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Effect of correlated noise on the choice of relative lengths between H1 and H2 for Advanced LIGO

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H1-H2 relative length trade: presence of long-term correlated noise

- **Optimal SNR combination of two co-located interferometer signals to extract strain signal**

The stochastic group analyzed the optimal manner in which to combine the strain signals from H1 and H2 when correlated noise is present [see Phys. Rev. **D** 70 062001 (2004) or http://www.ligo.caltech.edu/docs/P/P040006-05.pdf]. In that paper, starting from two measured signals,

$$\tilde{s}_{H_1}(f) = \tilde{h}(f) + \tilde{n}_{H_1}(f)$$
$$\tilde{s}_{H_2}(f) = \tilde{h}(f) + \tilde{n}_{H_2}(f)$$

the optimal combination

$$\tilde{s}_H(f) = \tilde{\alpha}(f)\,\tilde{s}_{H_1}(f) + (1 - \tilde{\alpha}(f))\,\tilde{s}_{H_2}(f)$$

is given by the following choice of $\tilde{\alpha}(f)$:

$$\tilde{\alpha}(f) = \frac{P_{H_2}(f) - P_{H_1 H_2}(f)}{P_{H_1}(f) + P_{H_2}(f) - (P_{H_1 H_2}(f) + P^*_{H_1 H_2}(f))} .$$

$P_{H_1 H_2}$ is the complex cross-power spectrum. Optimal means that, for a given signal, $\tilde{h}(f)$, the variance, or power of the combined signal, $\tilde{s}_H(f)$, is smallest. To simplify the analysis that follows, assume $P_{H_1}(f)$, $P_{H_2}(f)$, and $P_{H_1 H_2}(f)$ have similar shapes, so the paramters of the analysis are scalars, indpendent of frequency: if one wanted to carry the analysis further, it would be straghtforward to include frequency dependence. Under this assumption, we may write:

$$P_{H_2} \approx \beta^2 P_{H_1} \equiv \beta^2 P\text{, where } \beta \equiv \frac{L_1}{L_2} \geq 1 ,$$
$$\Gamma = \rho^2 = \frac{|P_{H_1 H_2}|^2}{P_{H_1} P_{H_2}} .$$

Then, $P_{H_1 H_2}(f) + P^*_{H_1 H_2}(f) = 2 P \beta \rho \operatorname{Cos}[\phi]$. ϕ allows for a possible phase difference between correlated noise in the two interferometers. Substituting, the optimal value of the weighting function $\tilde{\alpha}$ is given by:

$$\tilde{\alpha} = \frac{\beta^2 - \beta \rho \operatorname{Exp}[I \phi]}{(1 + \beta^2 - 2 \beta \rho \operatorname{Cos}[\phi])},$$

and the corresponding optimally combined noise power is given by,

$$\frac{P_H(\beta, \rho, \phi)}{P} = \frac{\beta^2 (1 - \rho^2)}{1 + \beta^2 - 2 \beta \rho \operatorname{Cos}[\phi]} .$$

It is then possible to consider how P_H varies with relative length of the shorter interferometer looking at it dependence on β. In practice, only $\beta = 2$ (present configuration) or β=1 (both machines are 4 km) need be considered. The figure below shows how the quantity $\frac{P_H}{P}$ depends on the coherence of the noise for 2 km:4 km ($\beta = 2$) and 4 km:4 km ($\beta = 1$) configurations.

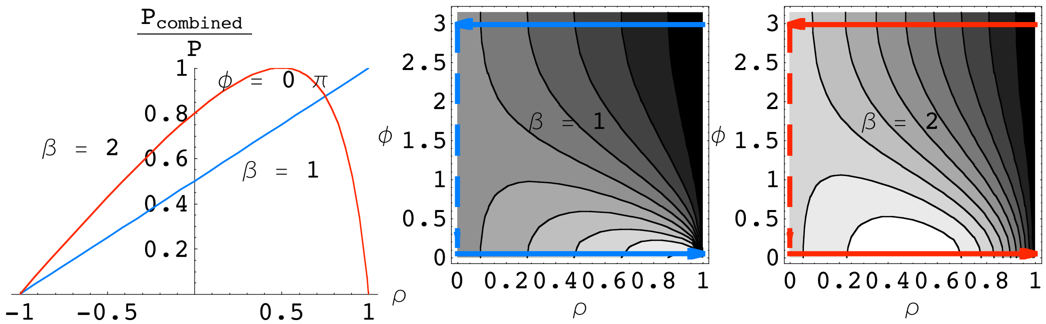

Figure 1. Left panel: Dependence of combined noise power on ρ for two values of β. $\beta = 2$ corresponds to the LIGO I configuration. Middle panel: contour plot of combined noise power as a function of both ρ and ϕ. The trajectory shown in blue corresponds to the the $\beta = 1$ plot in the left panel. Right panel: contour plot of combined noise power as a function of both ρ and ϕ. The trajectory shown in red corresponds to the the $\beta = 2$ plot in the left panel.

The following observations can be made:

(i) If the noise in the two interferometers is uncorrelated, then using two 4 km machines provides a better estimate of h.

*(ii) If the noise in the two interferometers is highly correlated (e.g., $\rho > 0.75$), then two unequal machines enable one to disentangle GW strain from correlated noise: the combined noise power for unequal machines $\to 0$ as $\rho \to 1$. For two machines of **equal** length, as $\rho \to 1$, the machines become indistinguishable and combining them offers no advantage.*

(iii) Anti-correlated noise ($\rho < 0$) can always be used to improve performance in either case; two 4 km machines provide better performance for all values of $\rho < 0$.

(iv) Statements (i) - (iii) apply when the noise correlations between the two machines are relatively real. For phases $\phi \neq 0$ and $\phi \neq \pi$, the situation is such that in all cases the 4 km: 4 km configuration outperforms the 2 km: 4 km pair.

Veto power: generation of a null stream from two interferometers

In addition to the optimal estimate of h,the data streams can be combined to produce a null stream that contains no GW signal. This is useful in helping to sort out the validity of a putative coincident detection. If the null stream shows a signal, then the likelihood that the putative detection is a real event is diminished accordingly. Of course, the ability to use the null stream depends on SNR of the event and the relative calibration accuracy between machines. The referenced paper discusses the null stream in detail. For the purposes of this discussion, what is of interest is the noise power in the null stream,

$$\tilde{z}_H(f) = -(\tilde{s}_{H_1}(f) - \tilde{s}_{H_2}(f))\sqrt{1 - \tilde{\alpha}(f) - \tilde{\alpha}(f) + 2|\tilde{\alpha}(f)|^2},$$

The noise power in the null stream is given by,

$$P_{zH}(f) = (P_{H_1}(f) + P_{H_2}(f) - (P_{H_1 H_2}(f) + P^*{}_{H_1 H_2}(f)))\,[1 - \tilde{\alpha}(f) - \tilde{\alpha}(f) + 2|\tilde{\alpha}(f)|^2].$$

Using the parametrization introduced above this becomes,

$$\frac{P_{zH}(\beta, \rho, \phi)}{P} = \frac{(-2\beta(1+\beta^2)\rho\,\mathrm{Cos}[\phi] + (1+\beta^4+2\beta^2\rho^2))}{1+\beta^2-2\beta\rho\,\mathrm{Cos}[\phi]}$$

Figure 2 presents the same information as in Figure 1, but for the null stream z_H. In the case of the null stream, equal length interferometers always outperform a configuration with a shorter machine. This is because forming the difference between the two signals brings in the noise power of the indiviudal machines with equal weight.

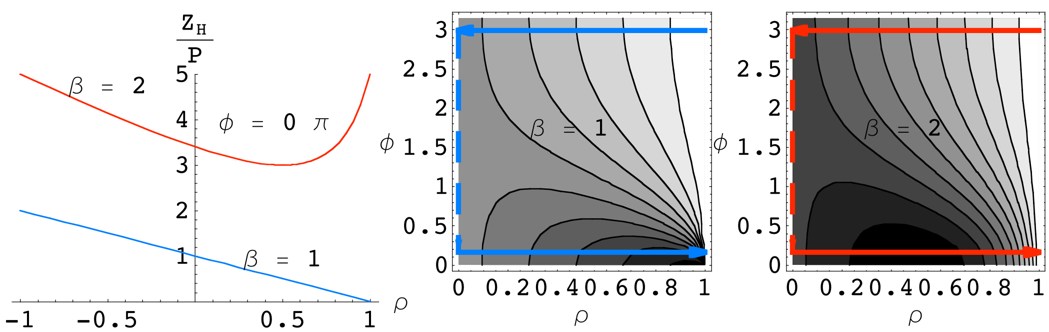

Figure 2. Left panel: Dependence of null stream signal noise power on ρ for two values of β. β = 2 corresponds to the LIGO I configuration. Middle panel: contour plot of null stream signal noise power as a function of both ρ and φ. The trajectory shown in blue corresponds to the the β = 1 plot in the left panel. Right panel: contour plot of null stream signal noise power as a function of both ρ and φ. The trajectory shown in red corresponds to the the β = 2 plot in the left panel.

■ Conclusions

This simple analysis considers the effect on the optimally combined signal from two interferometers when the length of the shorter interferometer is varied with respect to the longer one. Here, it was assumed that the spectra are similar, so that an overall scale factor can be used to characterize their differences. A general analysis requires consideration of the spectral character of the noise floor of the machines under consideration. Moreover, this analysis looks at the effect of length variation when the noise floors of the two machines are stationary. Only two cases are considered: the LIGO I design of 2 km: 4 km interferometers and a potential Advanced LIGO modification to this configuration the has two 4 km machines. This applies to the case of searching for a stochastic background using the two Hanford instruments.

Under these conditions, it appears to be the case that, except in the extreme (and unlikely) case that the two machines have almost completely correlated noise floors (i.e., $\rho \sim 1$), ***it is always better to have two long machines rather than a long machine and a short machine.***

■ Caveats

This analysis does not consider transient,burst-like instrumental artifacts.Such glitches are not characterizable by a (stationary) noise spectral density.Moreover,depending on the sources of such glitches,they may or may not be common-mode for machines of different lengths.One example of an artifact that could either be common or non-common mode is particulate shedding in the beam tubes that may occur either in the common or non-common sections of the beam tubes if the interferometers are of different length: for machines of identical length,all events will tend to become potnetially common mode.