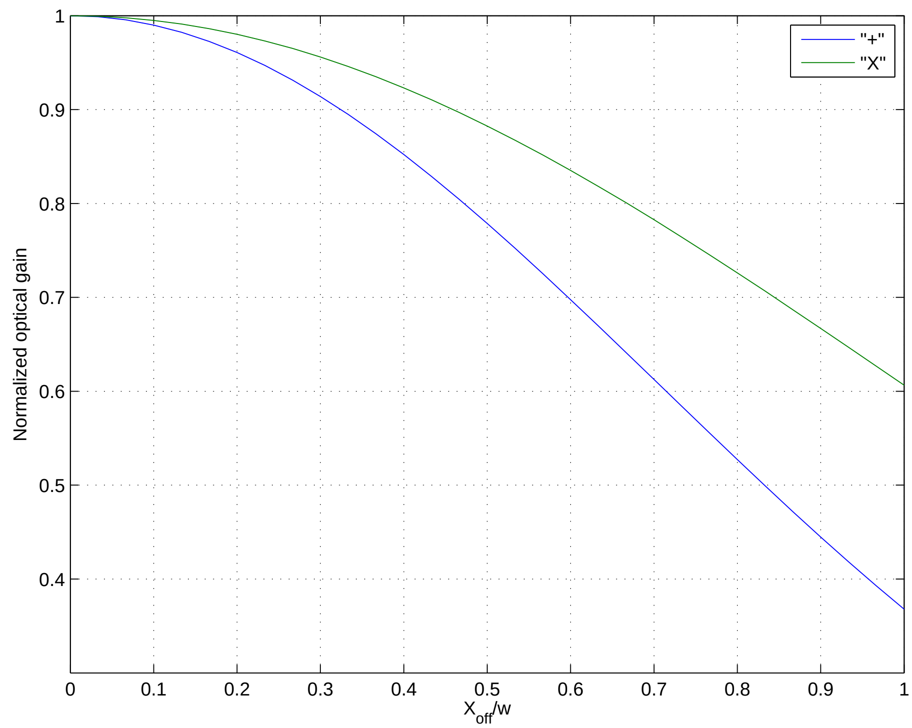

Figure 10: Normalized YAW optical gain for "X" (green) and "+" (blue) configuration against the beam displacement in YAW direction (i.e. $Y_{\text{off}} = 0$). Even though "X" looks better, note that, for "X", the PIT gain is also affected by almost the same amount. This makes the separation between YAW and PIT worse for "X".

gain per YAW off-centering for "+" is roughly twice as large as that for "X" (Fig.10). Since it is difficult to say which is worse, I would say both are equally bad in this respect.

4 Conclusion

The separation of PIT and YAW is worse in "X" oriented diode currently used by LIGO than in "+". A simple calculation showed that, only in "X" configuration, there is a crosstalk from YAW to PIT and from PIT to YAW via off-centering. This crosstalk becomes largest when the displacement is in the direction of $x = \pm y$. For example, if the off-centering is a quarter of the beam diameter, the crosstalk becomes 10%. Even though these numbers don't look catastrophic, they are not negligible, either, and can affect the performance of the interferometer when external disturbances are large and/or when recovering from loss of lock. Even when the disturbances are small, it would introduce the “breeding” of the noisier degree of freedom (typically PIT) into quieter one (YAW), resulting in the increase of the misalignment in the quieter degree of freedom.

Other things were also studied, i.e. optical gain reduction via off-centering (no clear winner), DC centering (“X” more annoying and problematic for DC centering servo), and LSC to ASC crosstalk via off-centering (“+” better than “X” for large off-centering, only marginally better for normal operating condition).

All in all, there’s no reason to prefer “X” to “+”, and there are some reasons to hate “X”. We should not use “X” configuration for future LIGO interferometers.

References

[1] H.Ward et al, Appl. Opt. 33, 0000 (1994).

[2] LIGO-T960111 Wavefront Sensor (1996).

[3] D.Anderson, Appl. Opt. 23, 2344 (1984).