
**Quadruple Suspension Dynamics Model
with Finite Elements
(Advanced LIGO)**

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Change Record:

Revision –00: Initial Release. Written 6 Dec 2005 (issued 26 June 2006). Note that this was work performed, and documented (to the extent given below) late last year but not completed. The work was started when neither the Matlab nor the Mathematica suspension models of the quadruple pendulum were matching initial response of the controls quadruple suspension. Eventually it was found that the largest discrepancy was the position of the effective wire flexure point. With beam elements modeling the wires, this was not a limitation of this finite element approach. The finite element model of the suspension was relatively successful, except that the lowest frequency pitch mode was at times found to be negative. This is either a model parameter problem or a numerical problem in the eigensolver.

1 Introduction

There are currently three models used for dynamics modeling of suspensions:

- A [Mathematica Model](#)¹ created by Mark Barton. This model has been used to model the Mode Cleaner triple suspension and the quadruple suspension².
- A Matlab Model³ created by Calum Torrie. This model has been extended and applied in design for the triple and the quadruple suspensions⁴.
- A Maple Model⁵ created by Mathew Husman.

All three of these models are based on closed form equations.

So why yet another approach to modeling? Recent measurements on the quadruple, controls prototype suspension did not (initially) match eigenfrequency predictions. Questions arose about the proper modeling of the effective wire flexure distance⁶ and the effect of non-vertical wires, blade spring stiffnesses other than in the principal bending direction (in-plane longitudinal, lateral and torsional stiffness), off-diagonal mass moments of inertia, non-flat blade spring geometry, geometry faithful to the CAD, blade tip mass, etc. All of these factors can be included in a finite element based numerical model.

There are (at least) two basic approaches to developing the finite element model:

- Geometry derived from the CAD 3D model. The virtue of this approach is that the model can be keyed to the CAD files used to create the physical parts. In principle a parametric 3D CAD model can be dynamically linked to the finite element model so that updates to the

¹ M. Barton, [Models of the Advanced LIGO Suspensions in Mathematic](#), LIGO-T020205-00.

² N. Robertson, M. Barton, J. Greenhalgh, [Cross-Coupling in Quadruple Suspension](#), T040143-00.

³ C. Torrie, “Development of Suspensions for the GEO600 Gravitational Wave Detector”, PhD Thesis, University of Glasgow, 2000. Description of the design of the GEO suspensions, with particular reference to an analytical model of them in Matlab.

⁴ N. Robertson, et. al., [Advanced LIGO Suspension System Conceptual Design](#), T010103-04.

⁵ M. Husman, “Suspension and Control for Interferometric Gravitational Wave Detectors”, PhD Thesis, University of Glasgow, 2000. Description of the design and control of the GEO suspensions, with particular reference to an analytic model of them in Maple.

⁶ N. Robertson, M. Barton, Technical Note on the Effects of Stiff Wires on the Behaviour of Suspensions, LIGO number pending, 9 Sep 2005.

CAD can propagate to the model; In practice this level of interoperability can be difficult to achieve.⁷

- Geometry generated from a command script with parameter values. The virtue of this approach is that the model can be updated quickly with a simple text edit (similar to editing the input parameters for either the Mathematica or Matlab models). Generating the eigensolution or transfer function for perturbations in parameter values is quick and easy.

I've describe both approaches in the following sections. The first approach was accomplished with I-DEAS and the second approach was attempted with ANSYS. However, either finite element tool could be used for either approach.

2 Approximations

Linear

Pre-stress, not geometrically nonlinear pre-load – assuming ideal loading to intended location

Etc.

Note approximations that are inherent & not inherent in the FEA approach

3 Geometry Derived from CAD

The quadruple suspension parts were developed as 3D parts and assemblies in SolidWorks. Since I-Deas does not directly import SolidWorks files, I converted. Of the available conversions⁸, I found IGES to work best. However it is essential to minimize the complexity of the assembly; Only include those parts essential for developing the model. Hidden parts in SolidWorks are not converted.

Once the geometry is imported into I-Deas, use the wireframe attach function to associate the finite element mesh with the assembly (or part). The following idealizations of the suspension were made in developing the mesh:

- Blade springs are modeled as plates (shells): The top surfaces of each blade spring solid part were selected for auto-meshing of shell elements (with bending, membrane and torsional stiffnesses). The thicknesses and material properties were keyed in manually.
- Wires are modeled as beams: The intersections of the wires with the clamp faces were 'found' by adding line and point features keyed to the imported geometry. Then beam elements (bending, torsion and extensional stiffness) were auto-meshed between the wire/clamp interface points. The cross-sectional geometry of each wire and the wire material properties were then keyed in manually.
- Suspension masses modeled as "lumped masses": The essentially rigid parts that comprise the suspension masses (upper mass, upper intermediate mass, penultimate mass and test

⁷ This seamless updating of CAD and FEM is possible within I-DEAS, but does not appear possible between our chosen CAD tool, SolidWorks, and ANSYS.

⁸ Of flavors generated by SolidWorks 2005 and read by I-Deas v9, STEP203 had 3 of the 6 blades facing the wrong way, STEP214 didn't work.

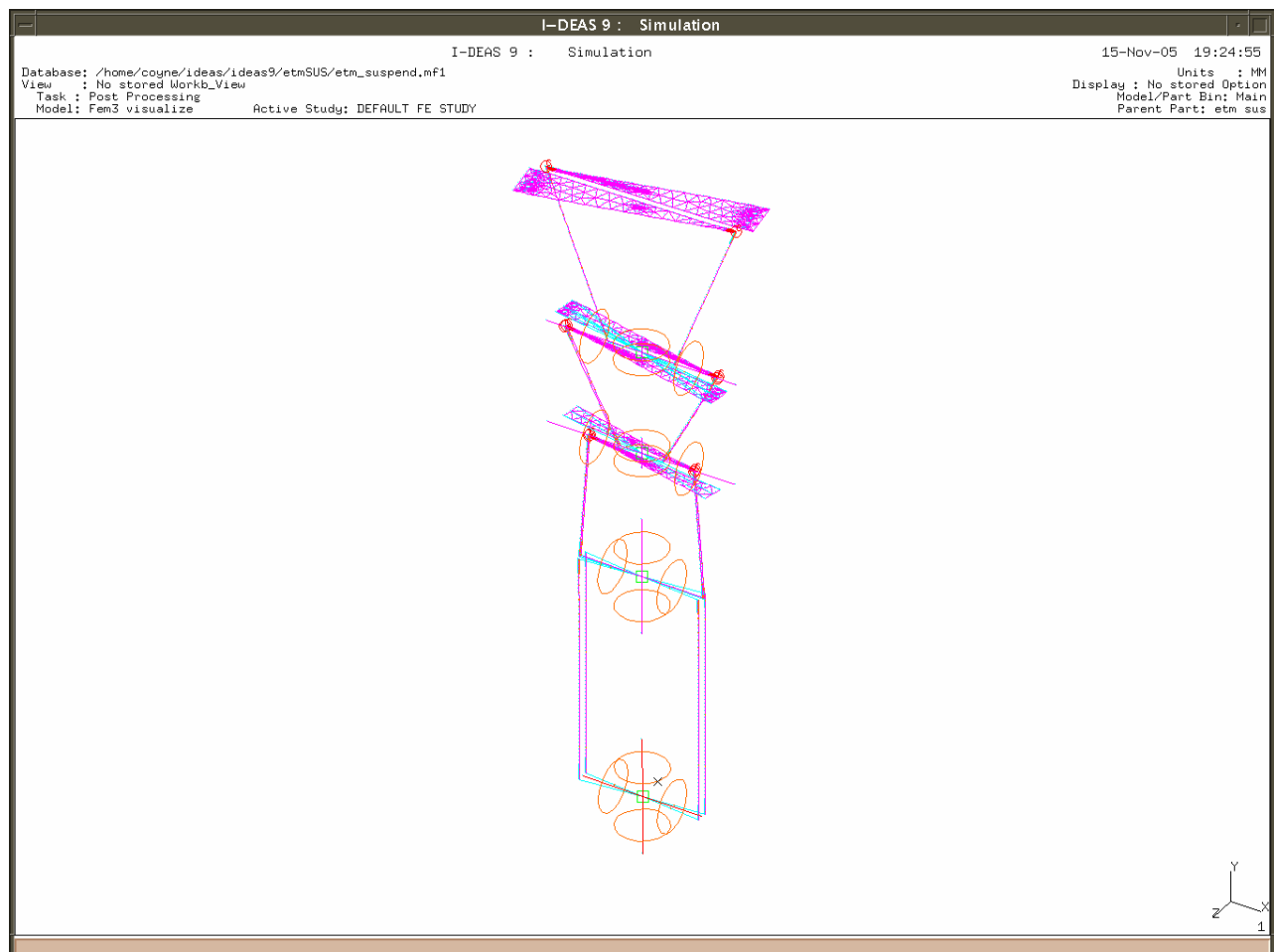
mass) are modeled as “lumped” (point) masses with a full mass moment of inertia matrix. The matrix was calculated in SolidWorks for each suspension mass, with the blade springs removed, and then keyed into I-Deas manually. The locations were established from the imported geometry.

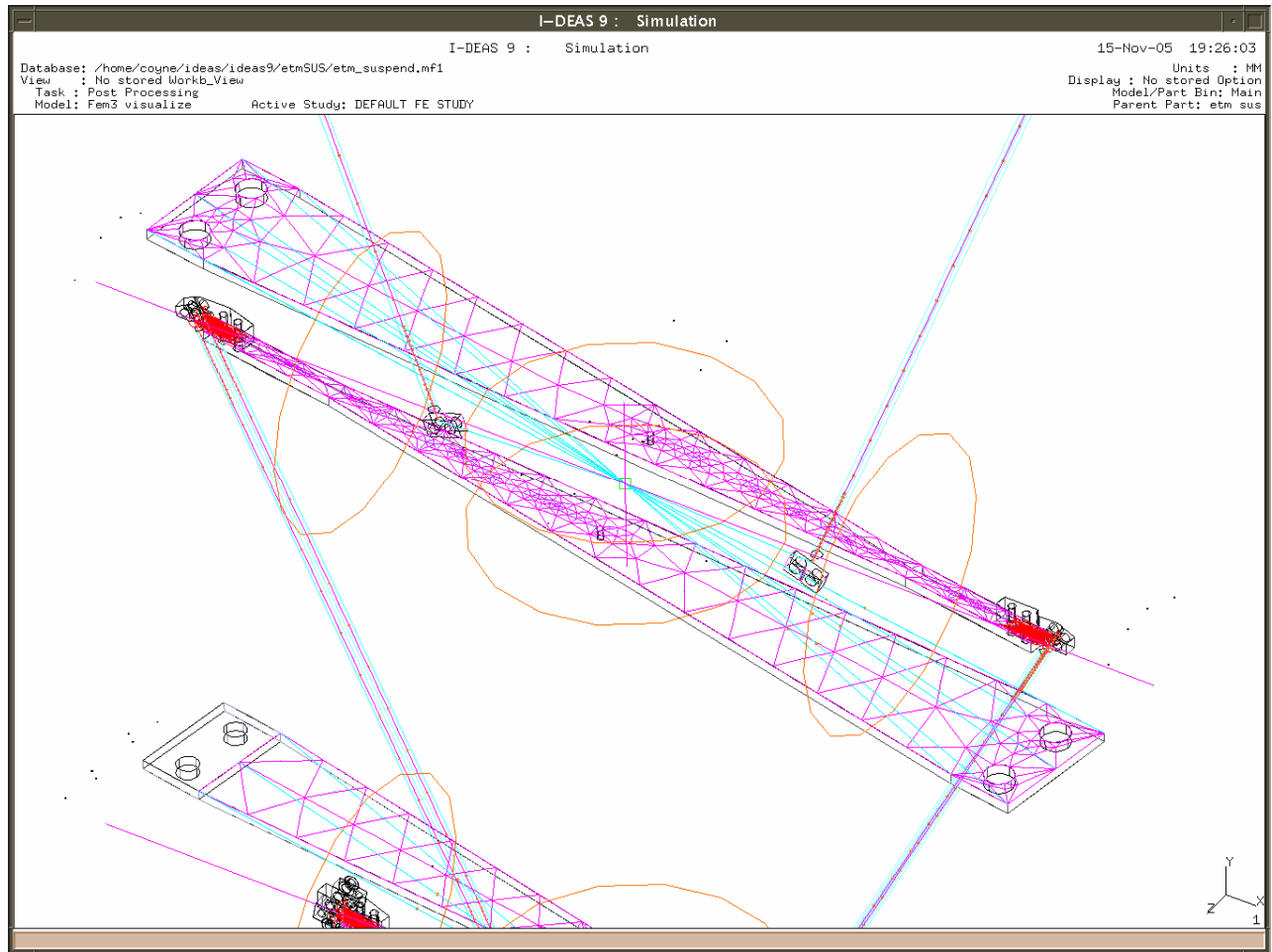
- Multipoint constraints (MPC): The point mass elements representing the suspension masses were then tied rigidly to the flexible elements (spring blade bases and wire ends) using multipoint rigid constraints which ensured that all 6 degrees of freedom of the suspension masses are imposed on the attachments. Since the MPC elements do not appear in any mode shape plots in I-Deas, stiff, low mass beams were added vertically and horizontally to each point mass (with dimensions equal to the physical extent of the actual suspension mass) to help visualize the motion of each suspension mass.

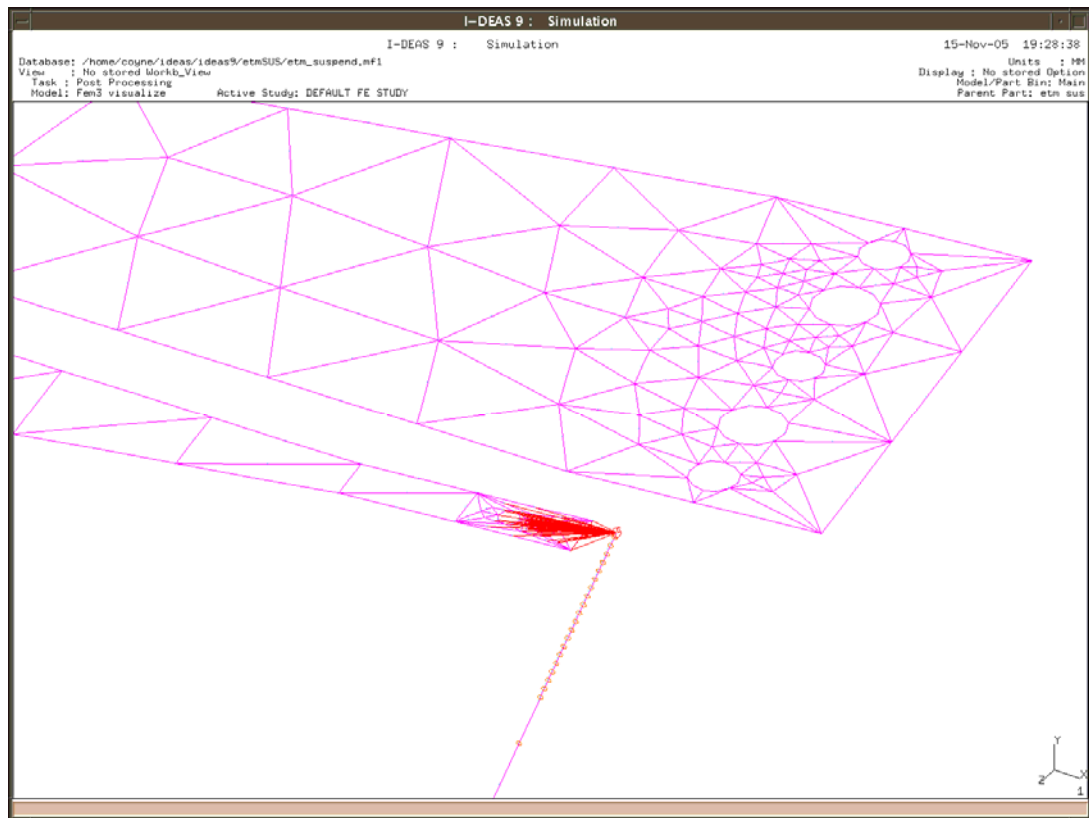
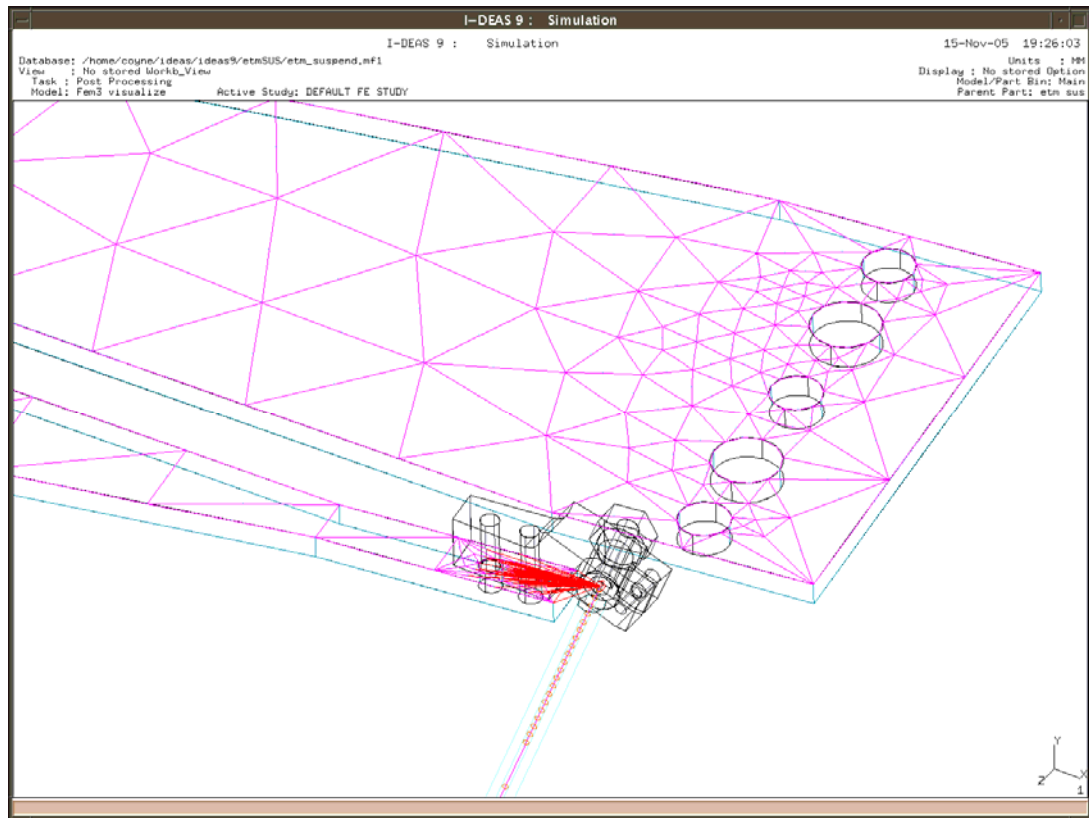
3.1 Model

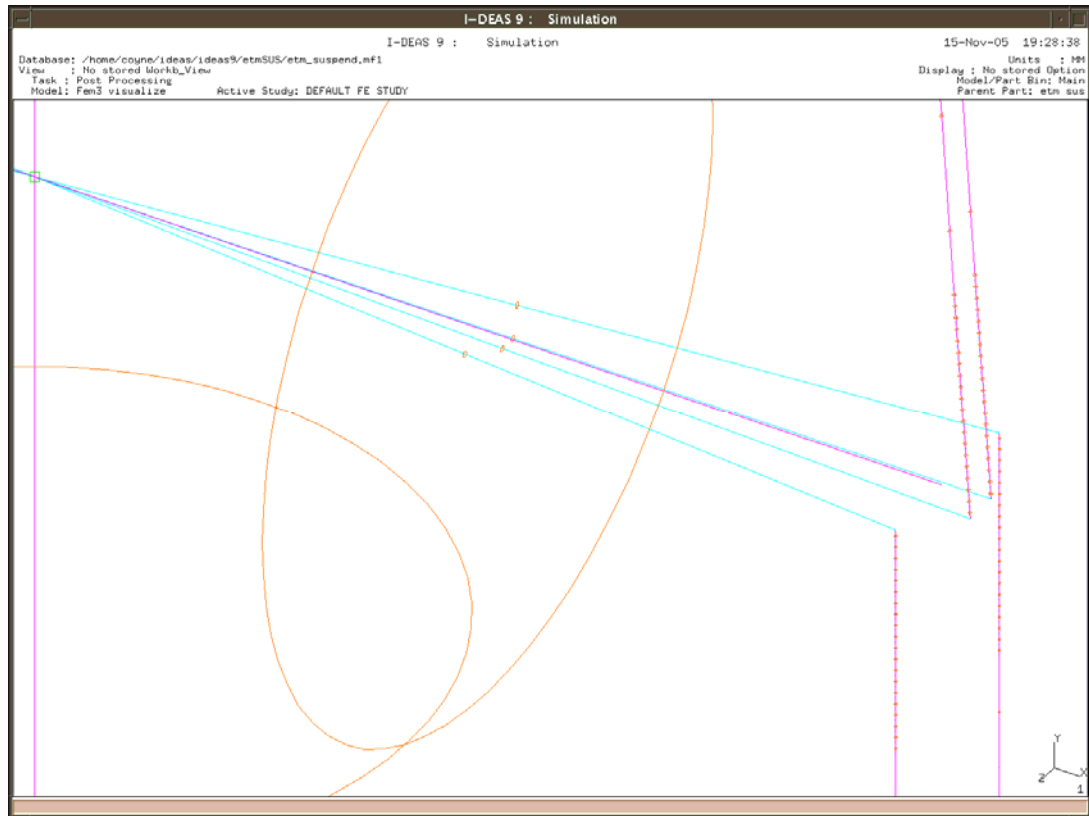
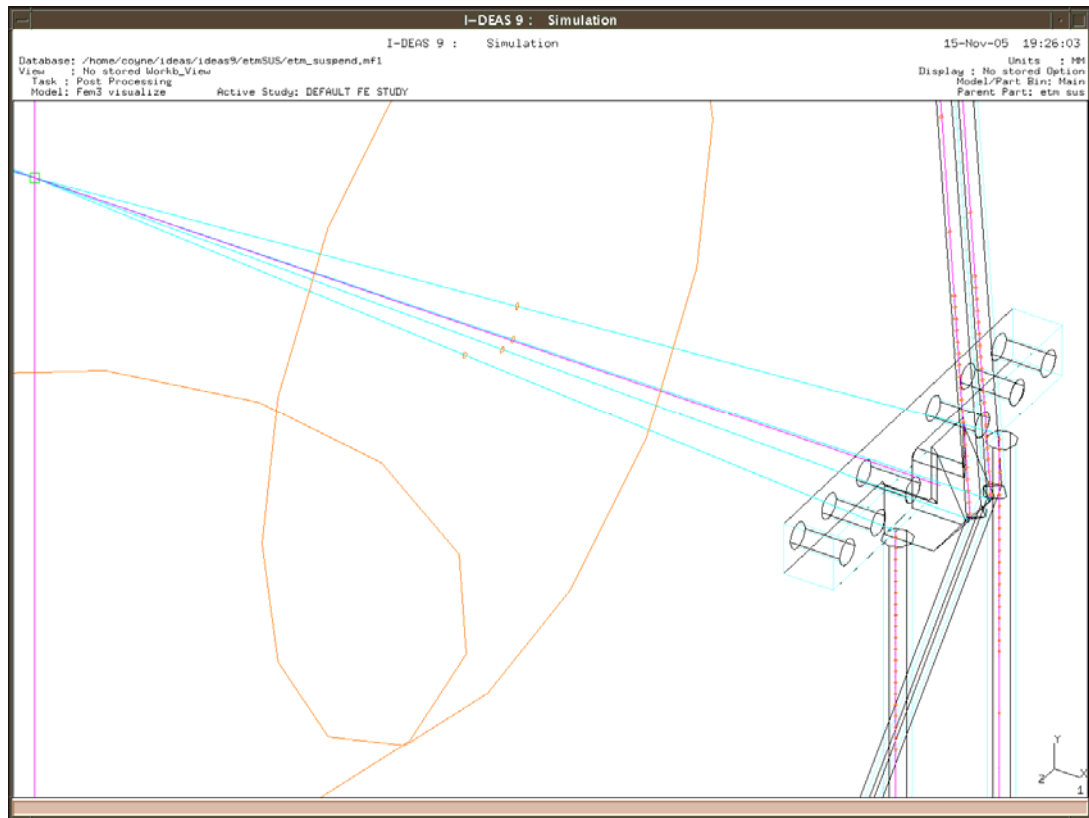
The model is depicted in the following Figures. More explanation and description ...

Figure 1: CAD Derived Finite Element Model









3.2 Eigensolution

Explain concerns with low frequency solutions – may have a numerical eigensolver problem.

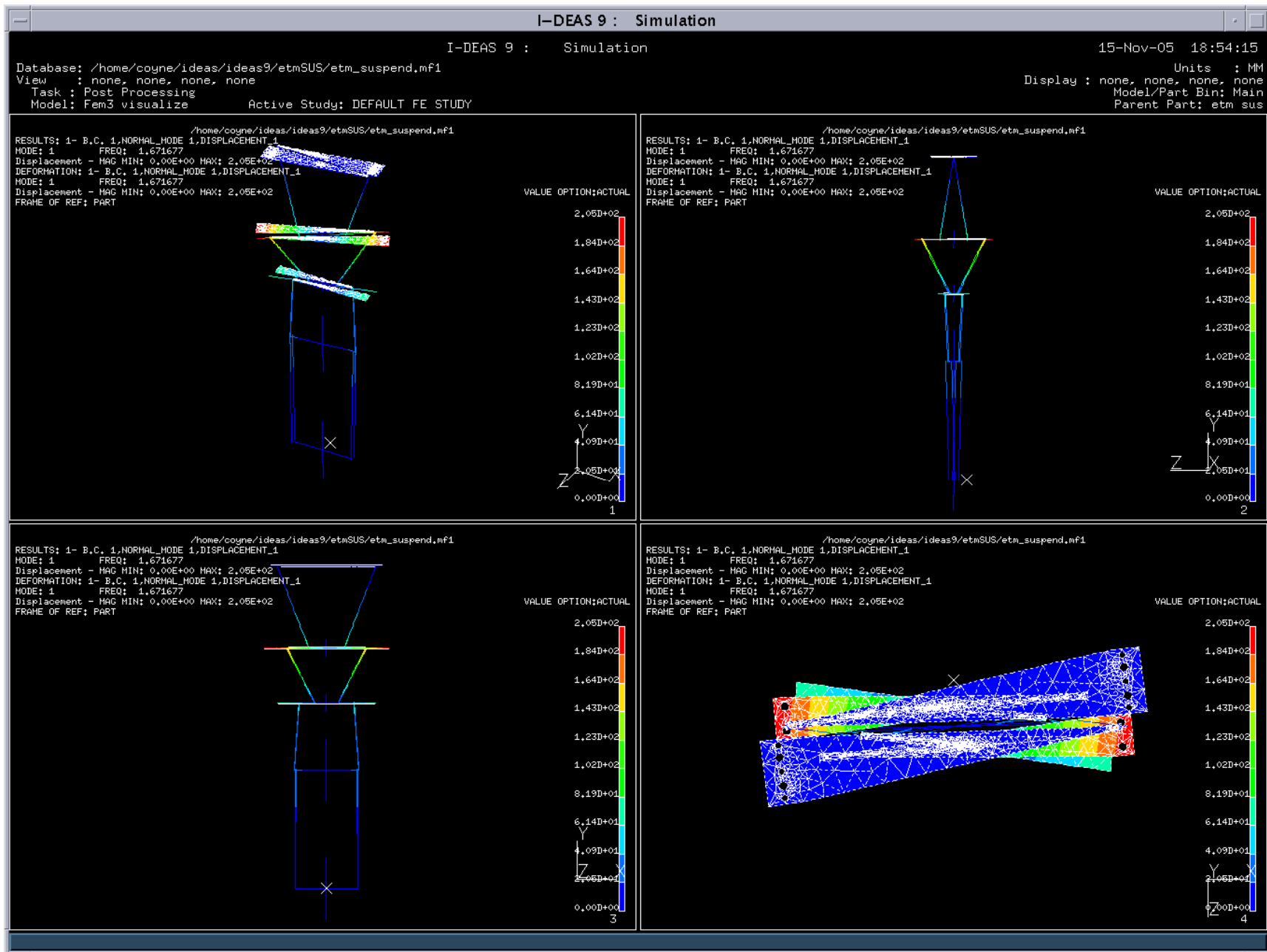
Table 1: Modal Frequencies

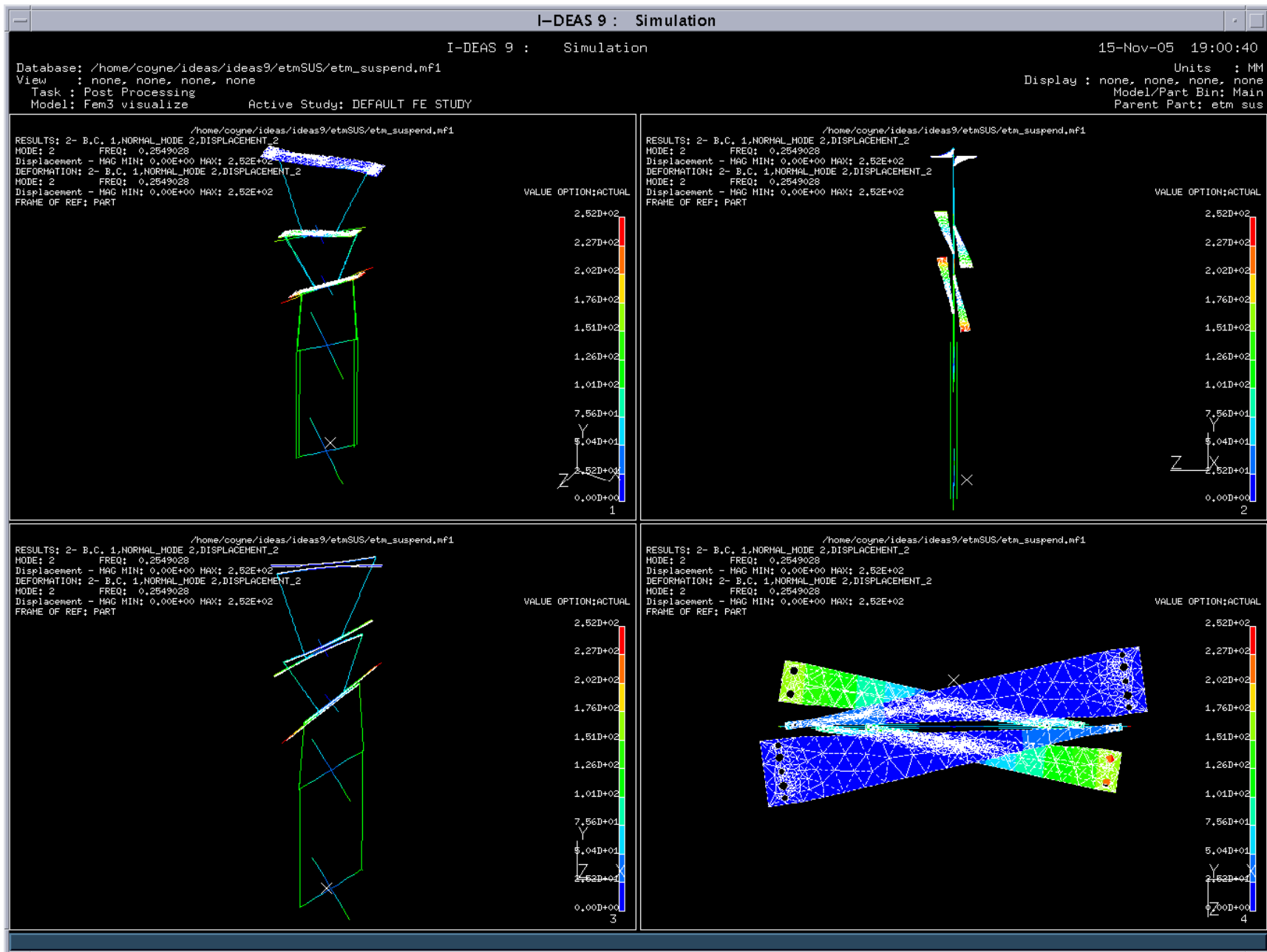
Mode #	Frequency(Hz)		Modal Prop.		
	Undamped	Mass	% X-Mass	% Y-Mass	% Z-Mass
*2	0.2549	1.32767e+06	3.58	0.00	0.00
*3	0.4358	4.73684e+07	0.00	0.00	77.20
*4	0.4707	2.41062e+07	68.87	0.00	0.00
*5	0.5408	525608	0.02	0.00	0.18
*6	0.5611	3.33127e+07	0.00	93.22	0.00
*7	0.9530	354642	0.00	0.00	0.00
*8	0.9879	9.72652e+06	0.00	0.00	15.25
*9	1.0346	9.75084e+06	17.46	0.00	0.00
*1	1.6717	342382	0.00	0.00	0.00
*10	1.7018	1.03984e+06	0.00	0.00	0.00
*11	1.7843	259162	1.04	0.00	0.01
*12	1.9703	2.59522e+06	0.05	0.00	5.05
*13	2.0580	676561	6.68	0.00	0.03
*14	2.1440	236195	0.27	0.00	0.01
*15	2.2637	2.22979e+06	0.00	5.22	0.00
*16	2.7155	271473	0.00	0.00	0.00
*17	2.8448	593596	0.01	0.00	0.00
*18	3.0861	400200	0.00	0.00	0.00
*19	3.2241	186502	0.14	0.00	0.01
*20	3.3555	843797	0.00	0.00	0.78
*21	3.6121	615741	0.00	0.43	0.00
*22	4.5777	1.03667e+06	0.38	0.00	0.00
*23	15.2017	3.3333e+06	0.00	0.00	0.00
*24	22.0533	908453	0.00	0.00	0.00
*25	64.6838	5238.6	0.01	0.00	0.00
*26	64.7360	5182.54	0.00	0.42	0.00
*27	93.4493	1746.49	0.00	0.00	0.00
*28	93.6721	1753.28	0.00	0.00	0.00
*29	112.9465	1153.3	0.00	0.00	0.00
*30	113.1198	1156.63	0.00	0.00	0.00

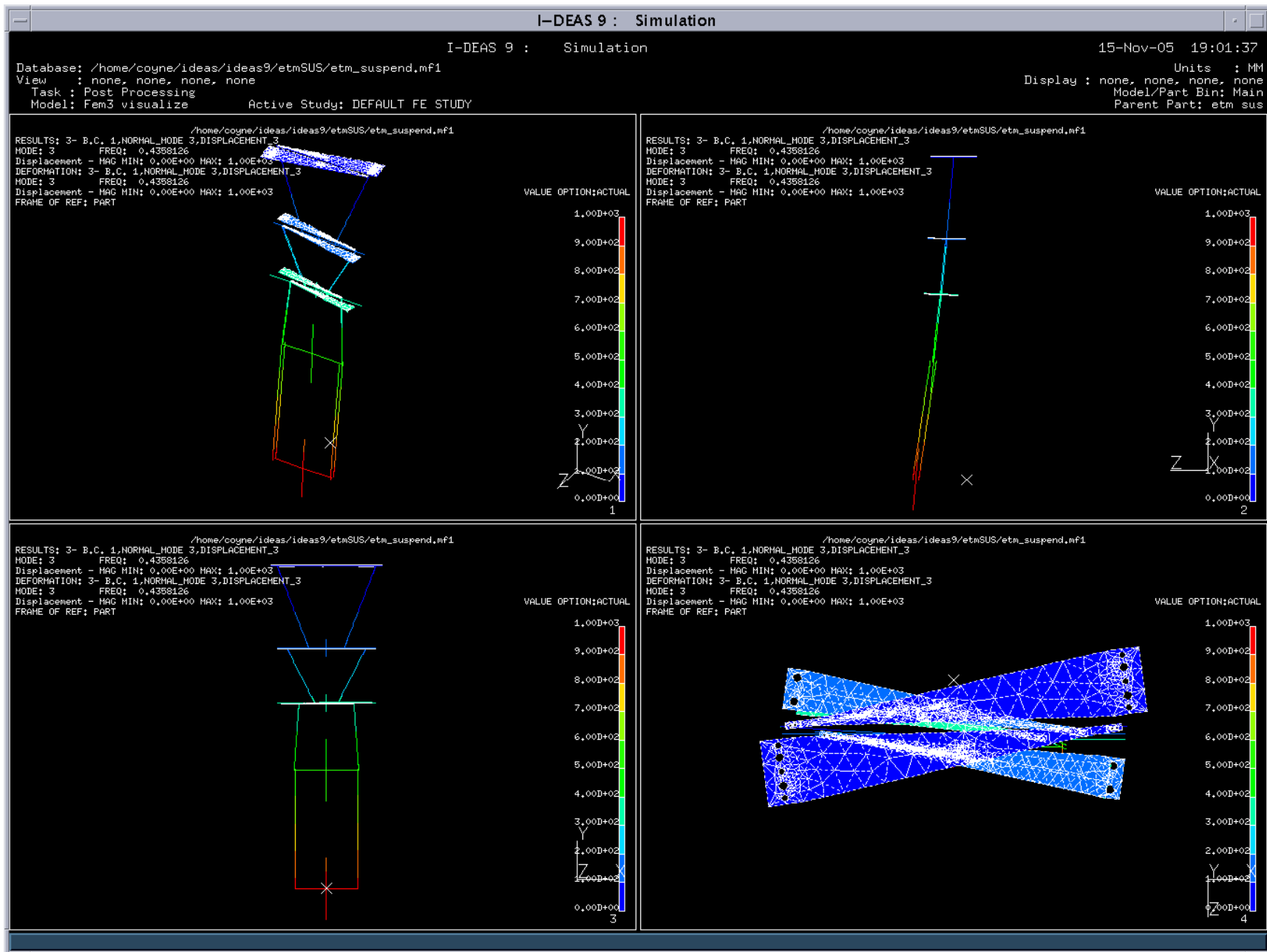
*26	64.7360	5182.54	0.00	0.42	0.00
*27	93.4493	1746.49	0.00	0.00	0.00
*28	93.6721	1753.28	0.00	0.00	0.00
*29	112.9465	1153.3	0.00	0.00	0.00
*30	113.1198	1156.63	0.00	0.00	0.00
*31	198.3211	940.398	0.00	0.02	0.00
*32	198.3726	945.893	0.00	0.16	0.00
*33	238.2189	25.3292	0.00	0.00	0.00
*34	238.2189	42.044	0.00	0.00	0.00
*35	238.2189	38.9075	0.00	0.00	0.00
*36	238.2189	77.564	0.00	0.00	0.00
*37	238.2189	25.3374	0.00	0.00	0.00
*38	238.2225	52.3369	0.00	0.00	0.00
*39	238.2225	52.3368	0.00	0.00	0.00
*40	238.2317	52.7298	0.00	0.00	0.00

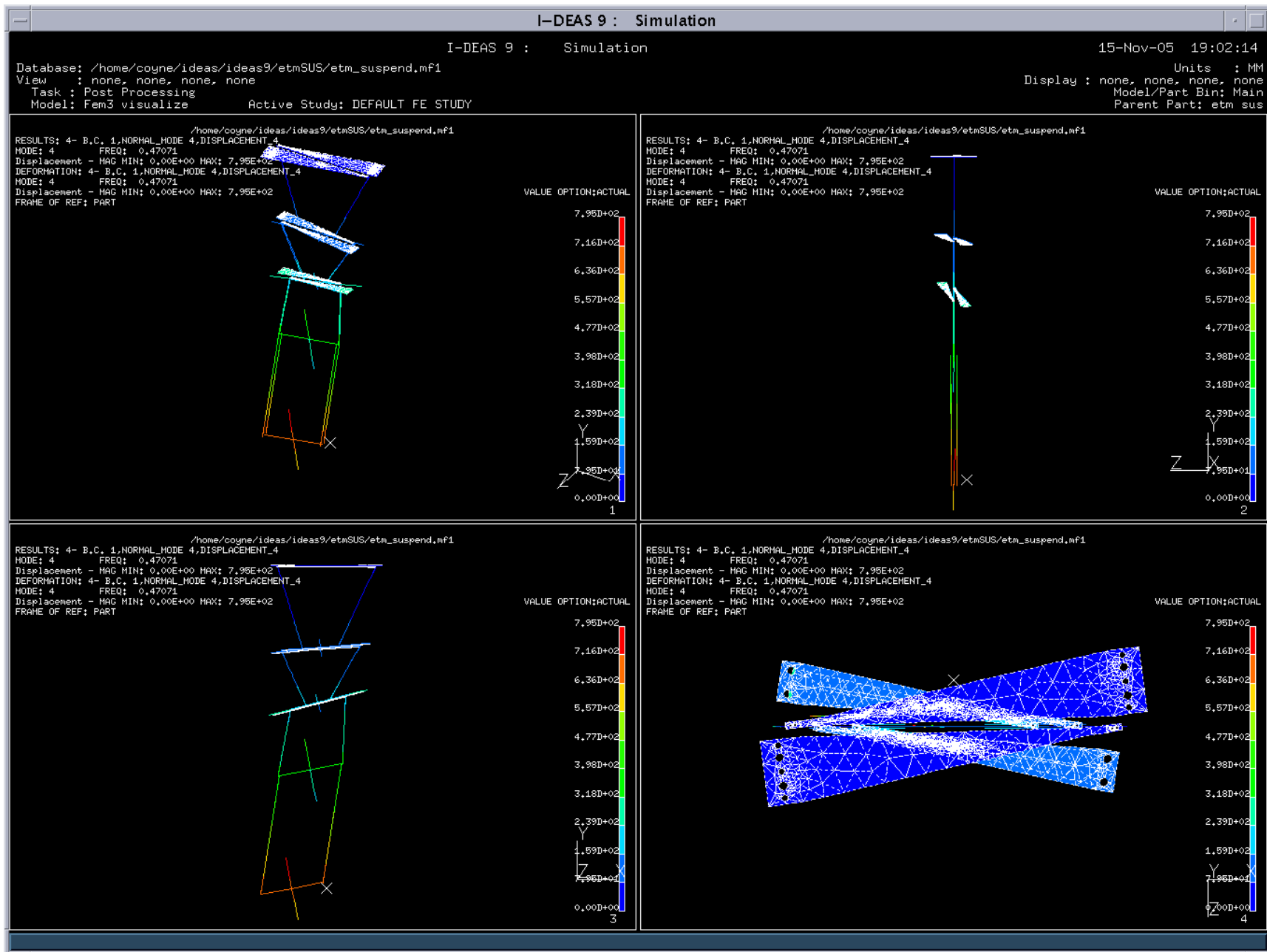
Mode shape plots are given in the following figures.

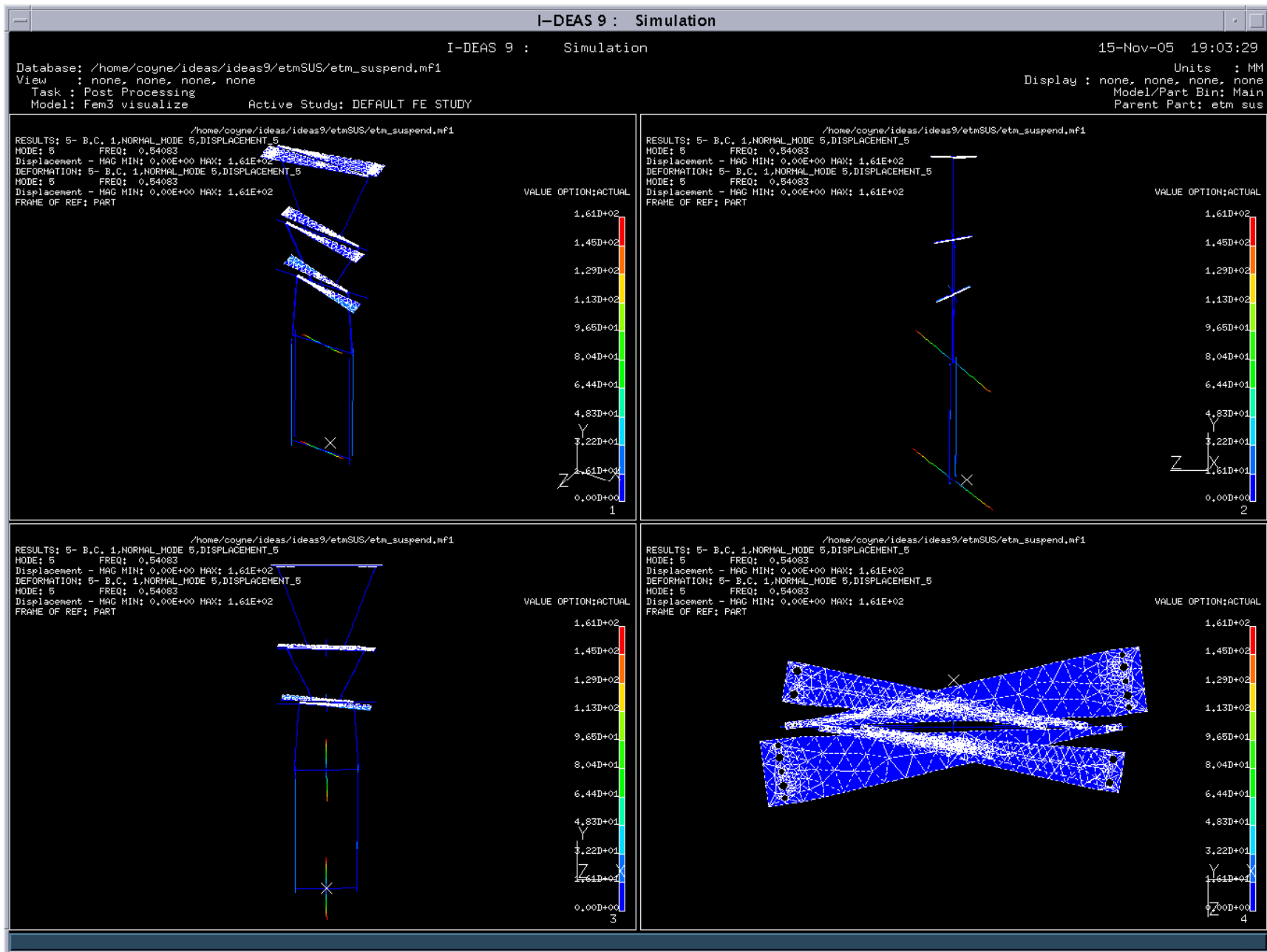
Mode shape movies for the first 33 modes follow.

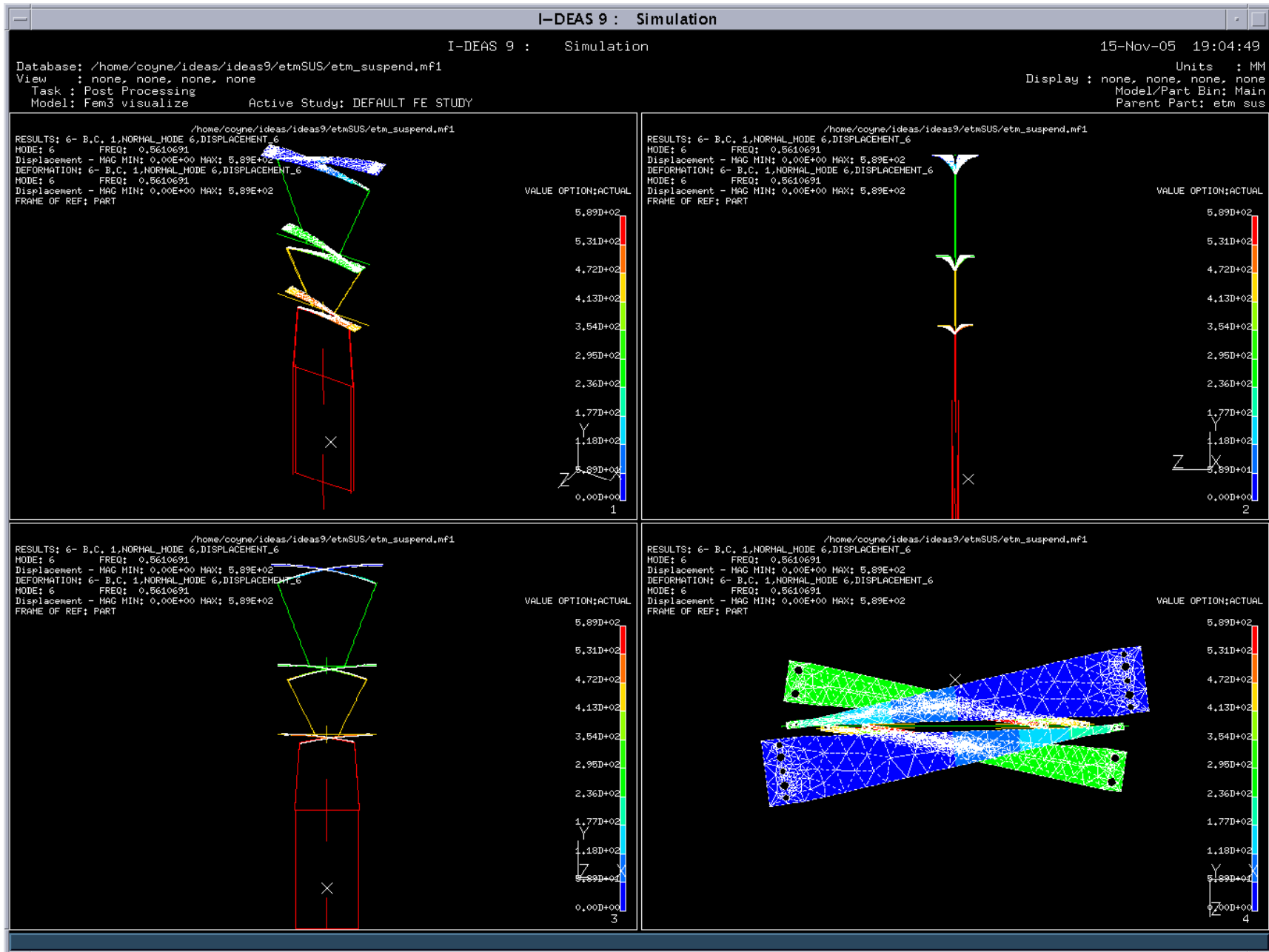


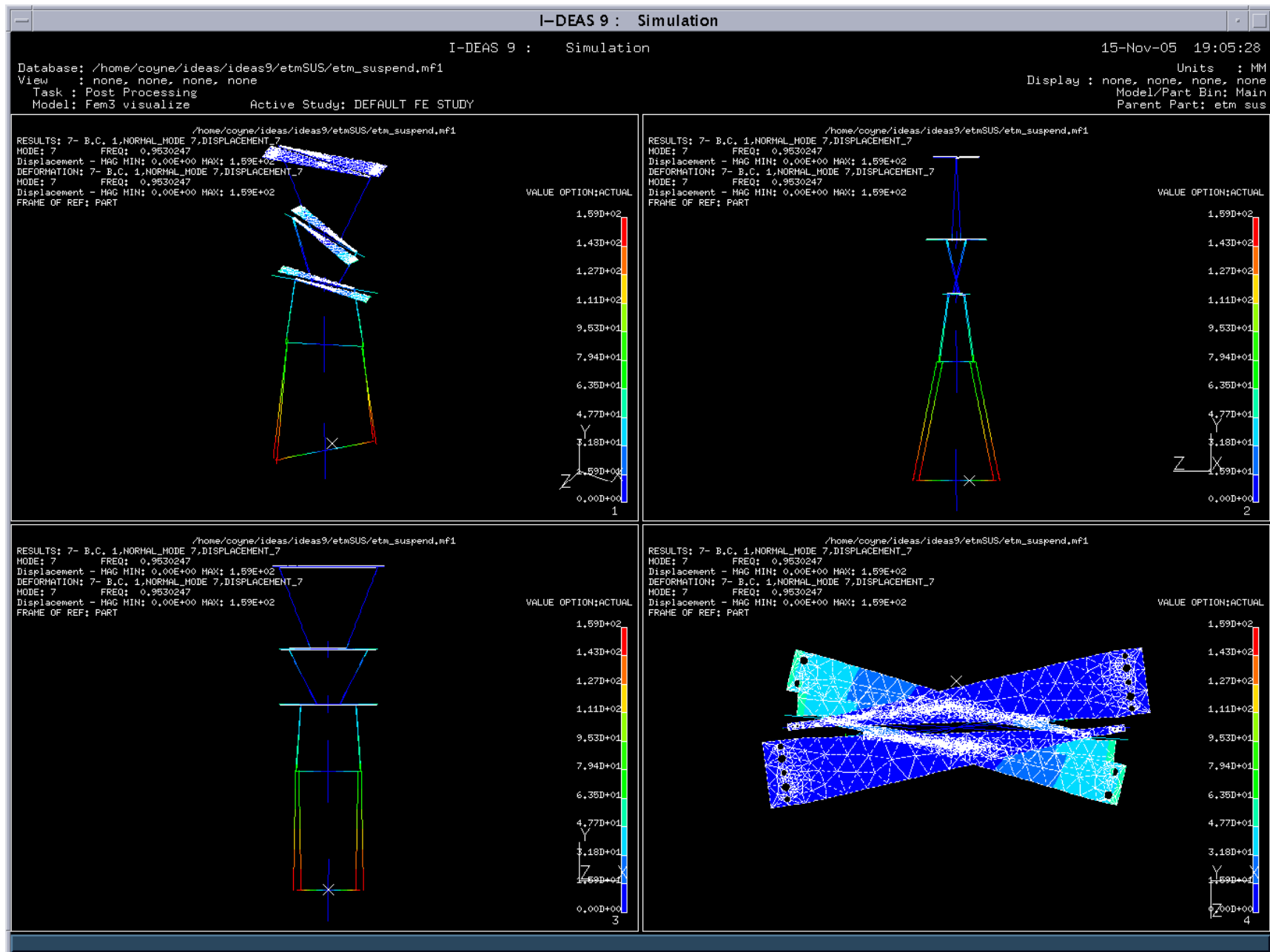


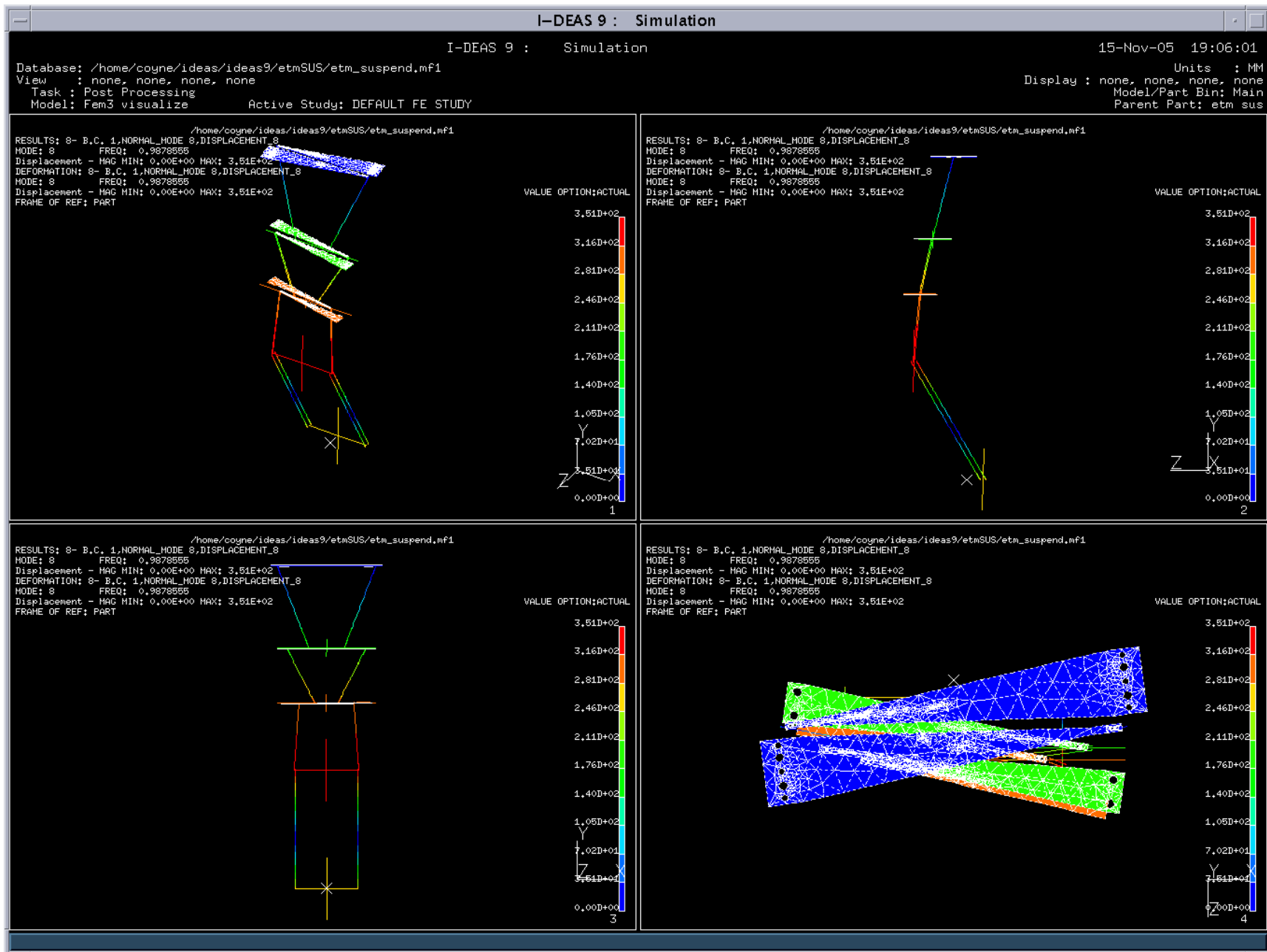


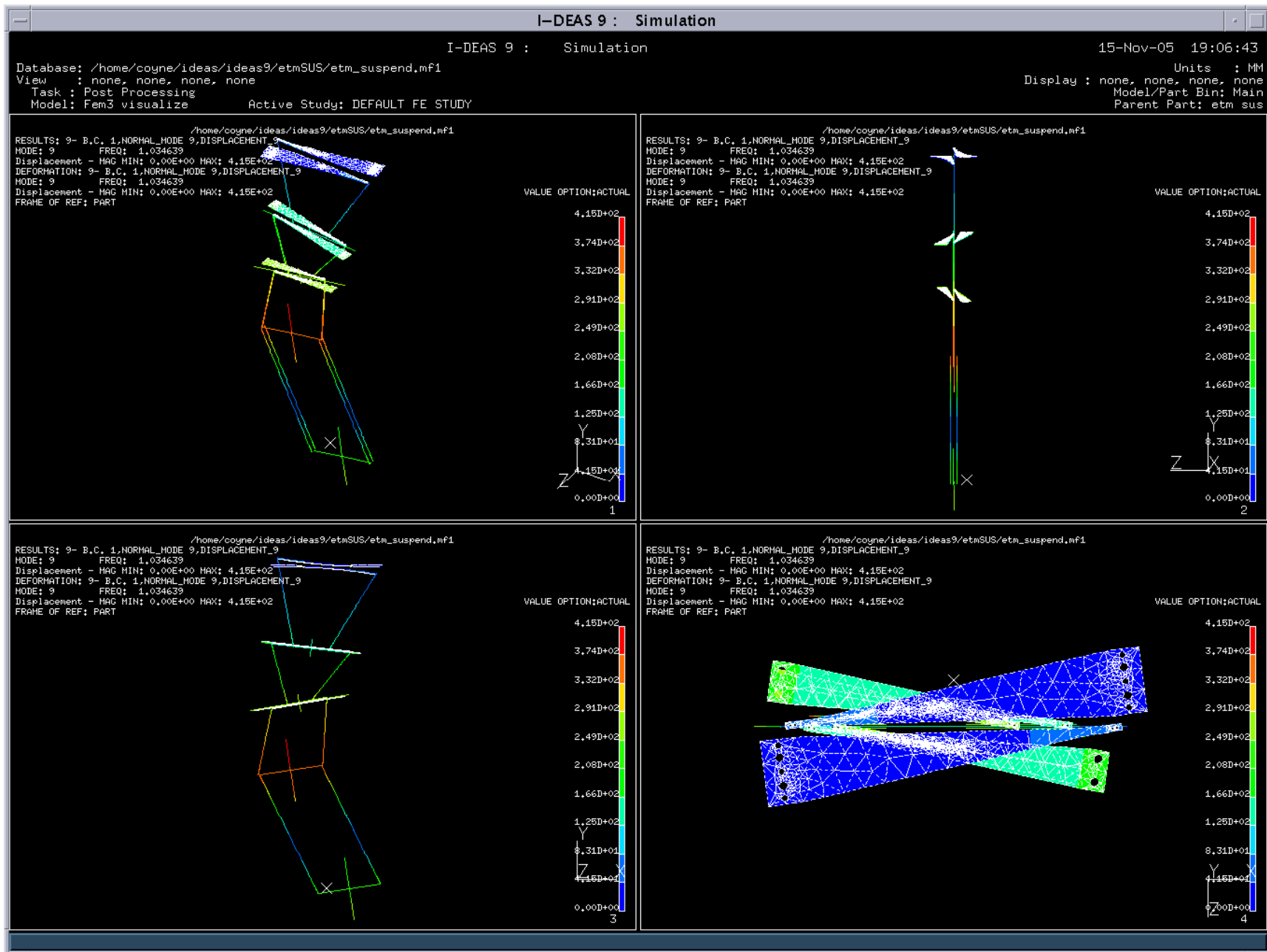


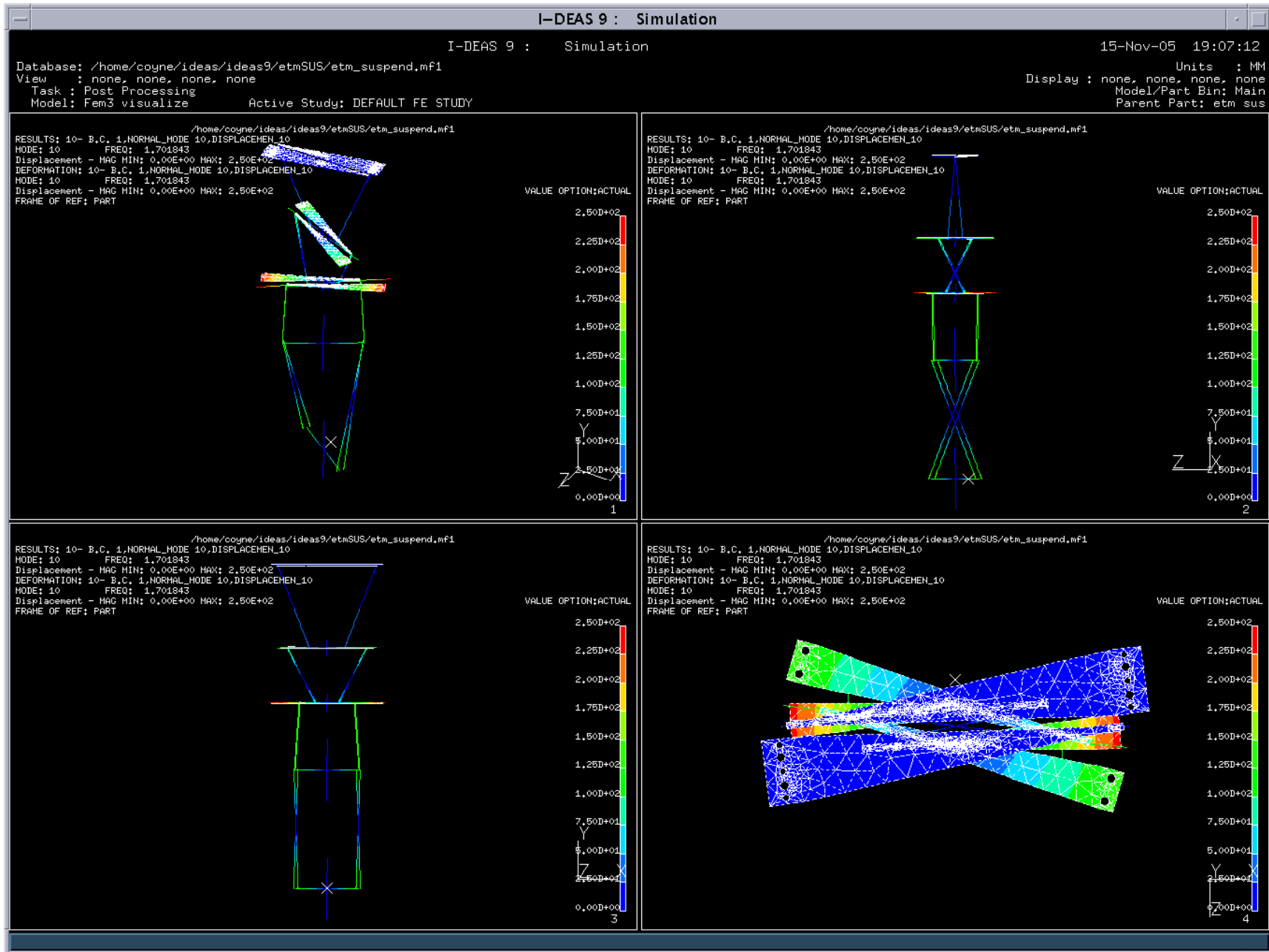


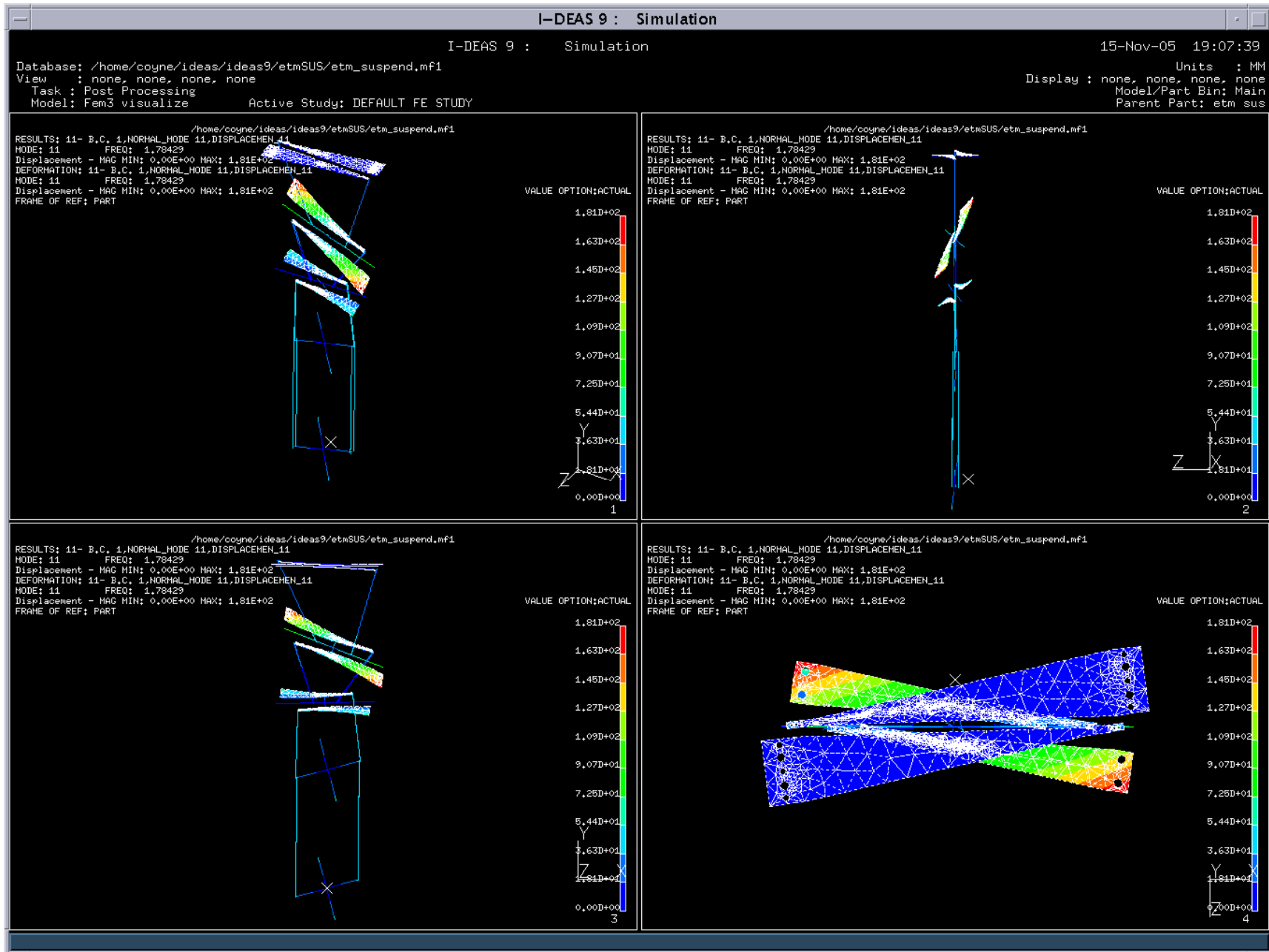


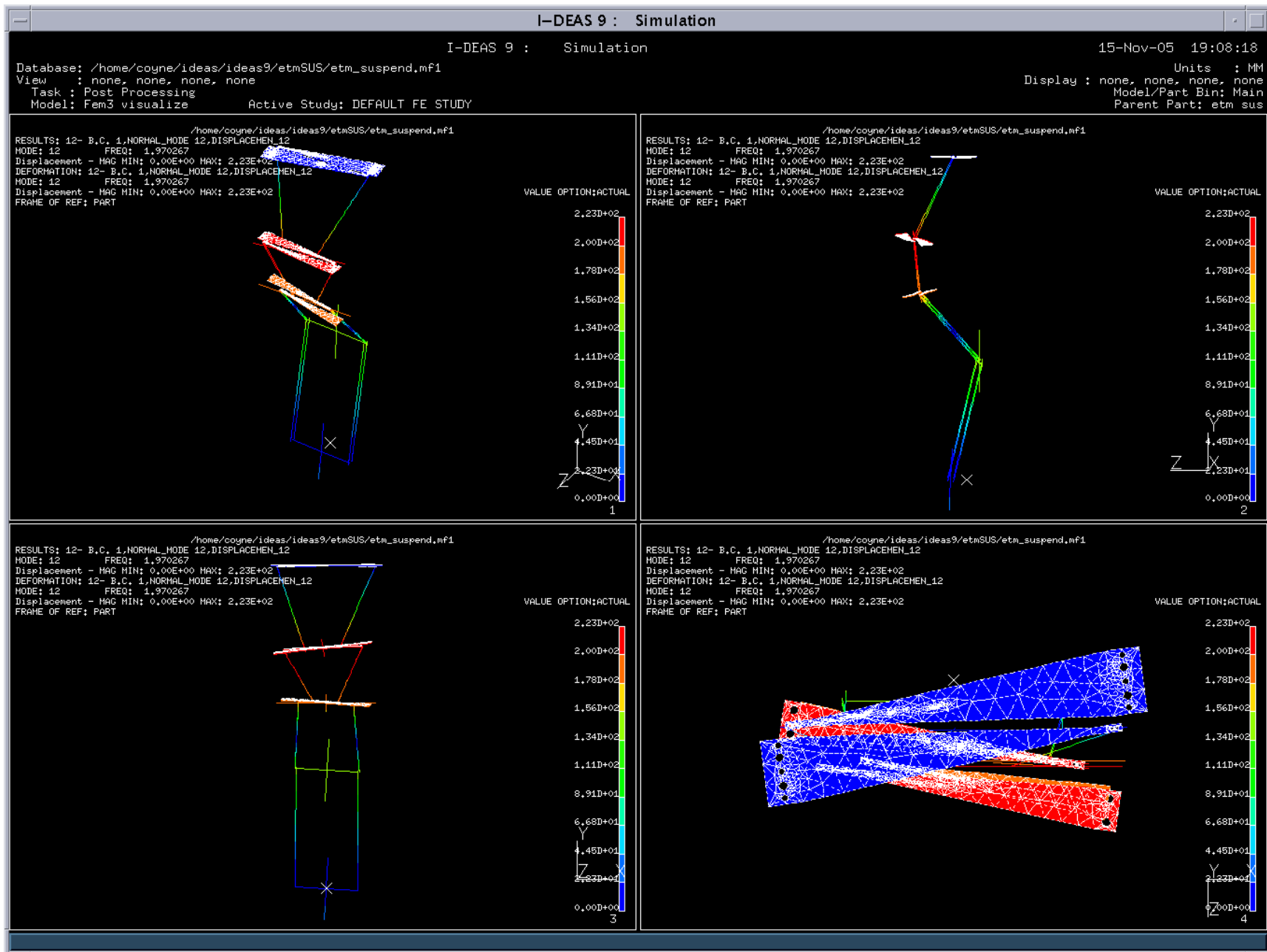


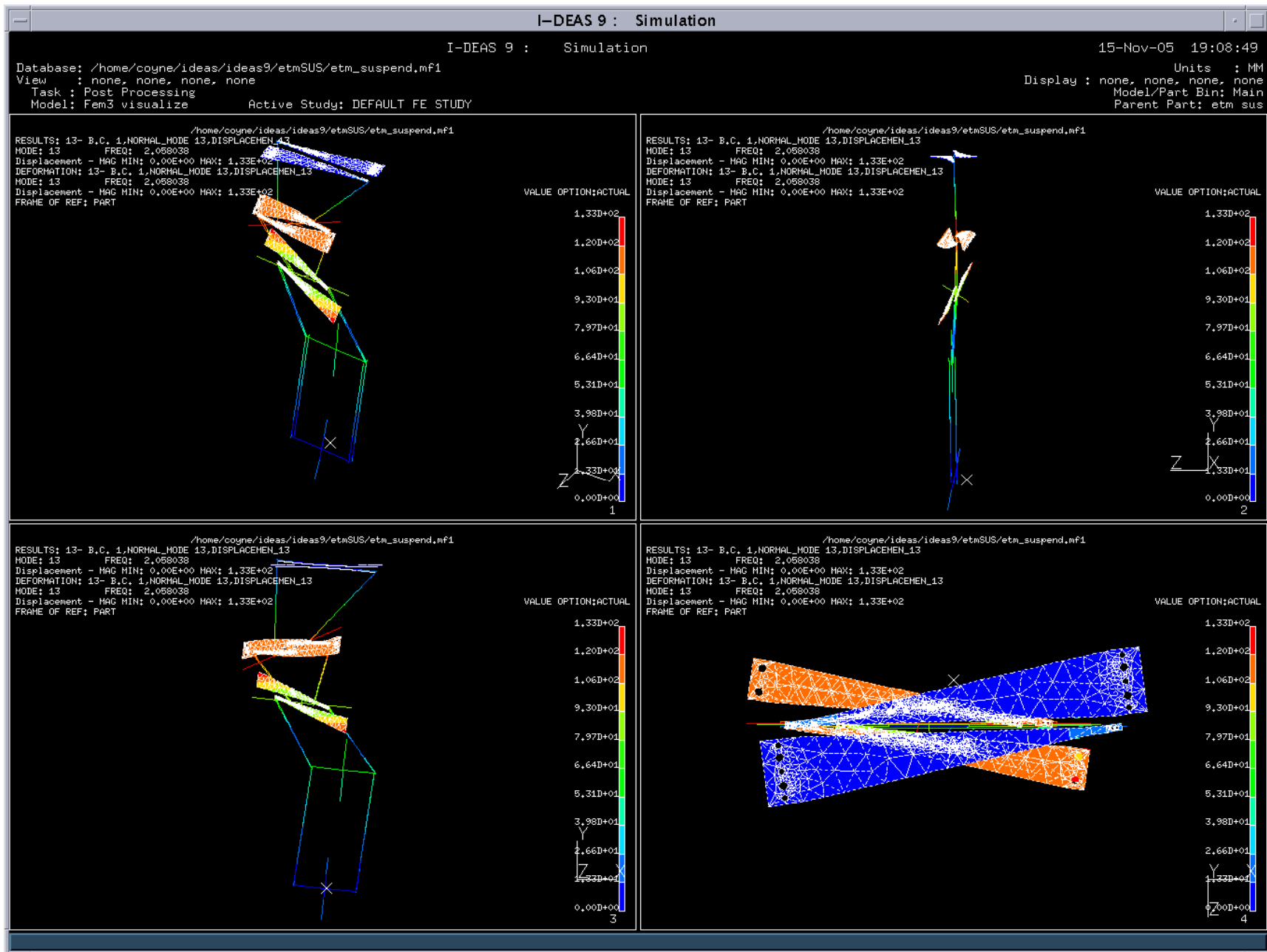


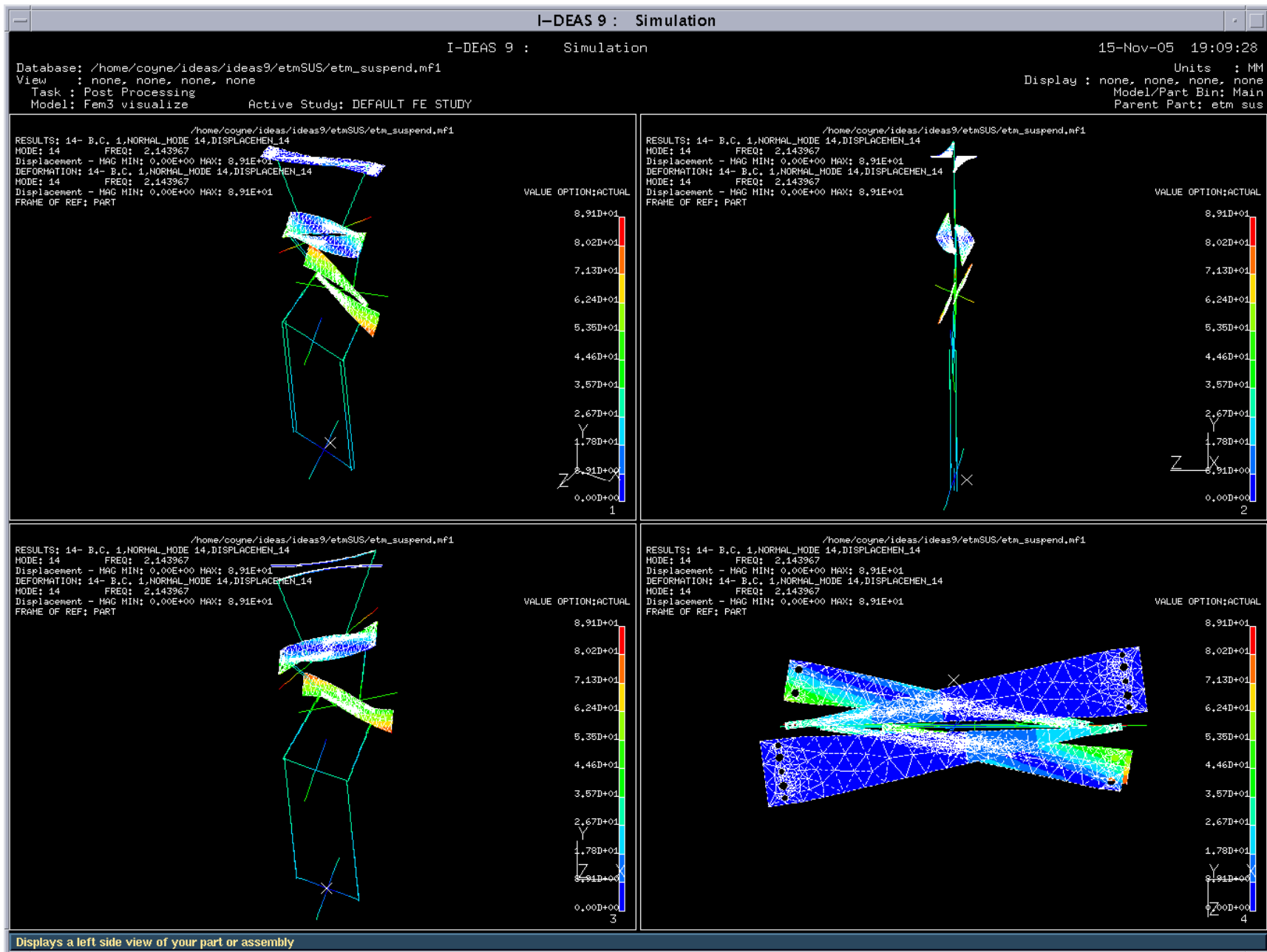


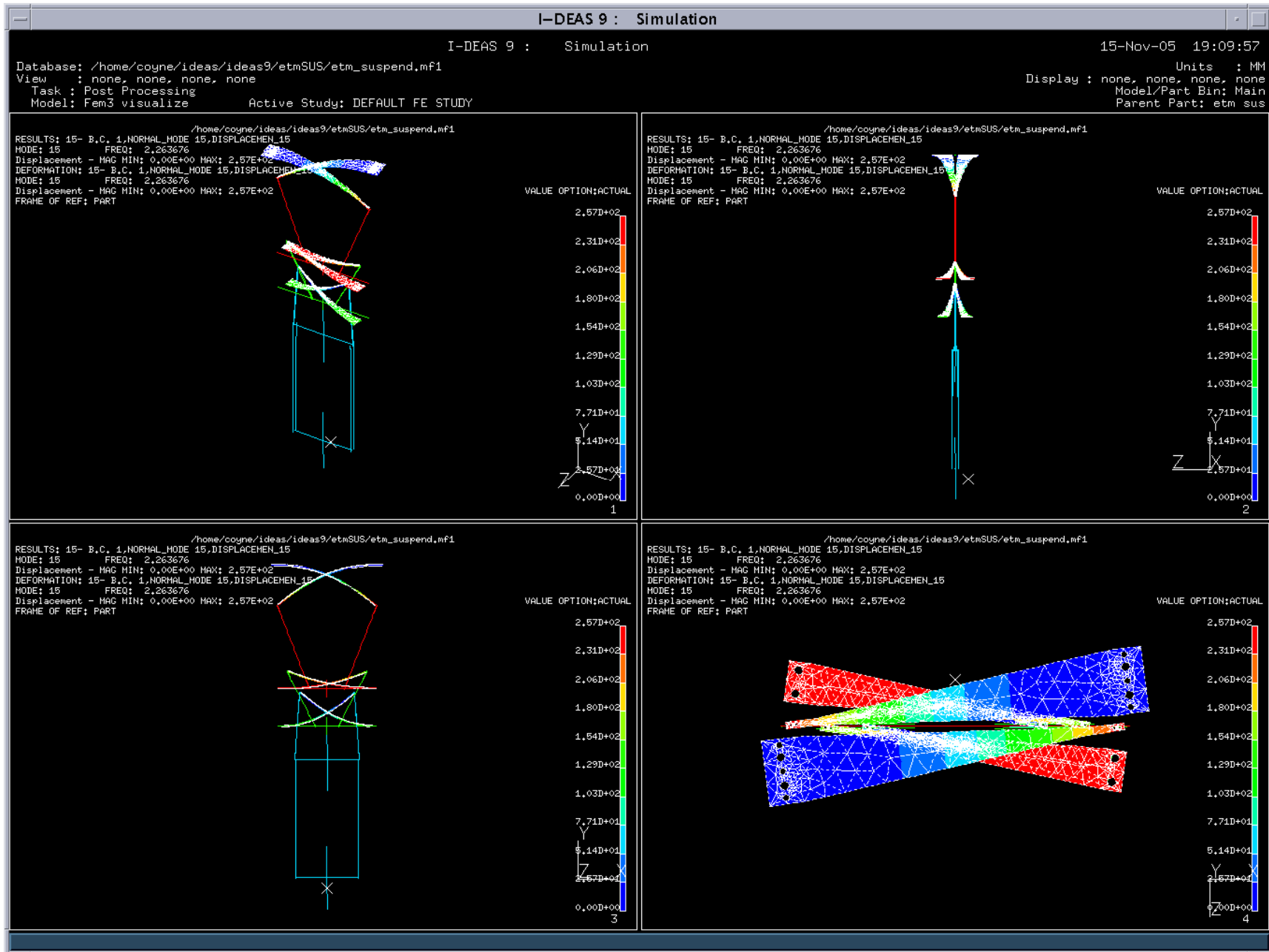


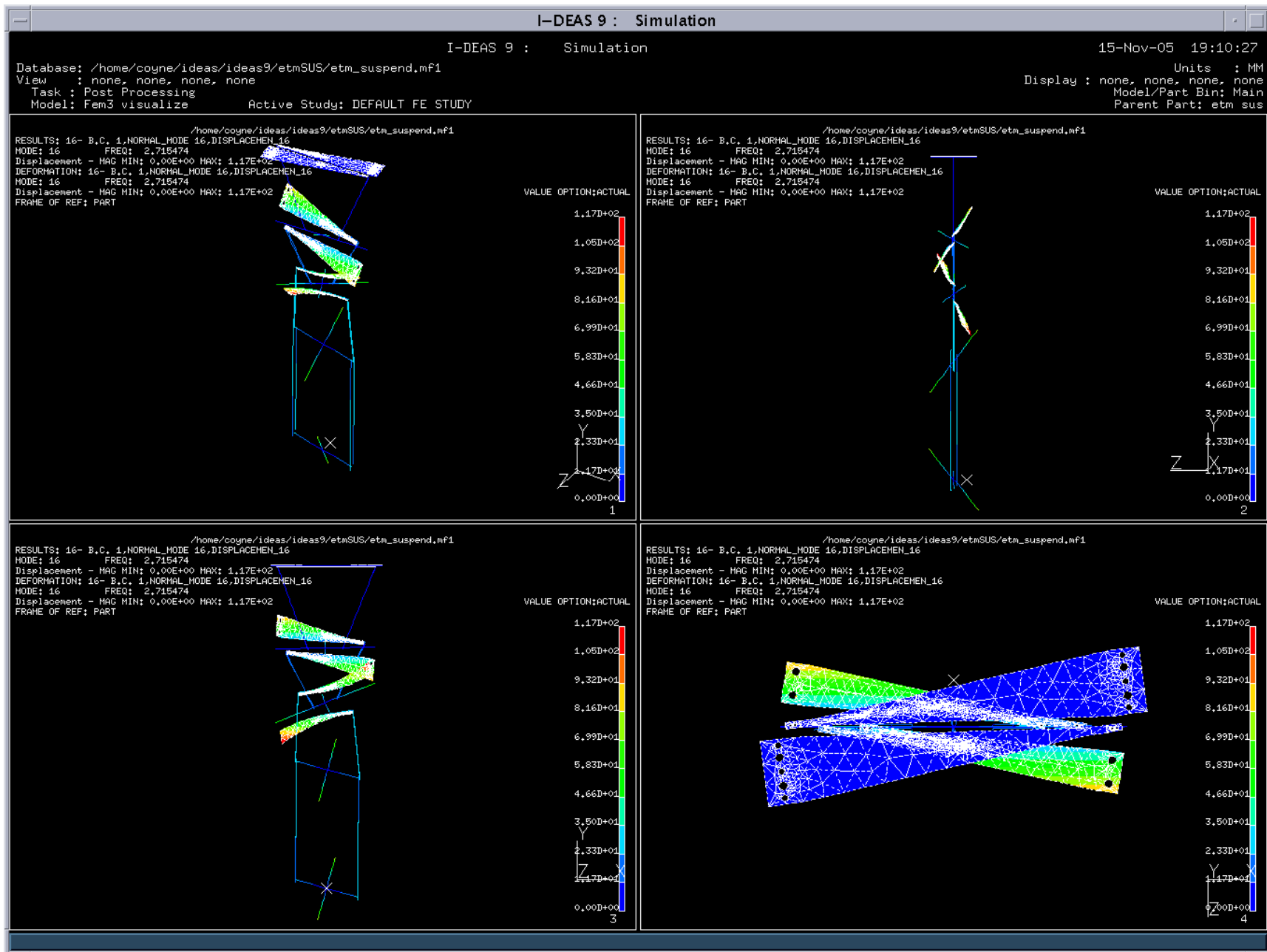


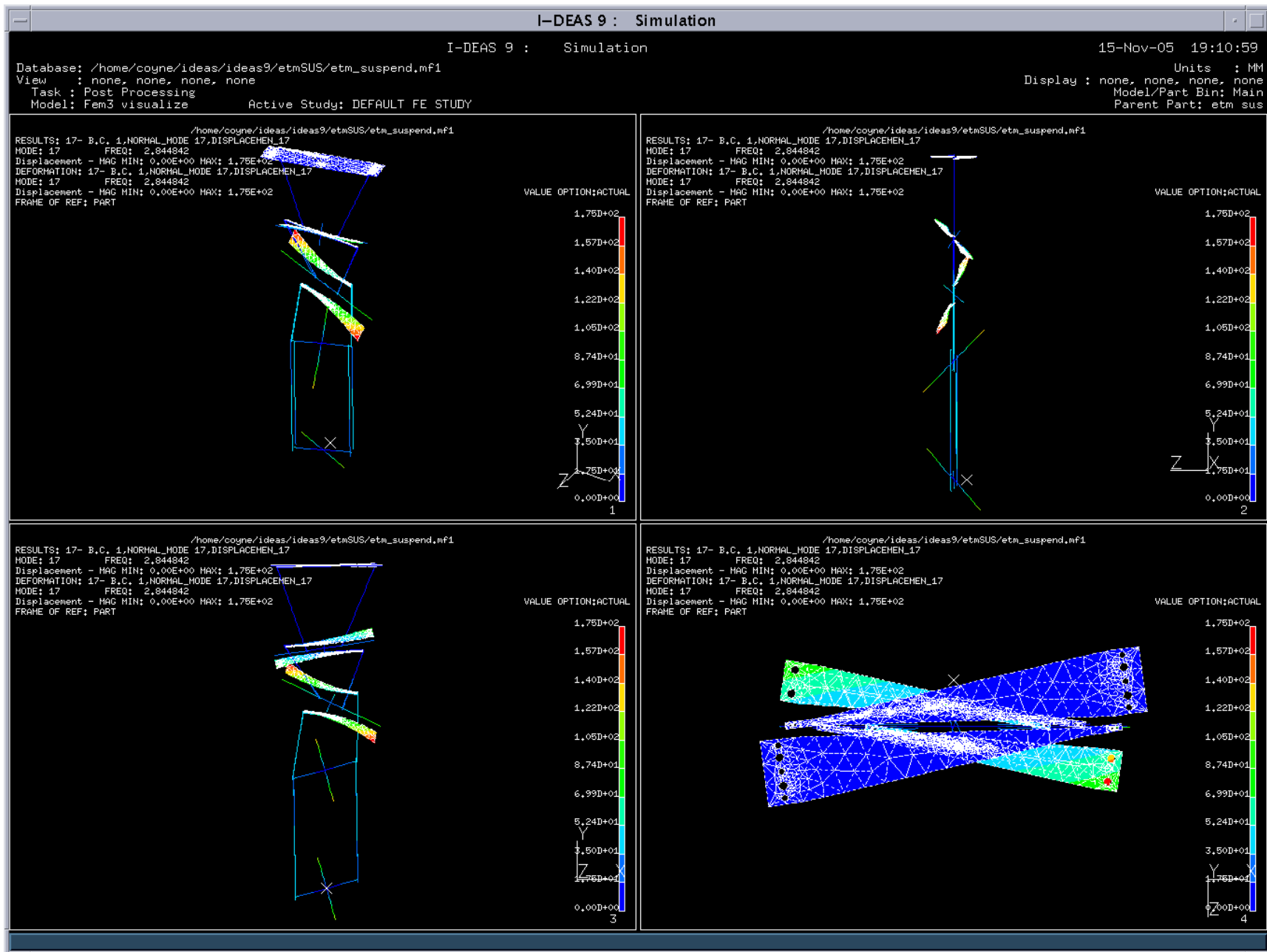


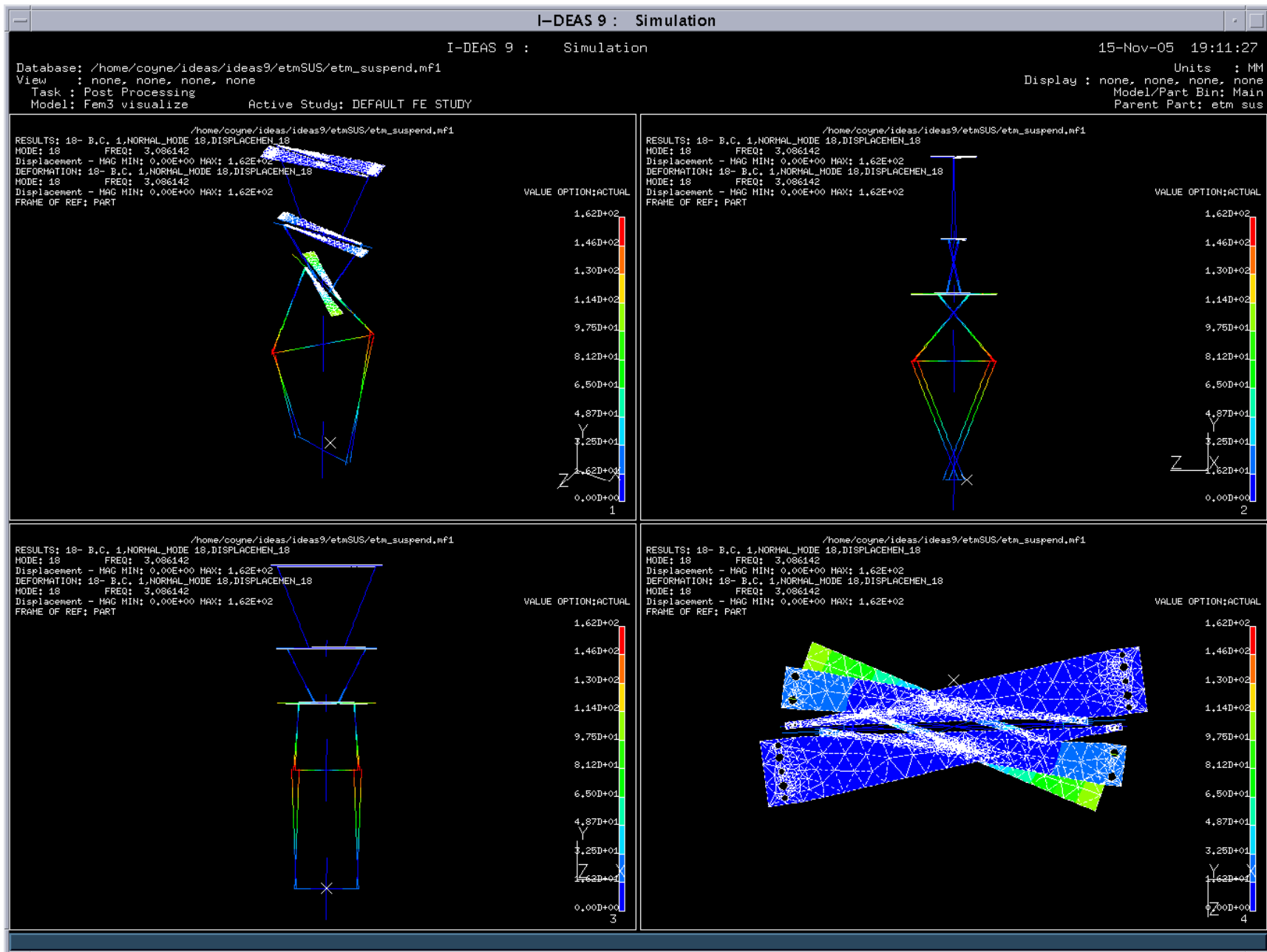


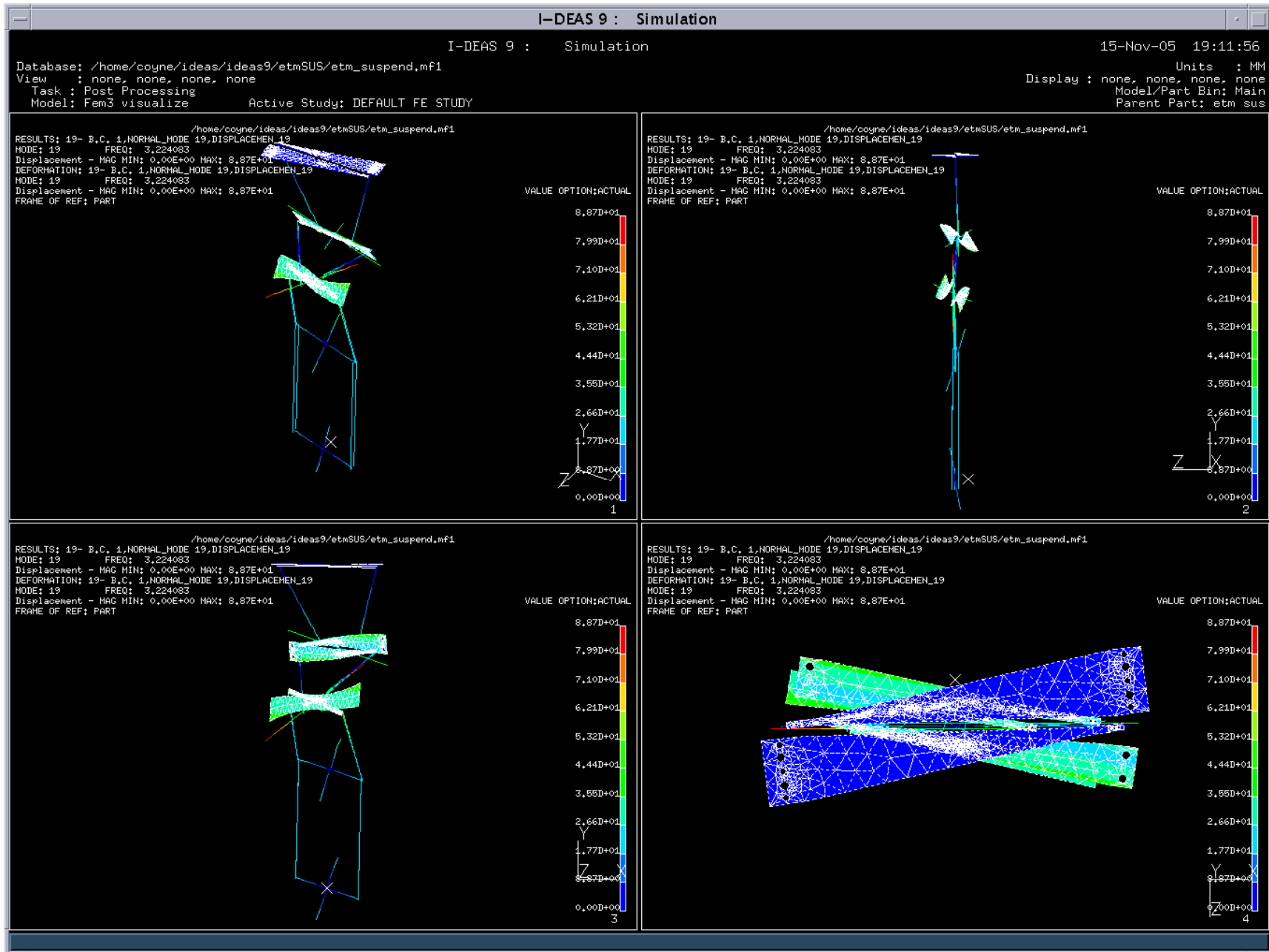


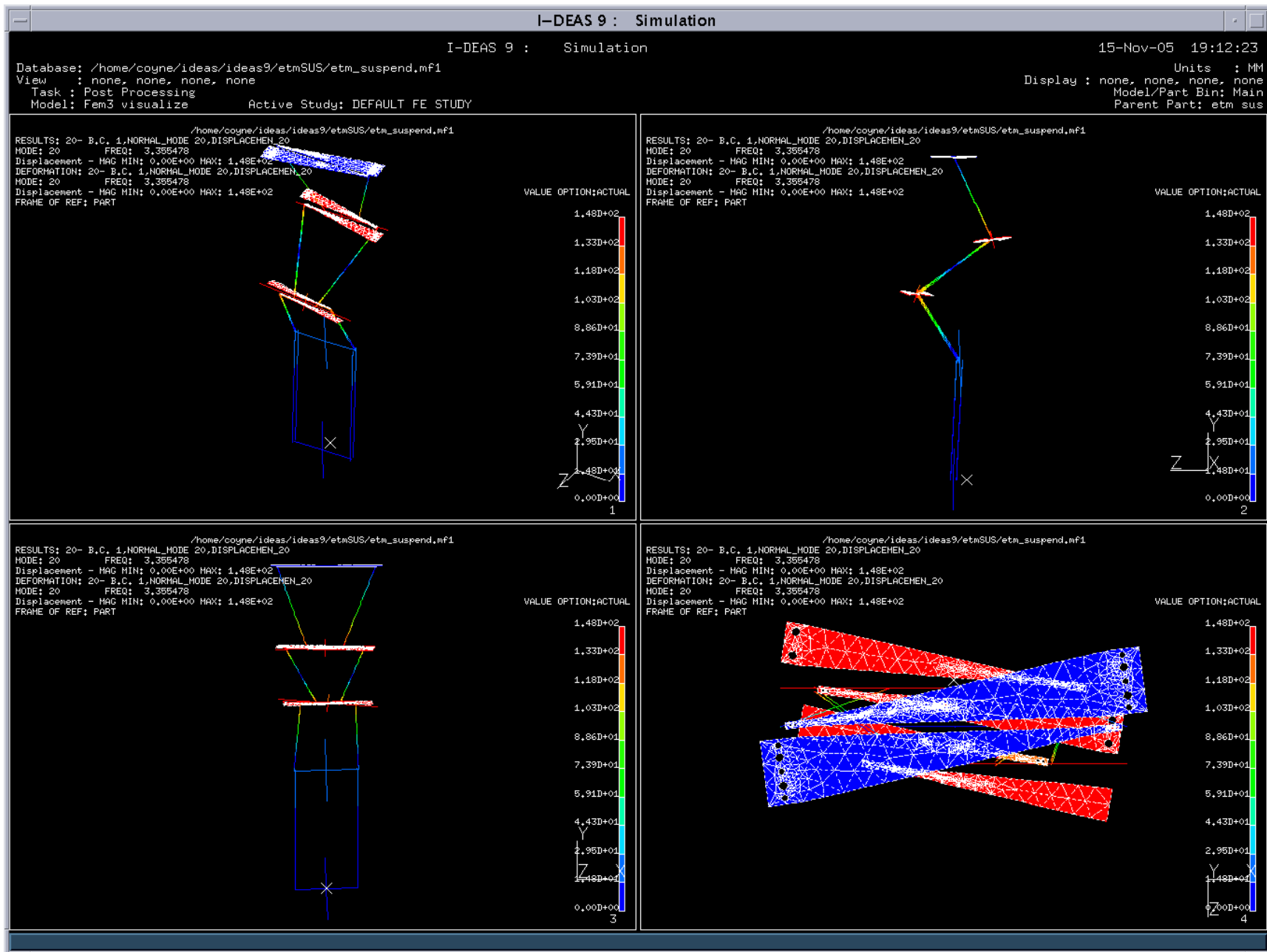


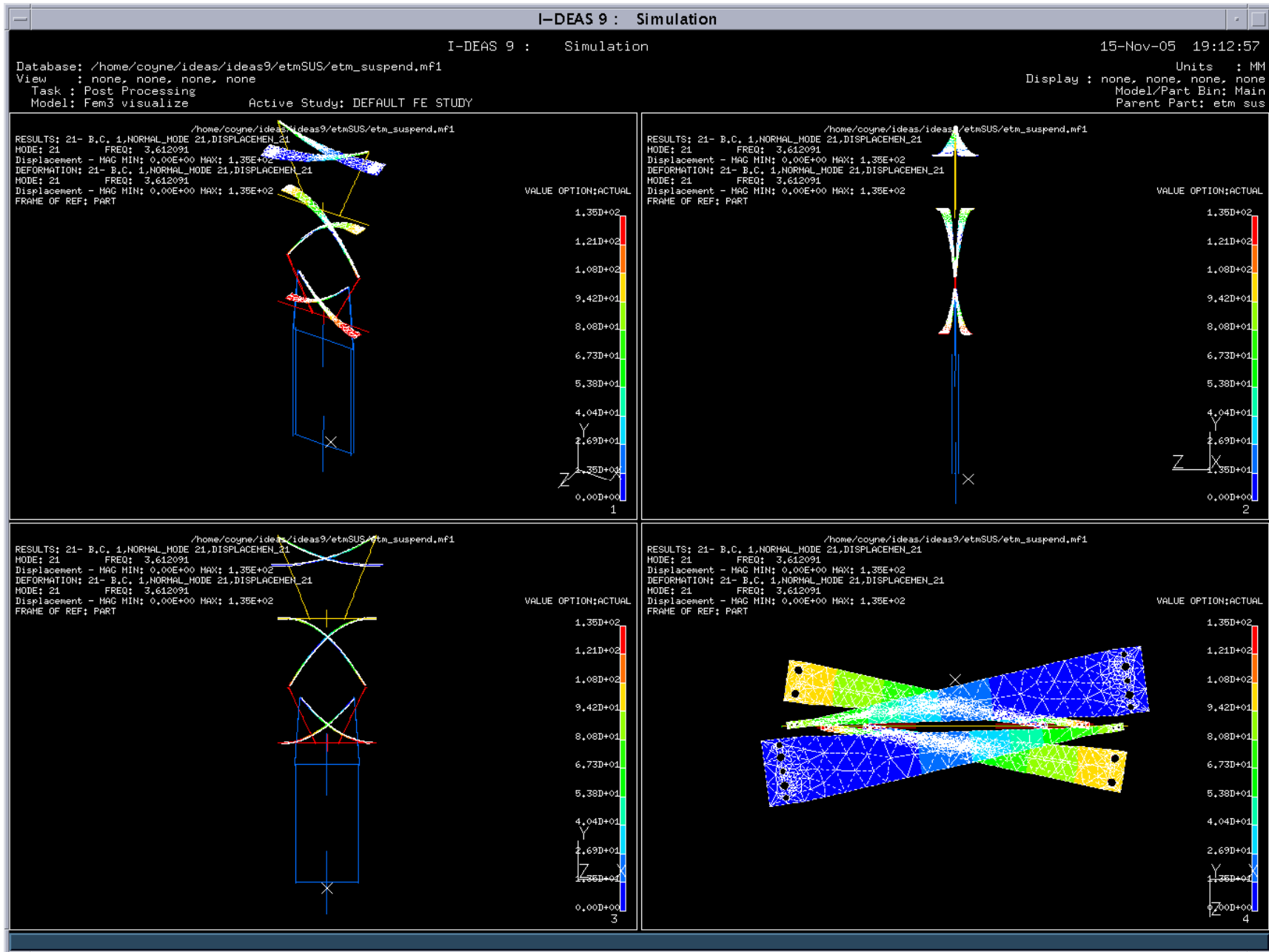


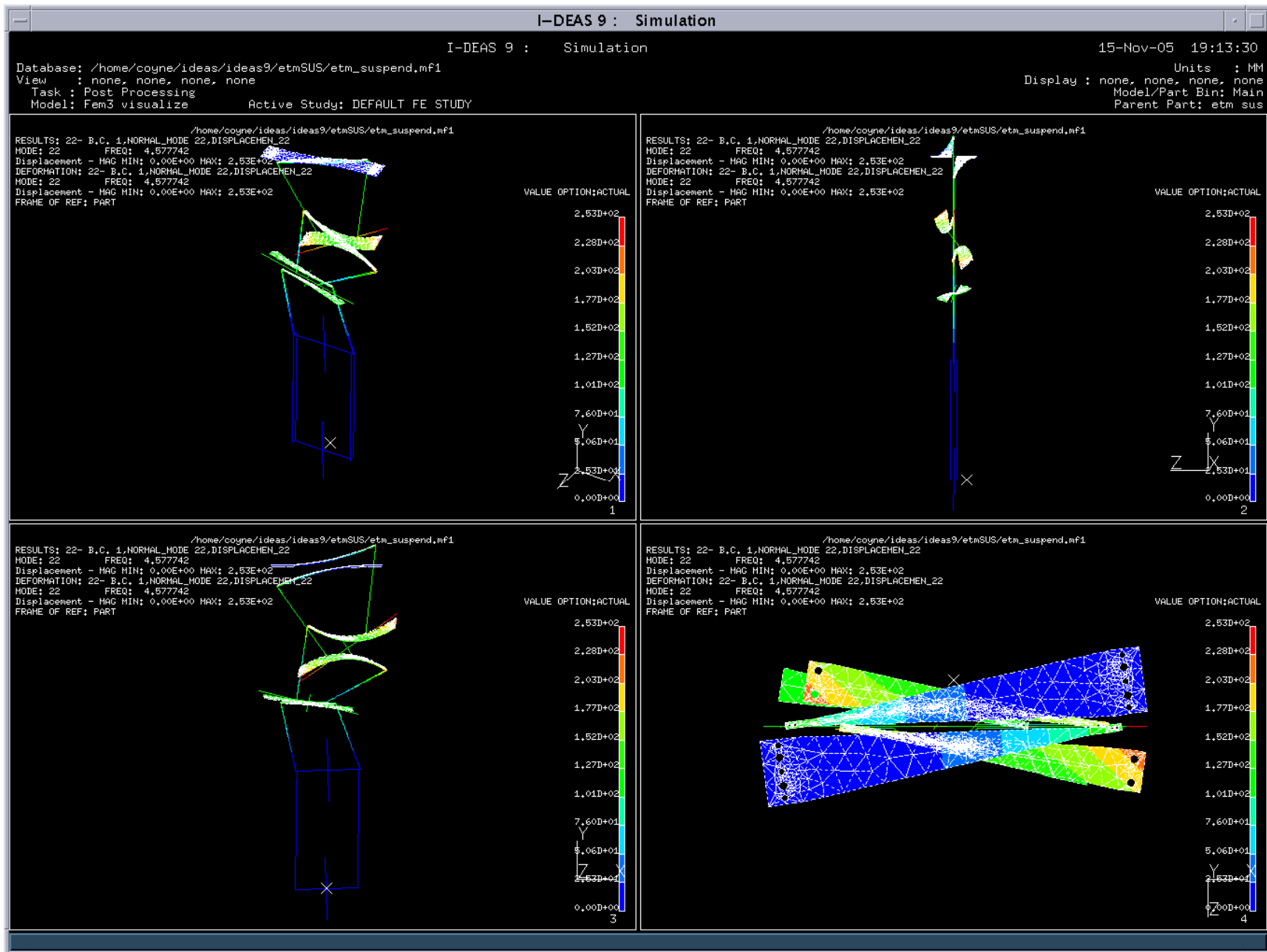


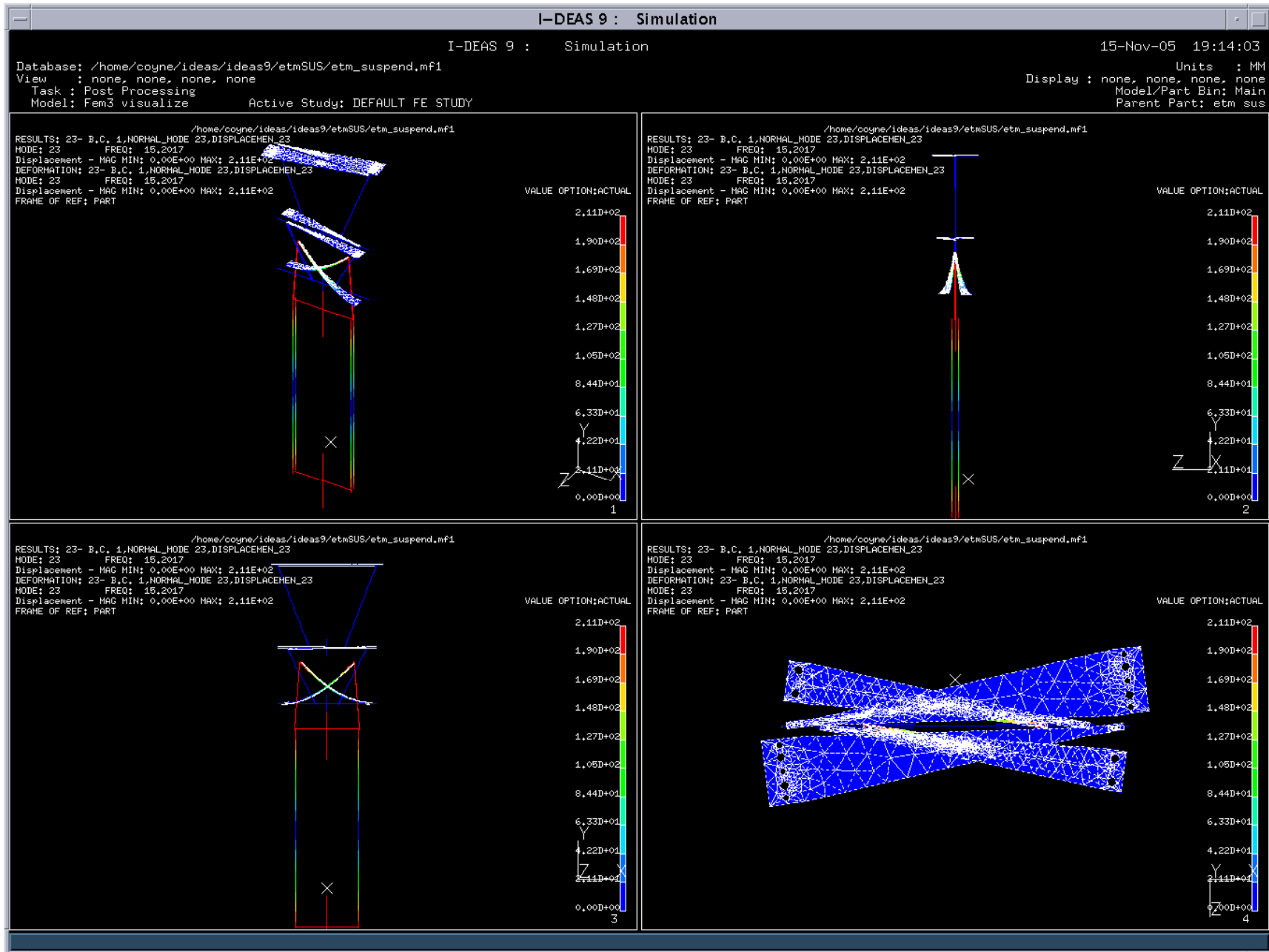


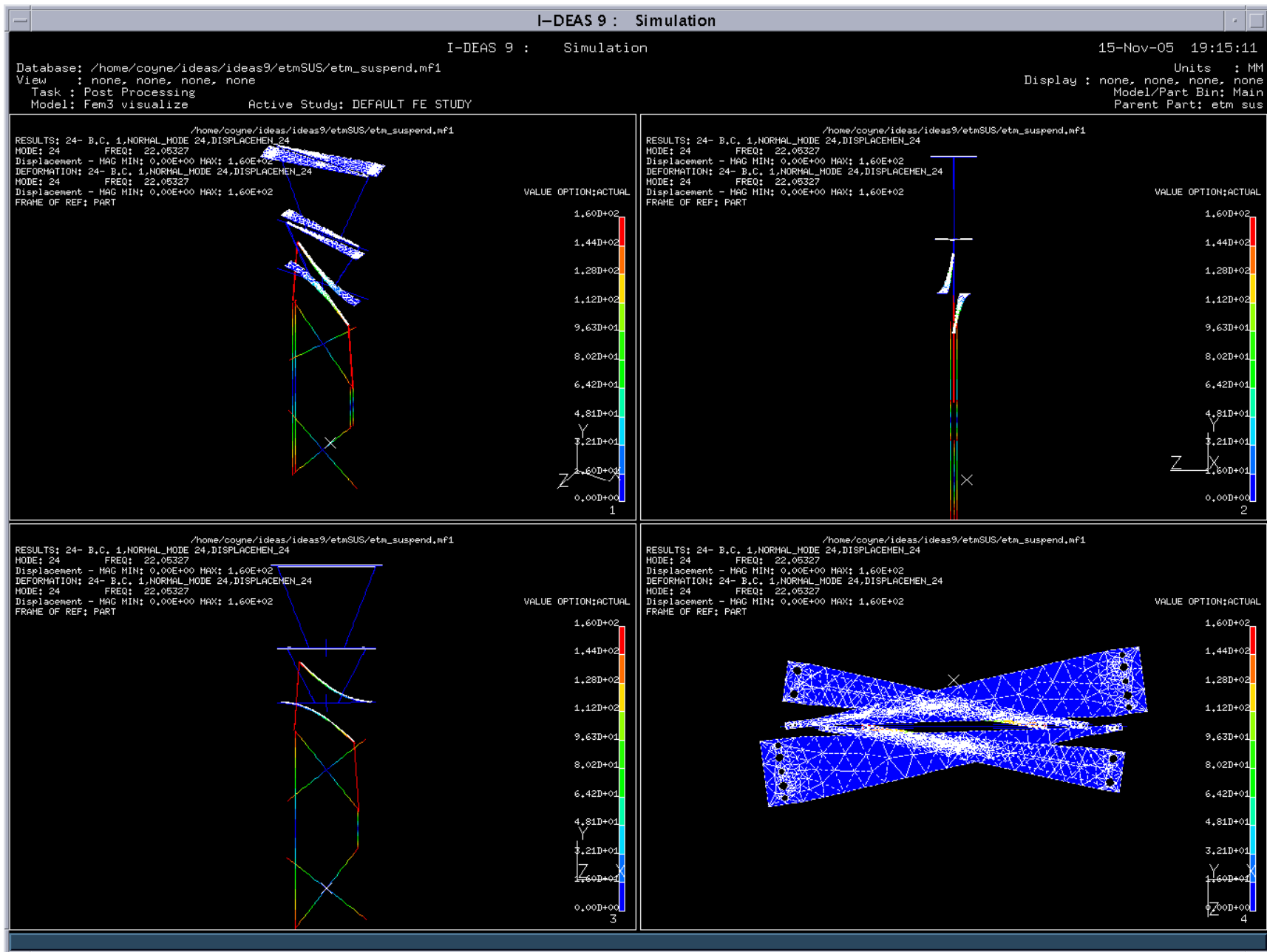




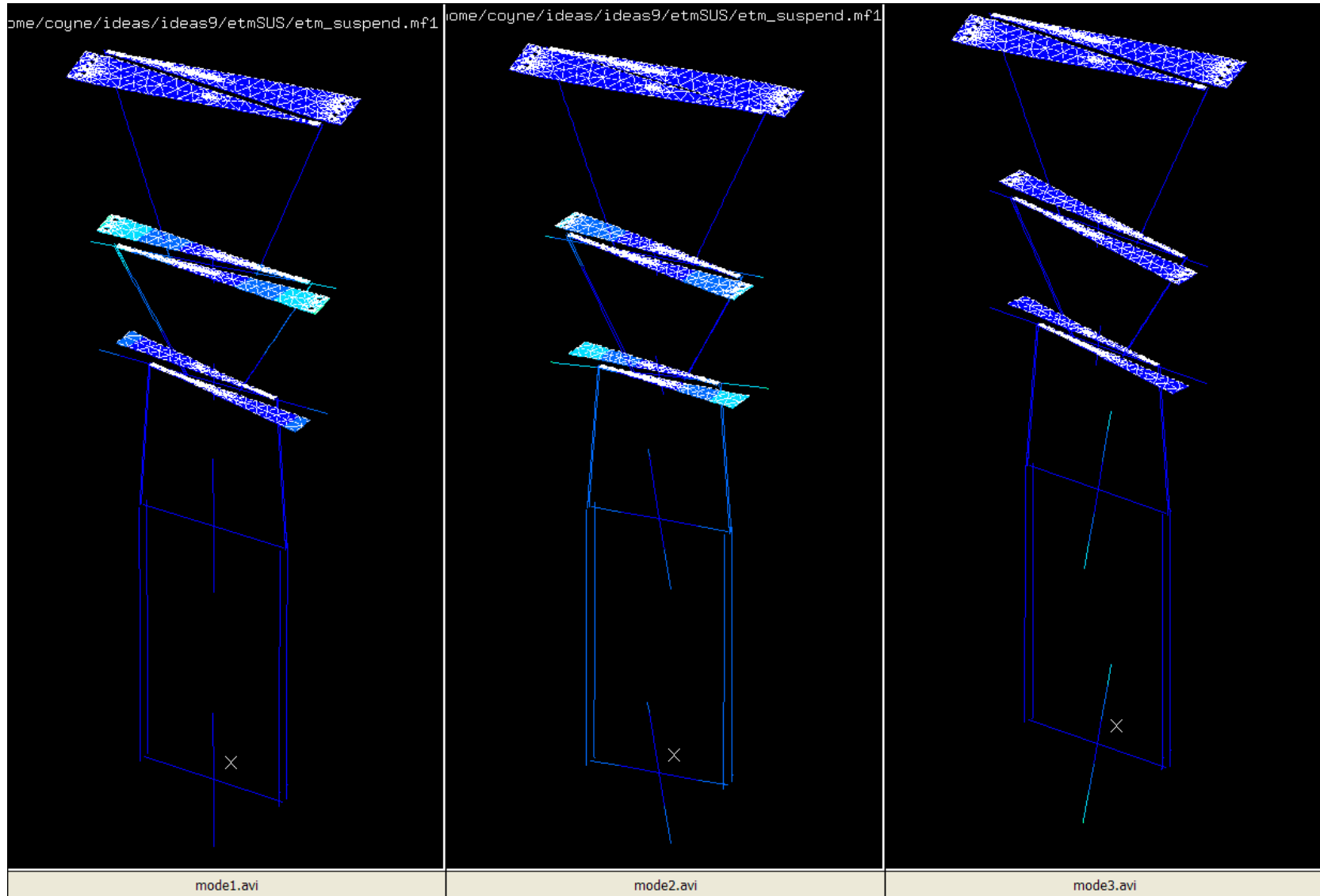


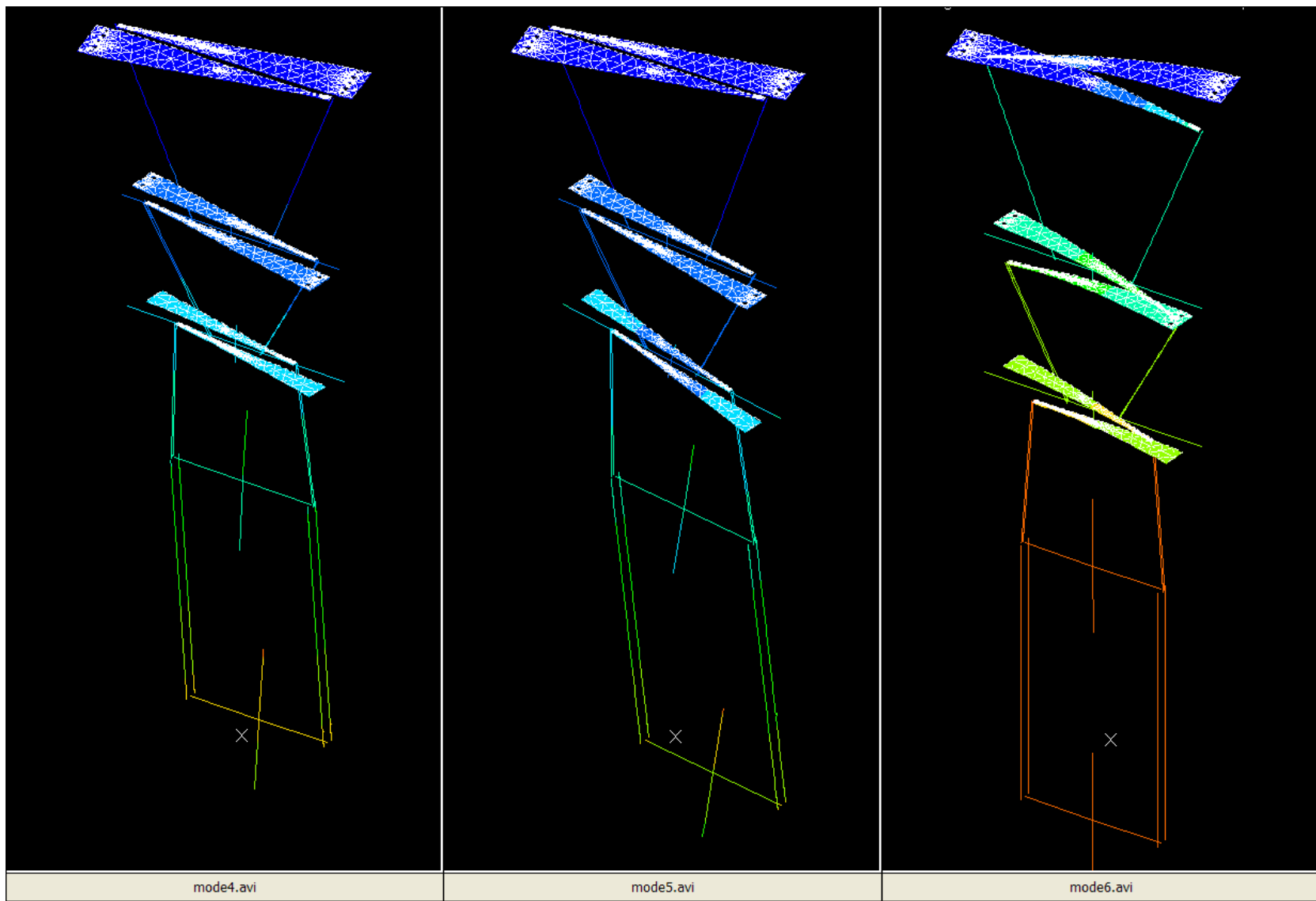


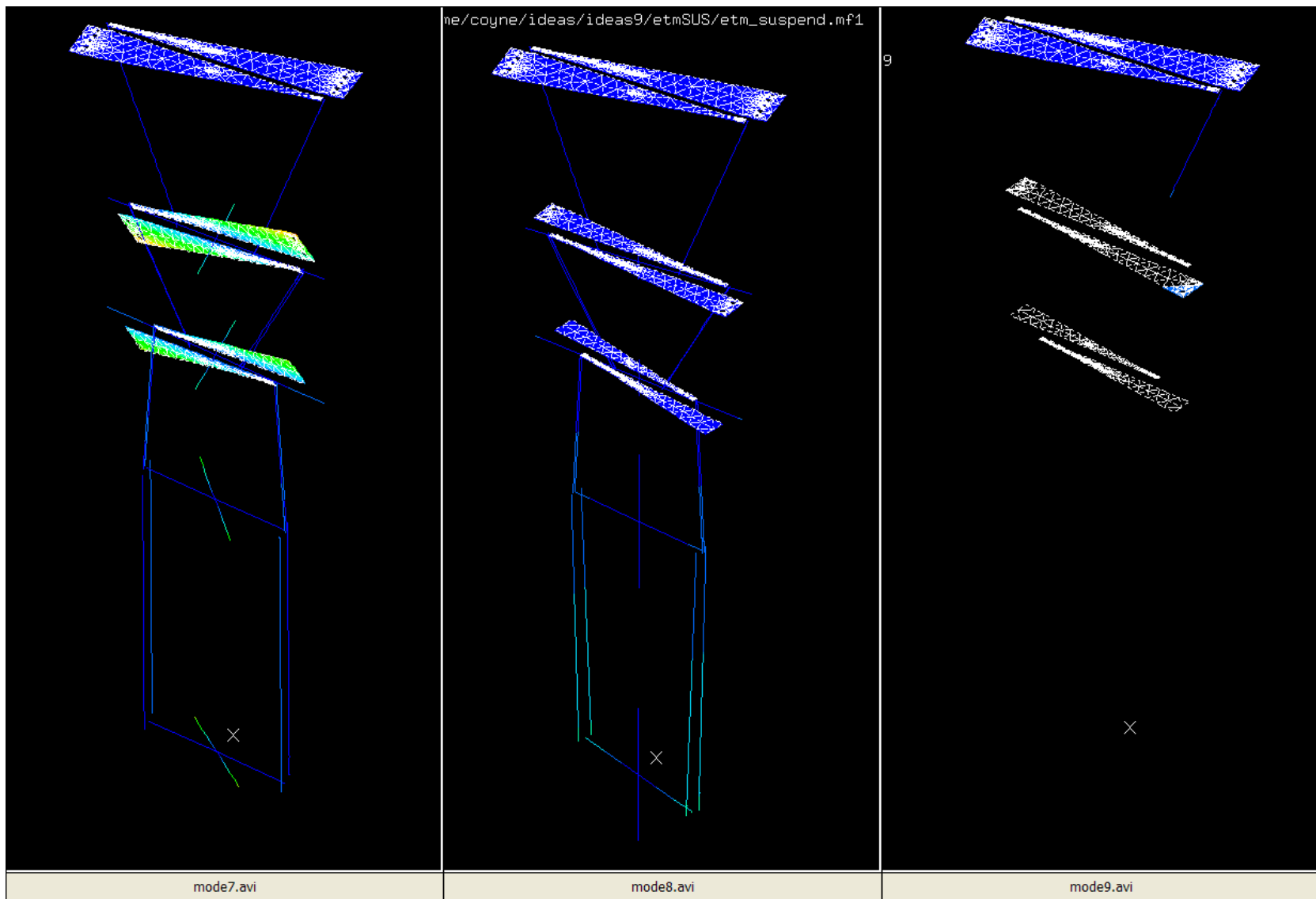


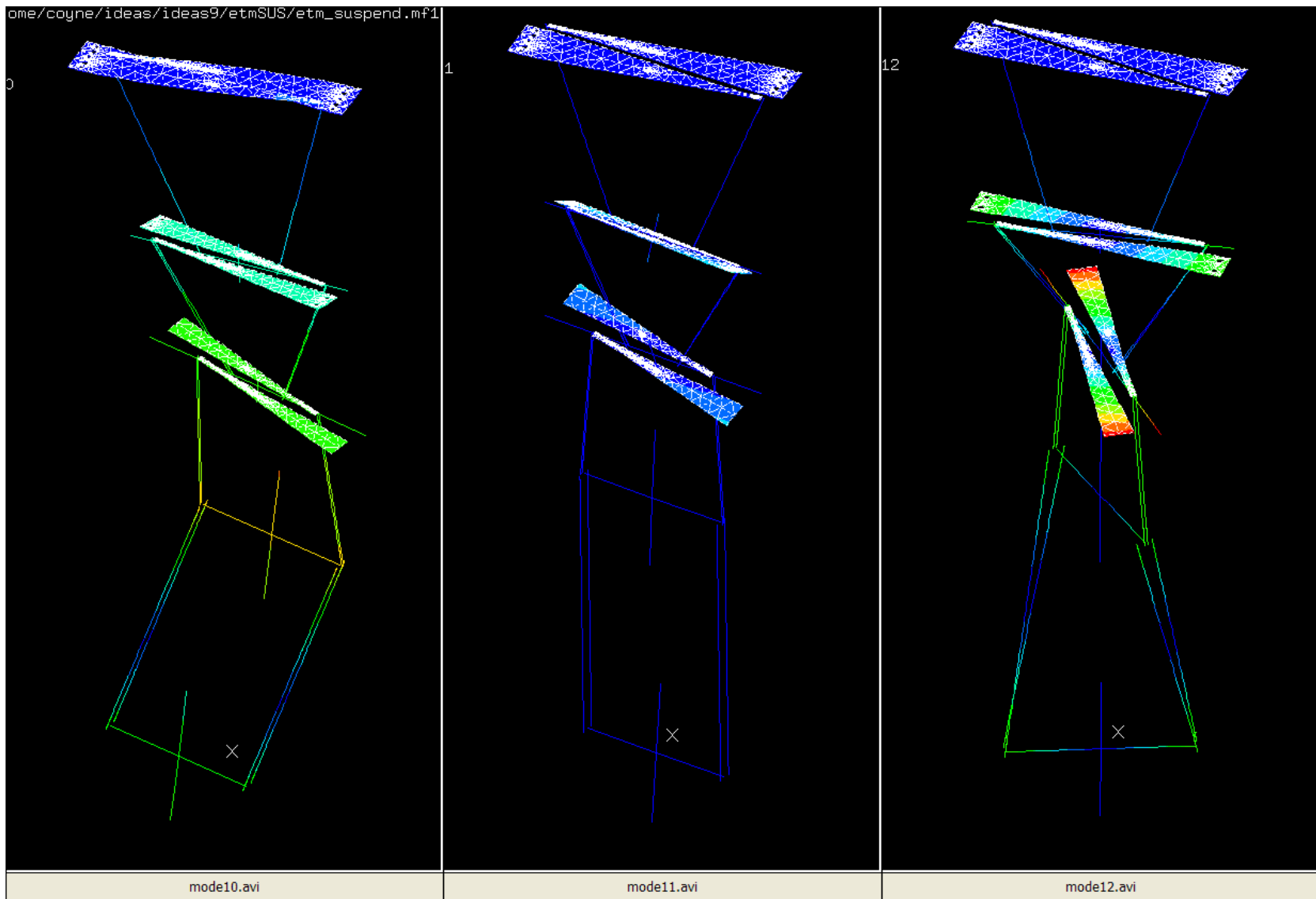


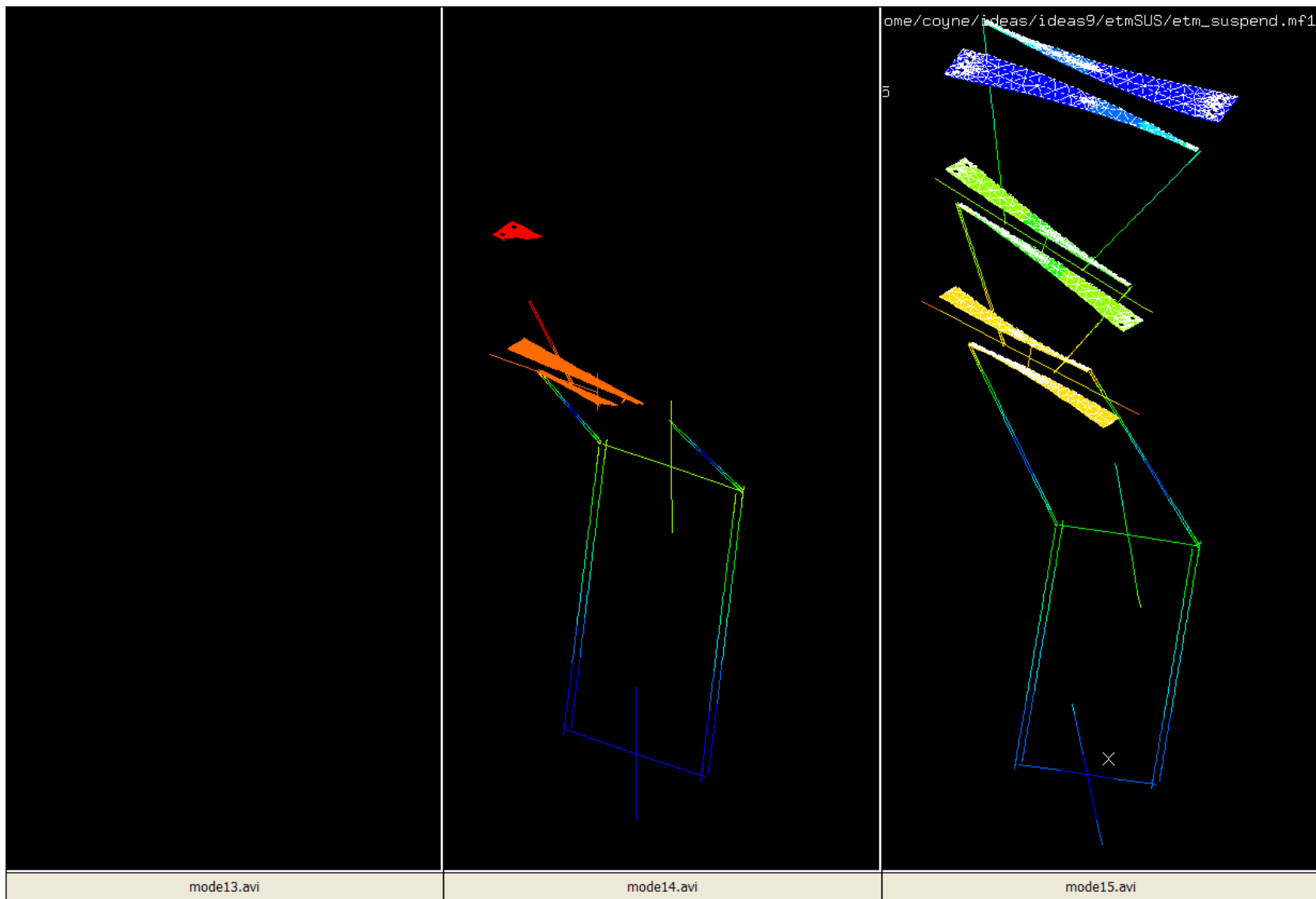
Mode shape movies are given in the following figures (just double click to launch viewer).

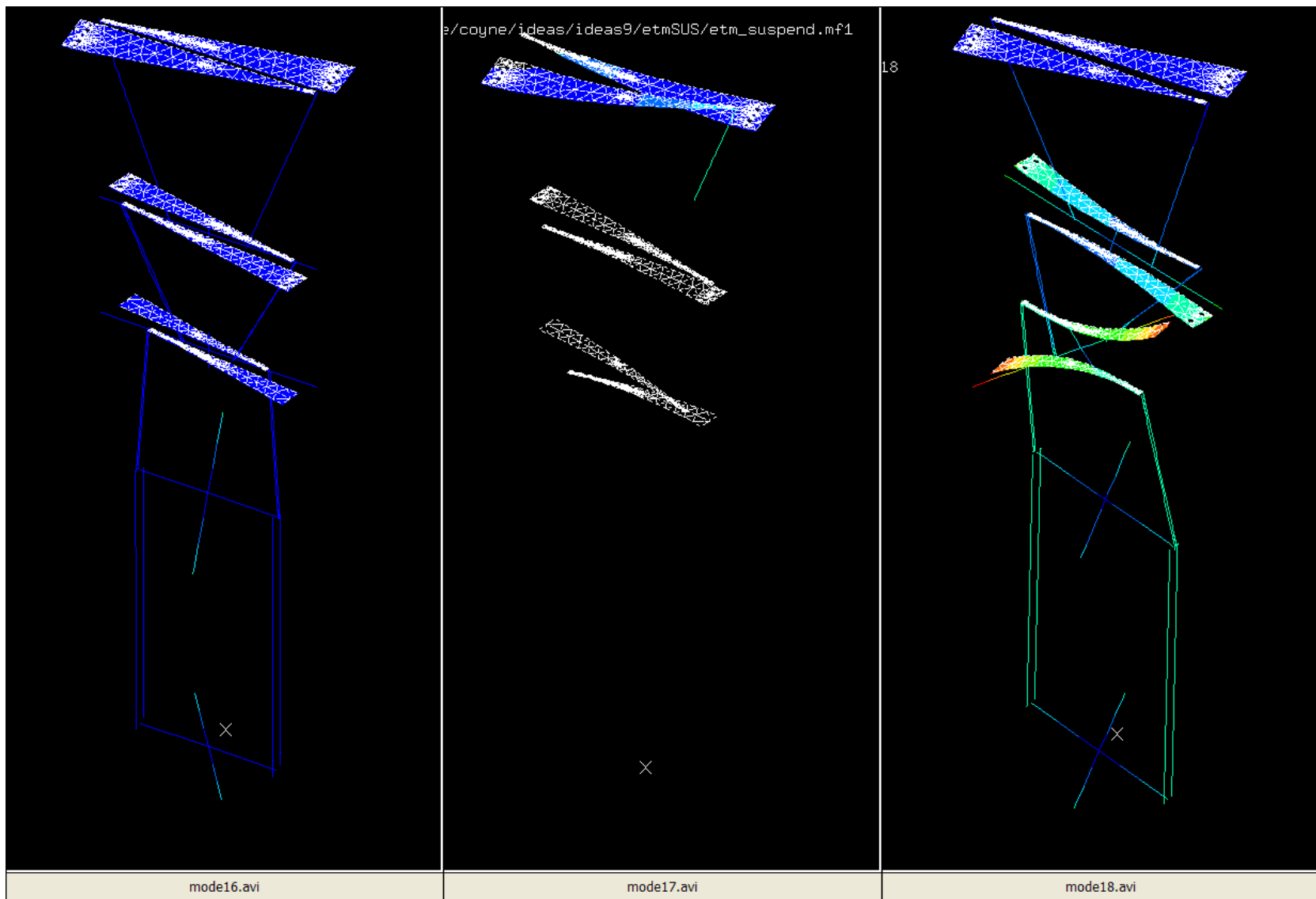


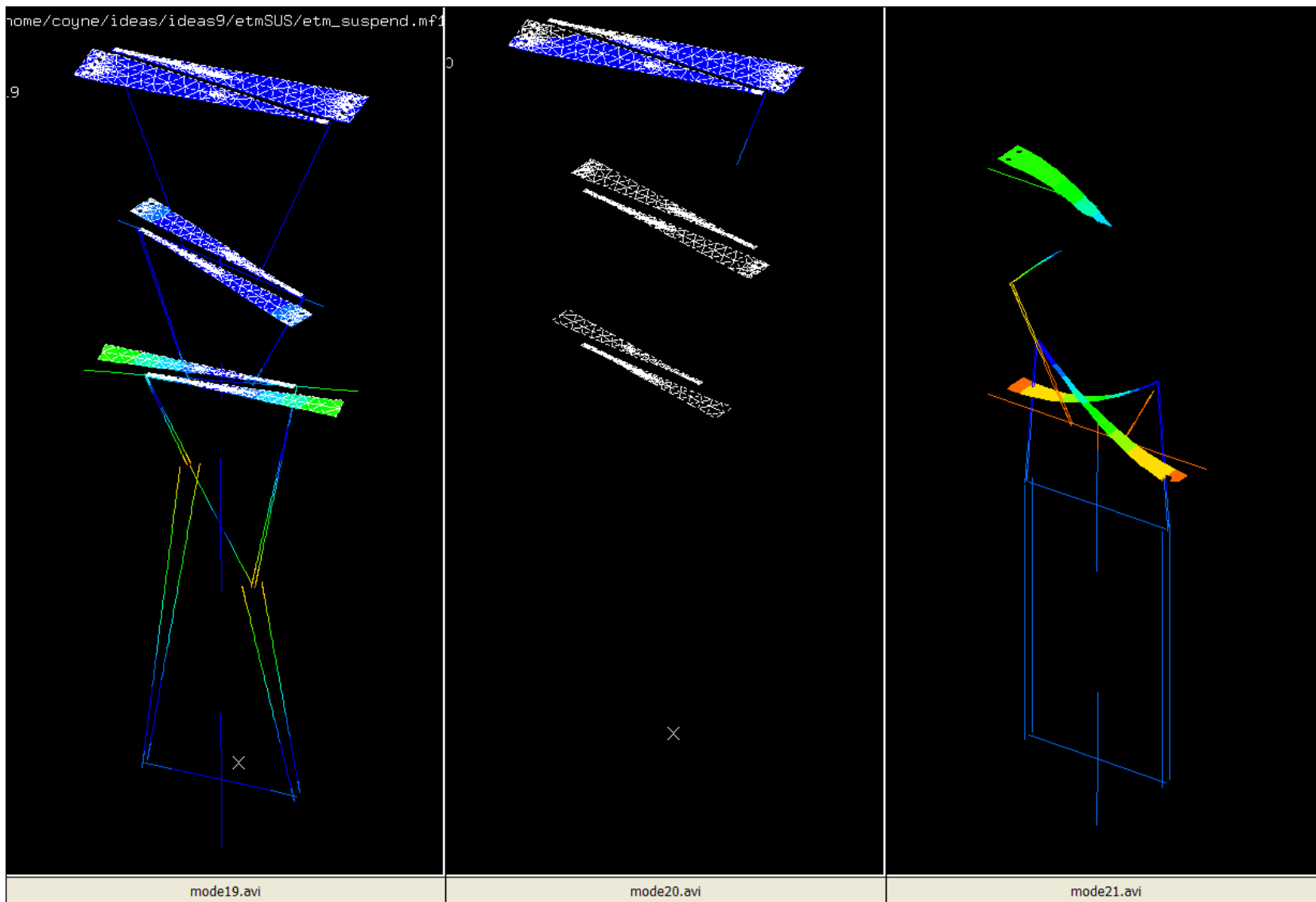








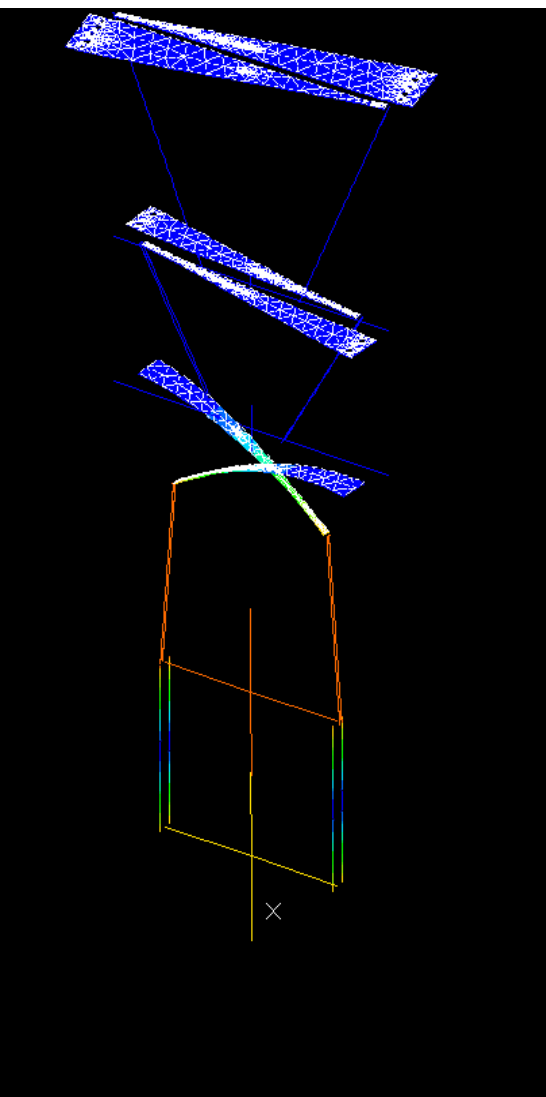




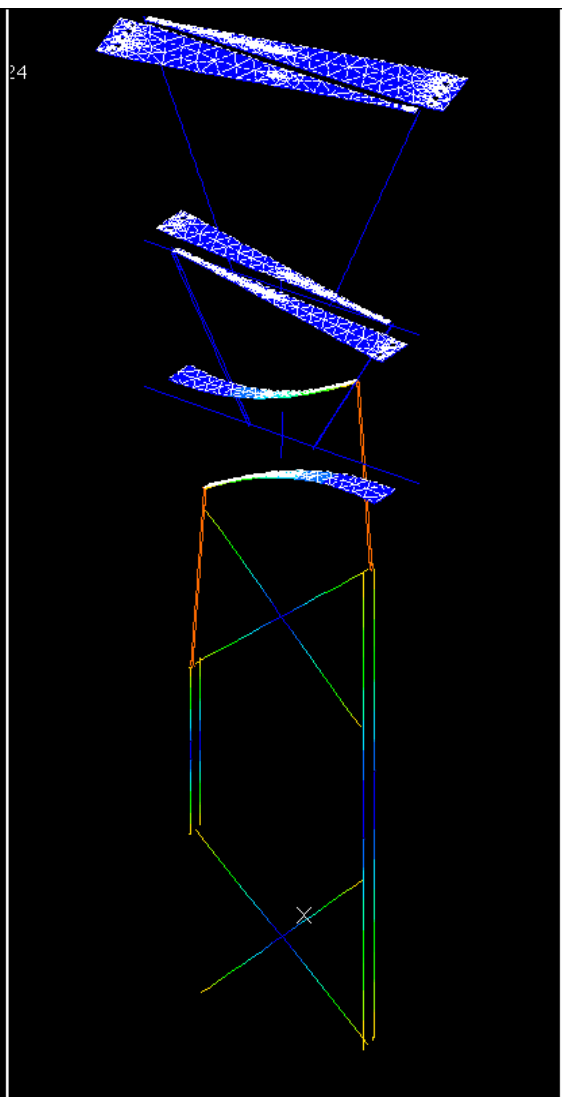
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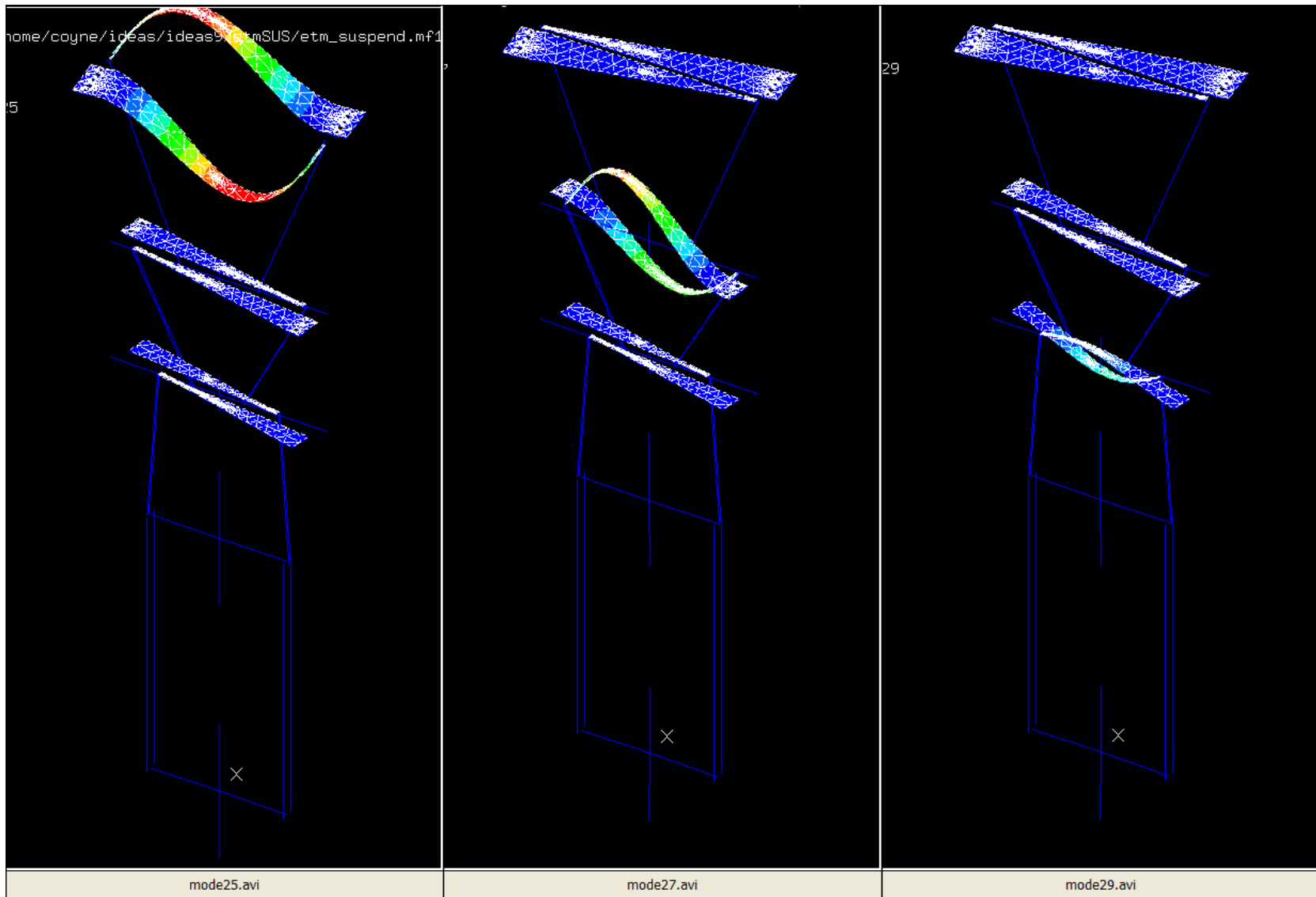
mode22.avi

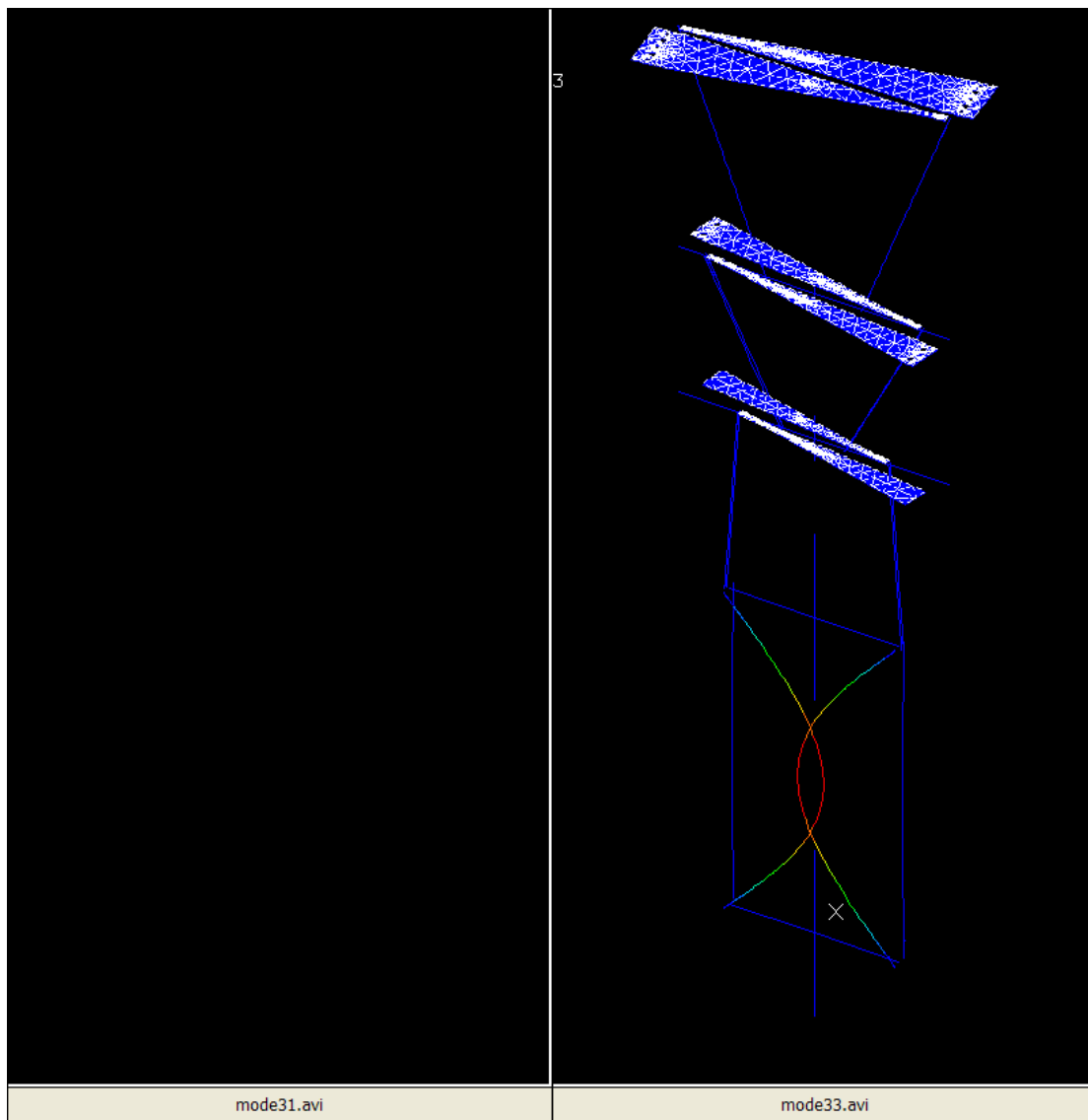


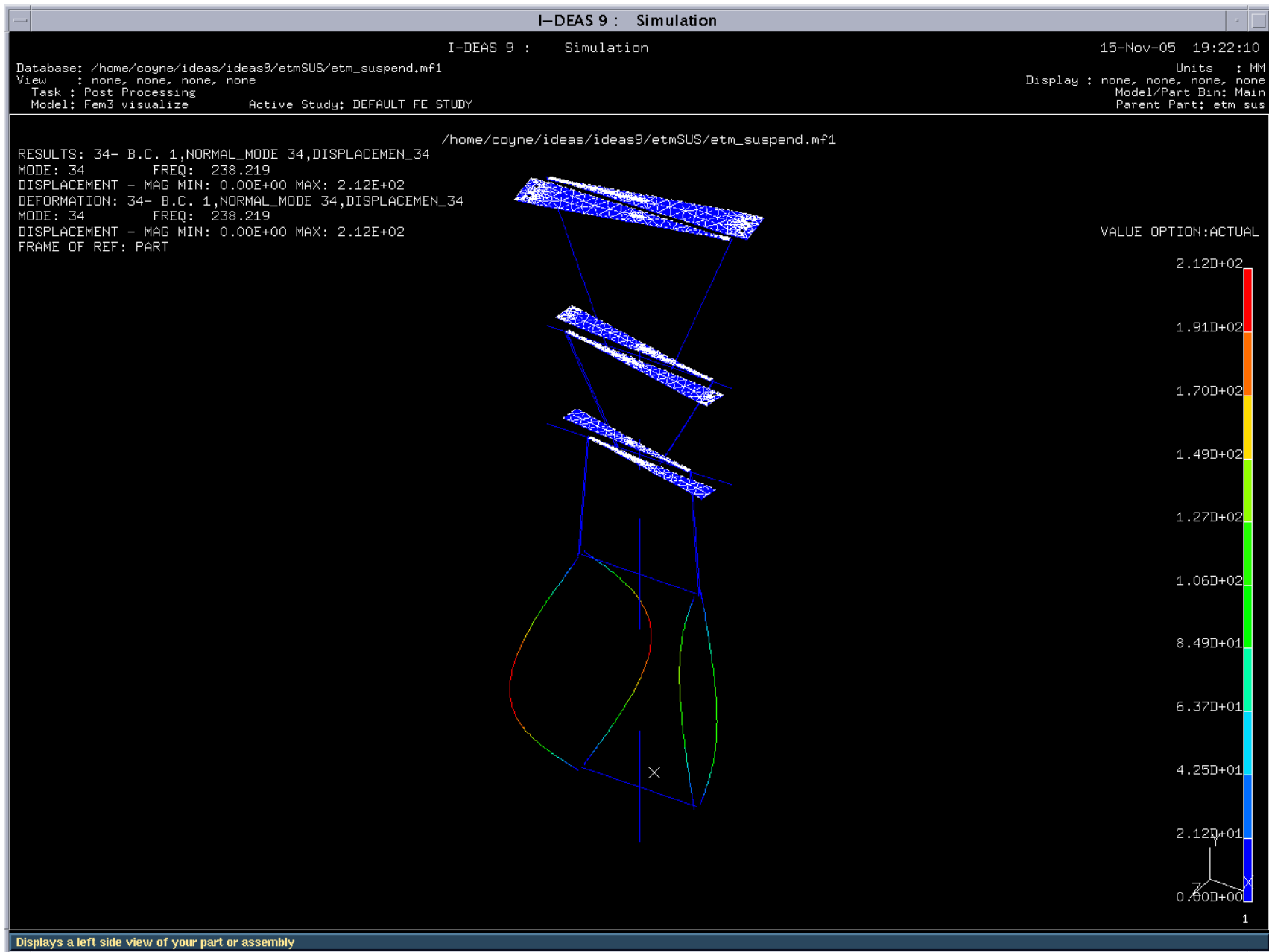
mode23.avi

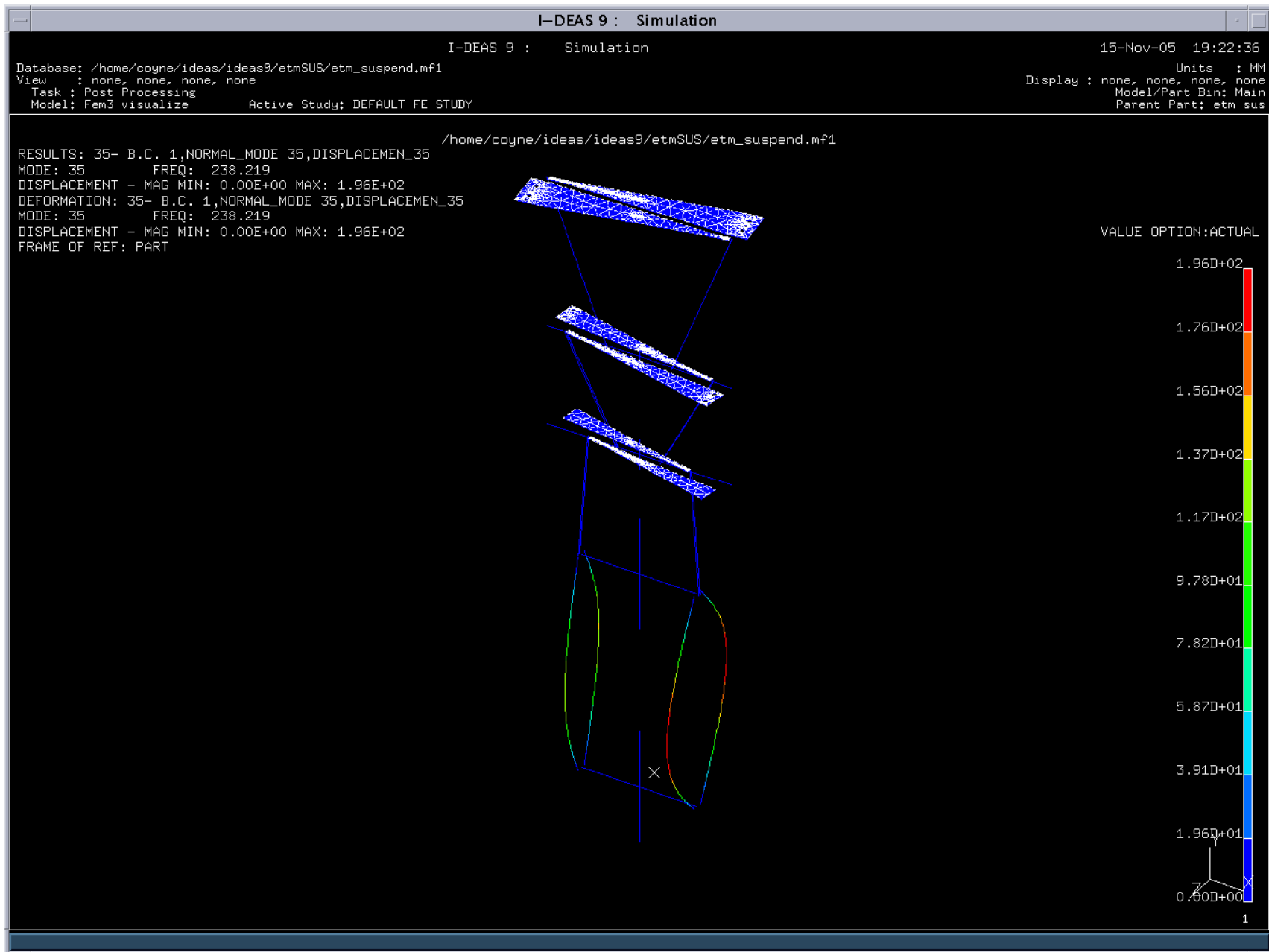


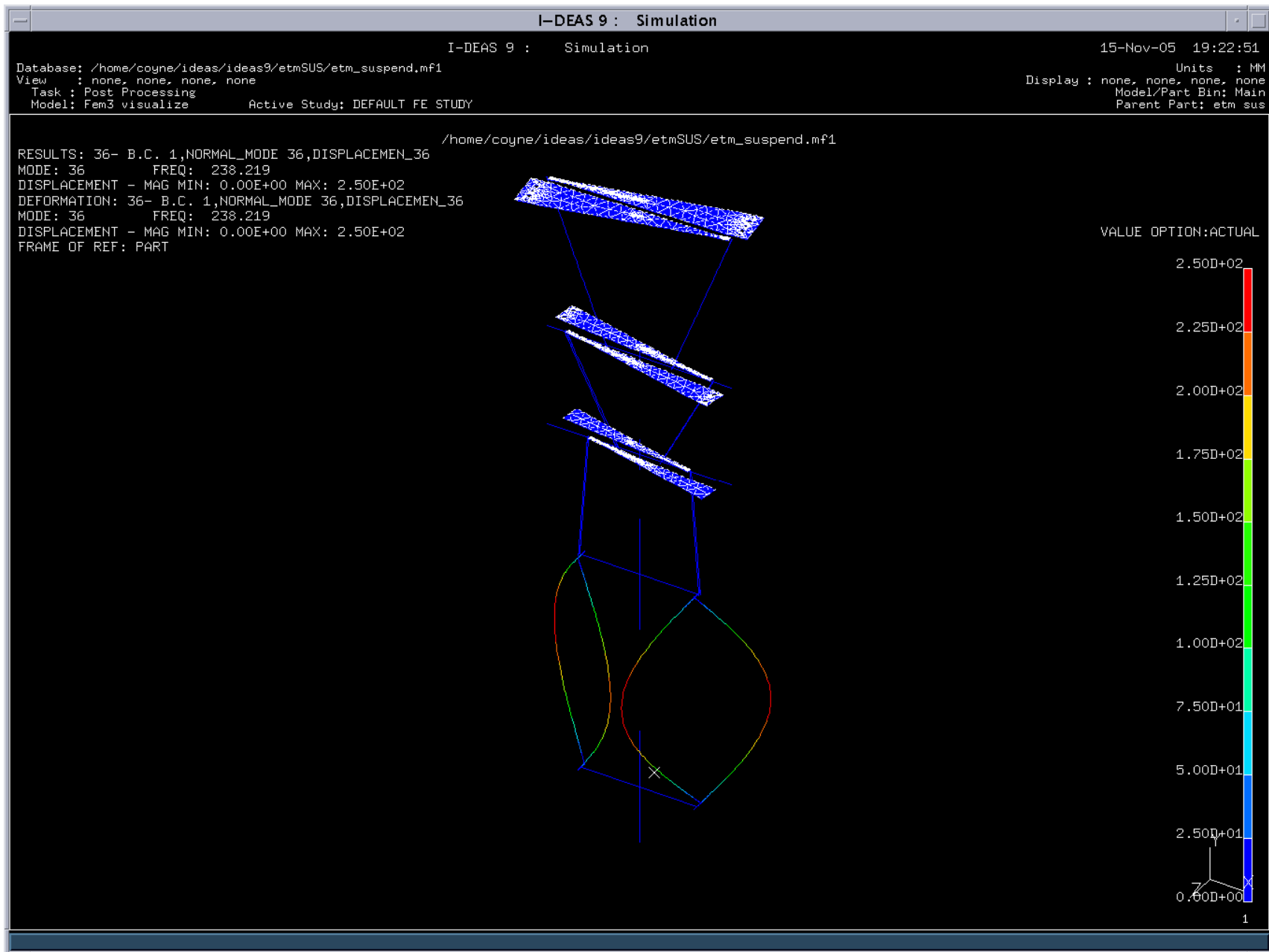
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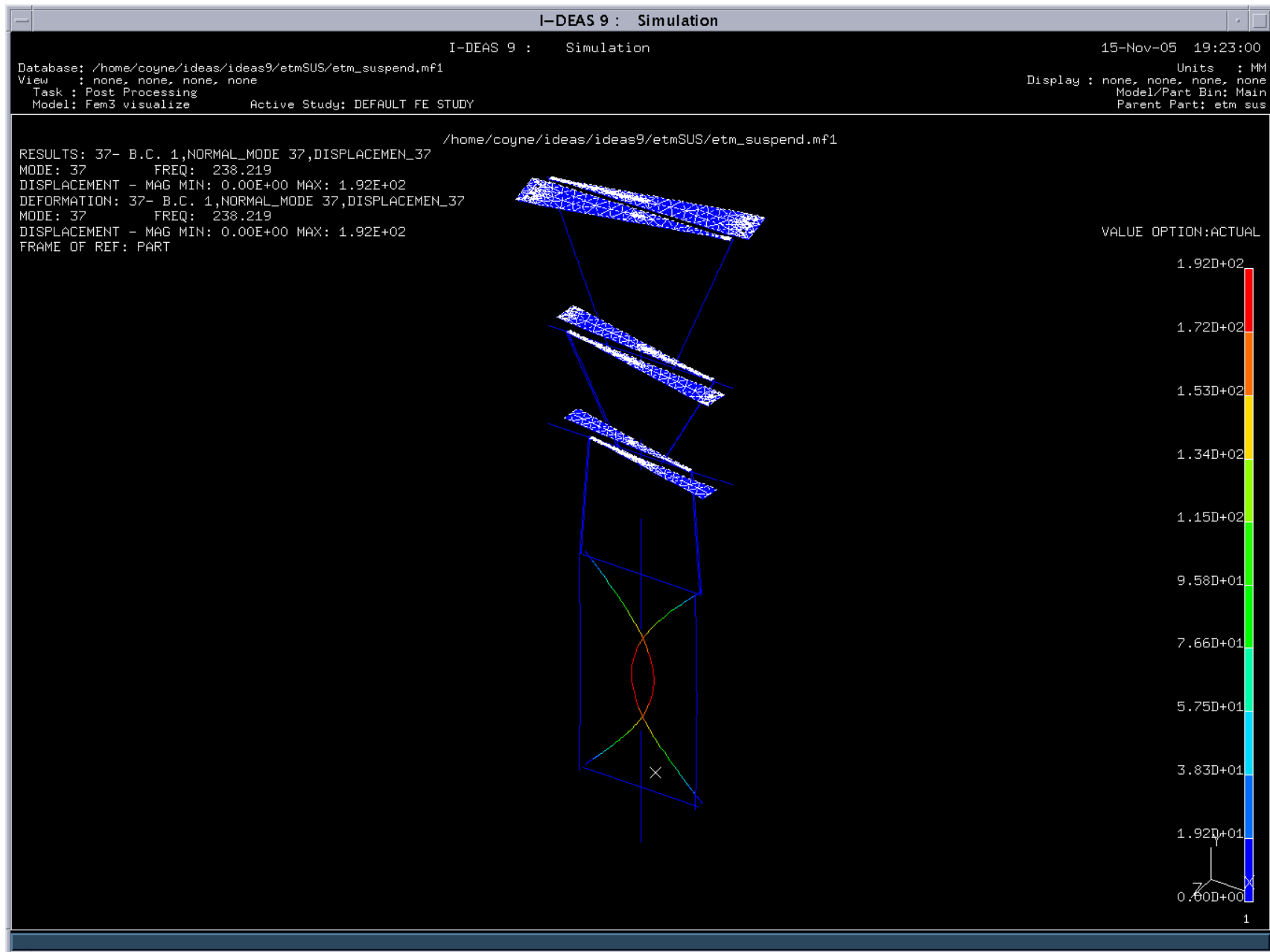


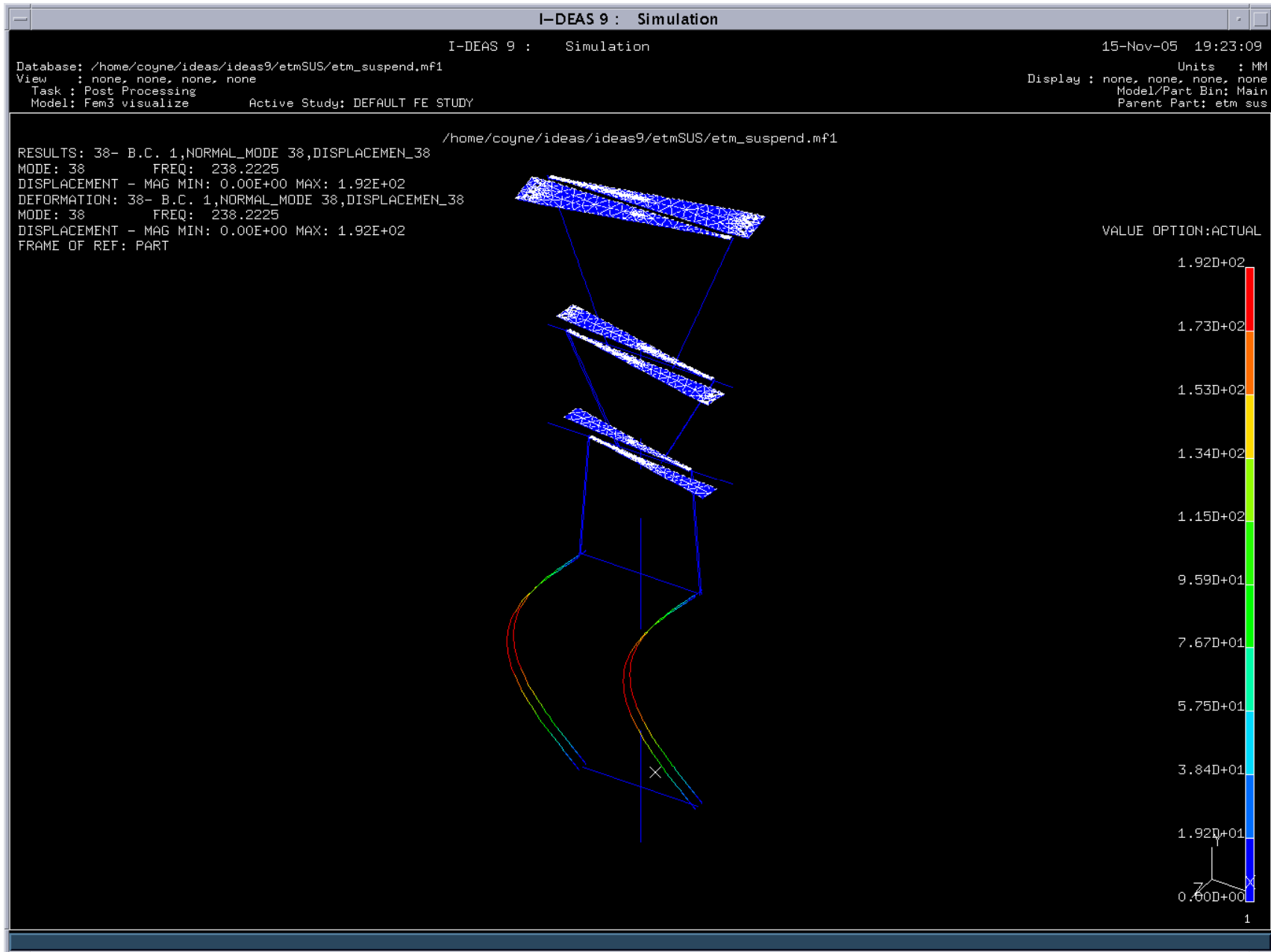


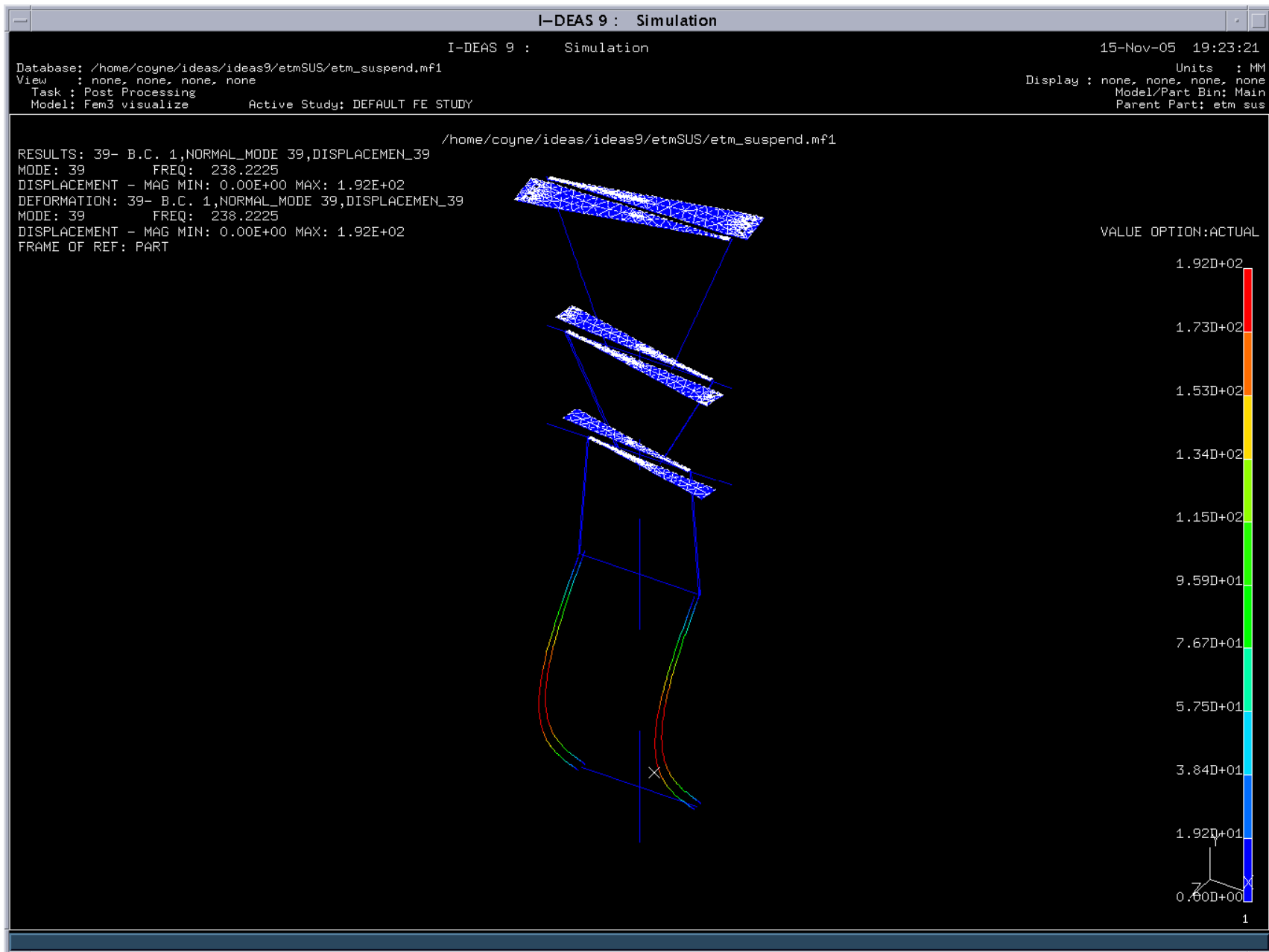


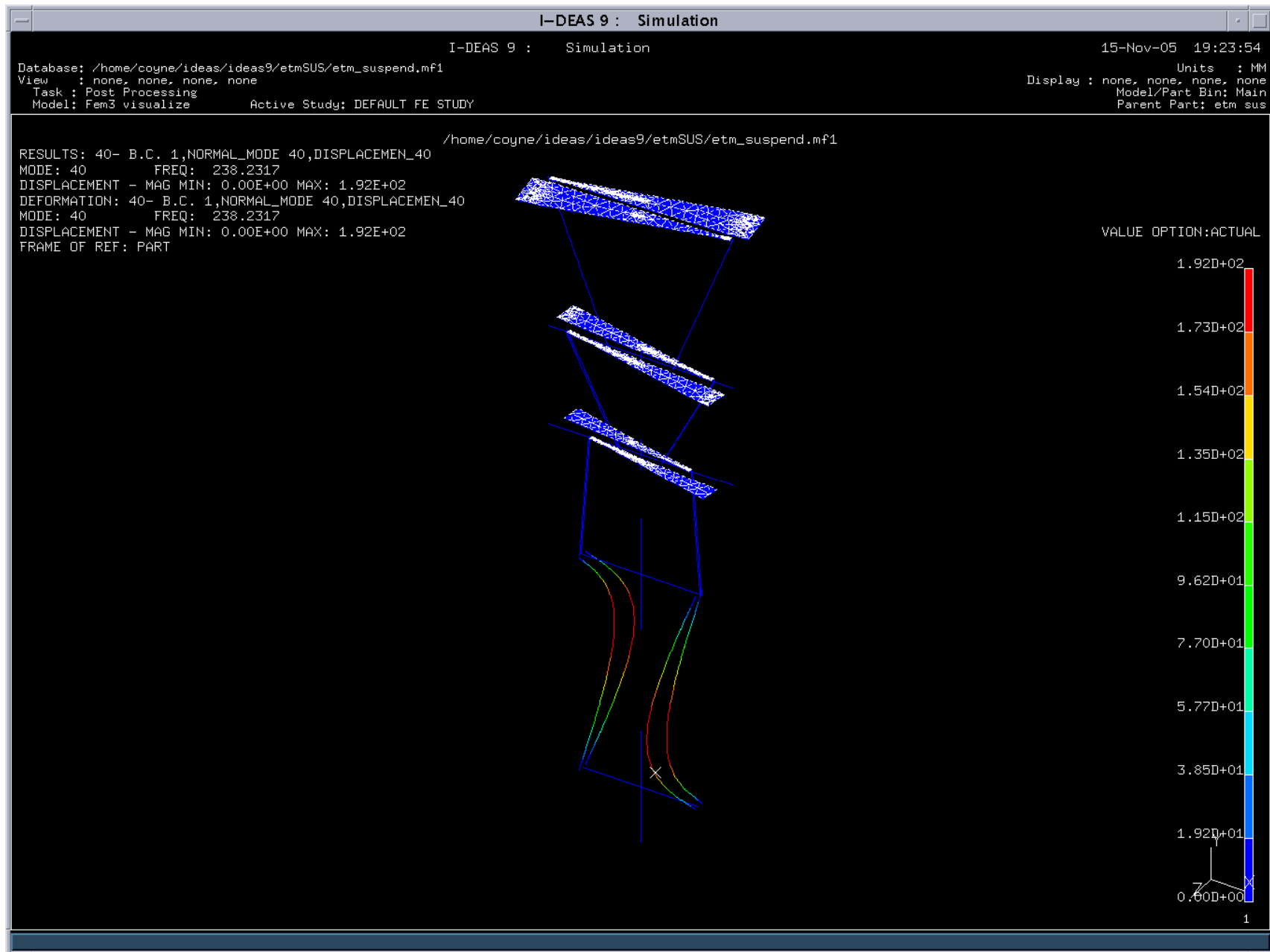










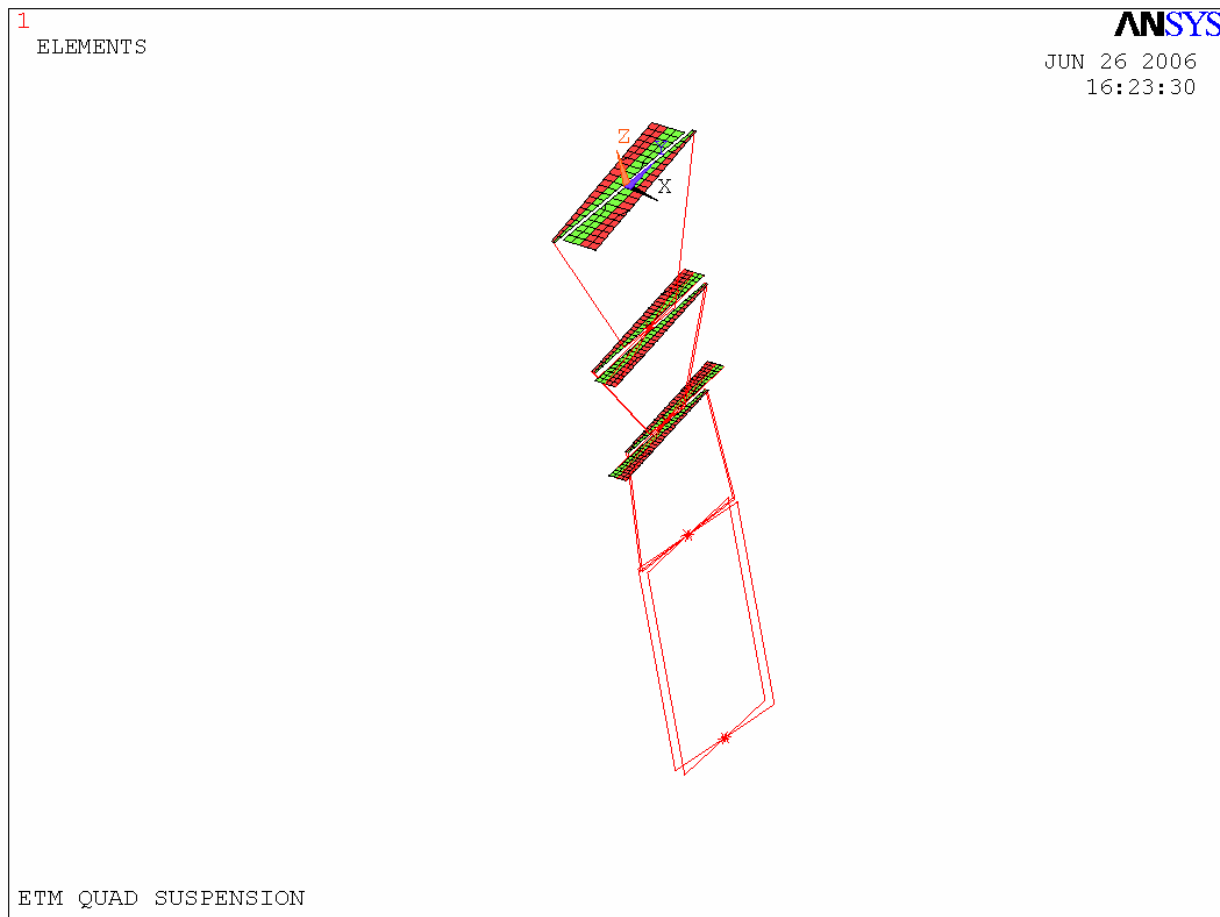


Include a Table of mass 6dof participation factors here

4 Script and Parameter Generated Geometry

The key parameters used were consistent with the values reported in the mode identification report⁹ for the quadruple controls prototype. These parameters and the terminology used in the script are consistent with the [pendulum naming conventions](#)¹⁰.

Using the Ansys macro script in Appendix A, the suspension model shown in the following figure was created.



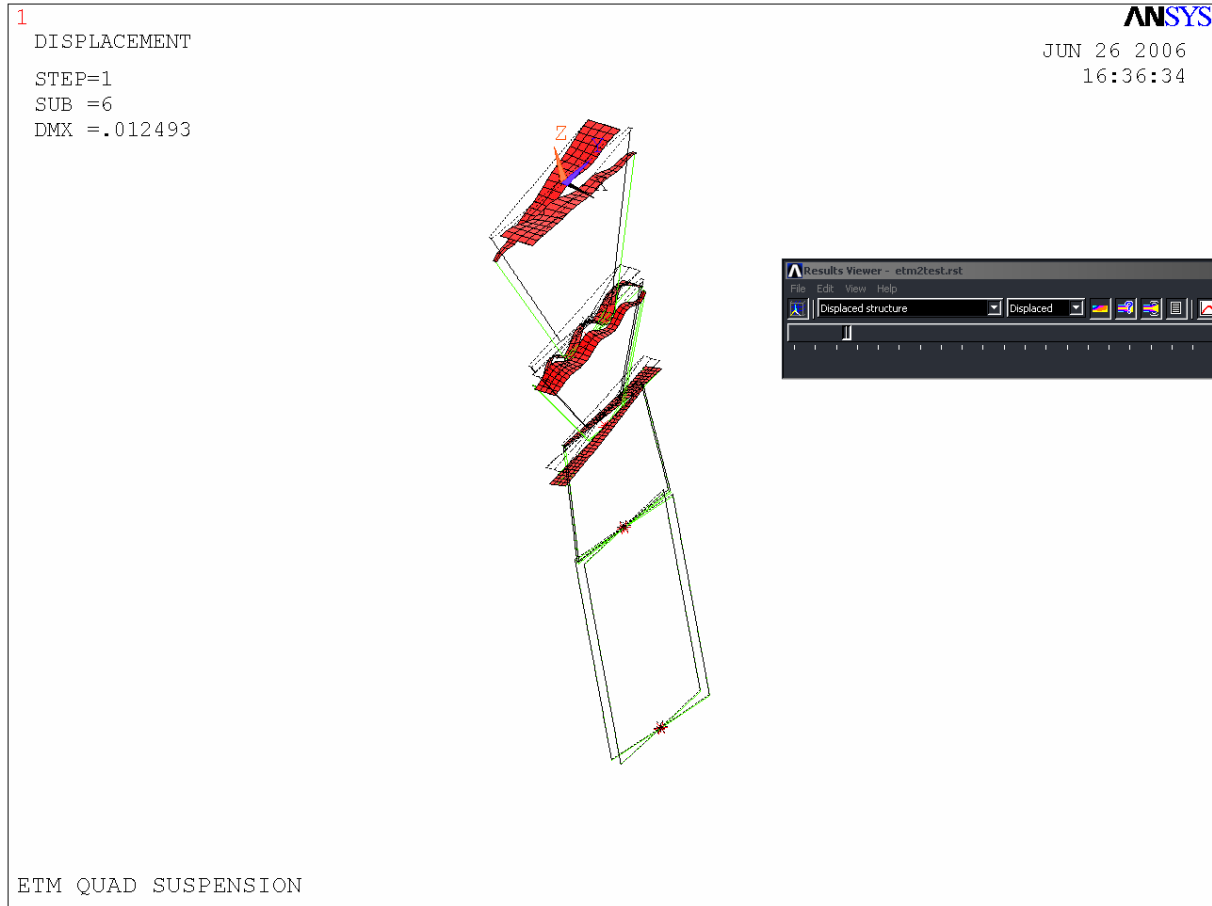
When pre-loaded and then the eigenvalue analysis is performed, some of the modes seem reasonable, such as the one in the following figure (although in-plane blade spring compliance seems higher than actual).

⁹ M. Barton, Quad Pendulum Controls Prototype – Identification of Modes, LIGO-pending number, 3 Nov 2005.

¹⁰ M. Perreux-Lloyd, [Pendulum Parameter Descriptions and Naming Convention, LIGO-T040072-01](#).



However some of the modes, such as the one in the following figure, have high bending modes in the blades at low frequencies, which is not realistic. Either there is a model error or the eigensolver is having difficulties.



5 Appendix: Ansys Macro to generate Quadruple Pendulum Suspension


```
1 !*****
2 ! etmsus.mac
3 !*****
4 ! macro to analyse natural frequencies of an ETM quad suspension
5 !
6 !Dennis Coyne 28-nov-2005
7 !
8 finish
9 /CLEAR,START
10 /COM,ANSYS MODEL OF A SINGLE ETM QUAD SUSPENSION CHAIN
11 /PREP7
12 /TITLE,ETM QUAD SUSPENSION
13 !*****
14 !*      GEOMETRIC PARAMETERS
15 !*****
16 ! values of parameters
17 ! SI units (m,N)
18 ! naming per T040072-01, plus extensions explained below
19 ! coordinate system per T040072-02 with origin at center of top blades
20
21 ! top blades (D040298)
22 lnb=0.480
23 anb=0.095
24 bnb=0.010 ! blade tip width
25 cnb=7.5*3.14159/180 ! blade centerline angle to y-axis (rad)
26 hnb=0.00429
27 nn0=0.25
28 !*
29 ! middle blades (D040297)
30 llb=0.4207
31 alb=0.059
32 blb=0.010 ! blade tip width
33 clb=7.5*3.14159/180 ! blade centerline angle to y-axis (rad)
34 hlb=0.0046
35 n0=0.200
36 !*
37 ! bottom blades (D040296)
38 llb=0.370
39 a2b=0.049
40 b2b=0.010 ! blade tip width
41 c2b=7.5*3.14159/180 ! blade centerline angle to y-axis (rad)
42 h2b=0.0042
43 n2=0.140
44 !*
45 ! Top Wires
46 ! zw1 is the height coordinate of the top wire attachment to the top mass
47 ! zcg1 is the height coordinate of the c.g. of the top mass
48 ! zbl is the height coordinate of the neutral axis of the middle blades
49 nn1=0.090
50 ln=0.450
```

```
51 rn=0.00052
52 dm=0.001
53 dn=0.001
54 zw1=-SQRT(ln**2-(nn0-nn1)**2)
55 zcg1=zw1-dm
56 zbl=zcg1-dn
57 !*
58 ! Middle Wires
59 ! zw2 is the height coordinate of the middle wire attachment to the UI mass
60 ! zcg2 is the height coordinate of the c.g. of the UI mass
61 ! zb2 is the height coordinate of the neutral axis of the bottom blades
62 n1=0.060
63 l1=0.3085
64 r1=0.00035
65 d0=0.001
66 d1=0.001
67 su=0.003
68 zw2=zb1-SQRT(l1**2-(n0-n1)**2)
69 zcg2=zw2-d0
70 zb2=zcg2-d1
71 !*
72 ! Penultimate Wires
73 ! zw3 is the height coordinate upper (penultimate) wire attachment to the PM
74 ! zcg3 is the height coordinate of the c.g. of the PM
75 ! zb3 is the height coordinate of lower (TM) wire attachment to the PM
76 n3=0.1635
77 l2=0.3400
78 r2=0.00031
79 d2=0.001
80 d3=0.001
81 si=0.003
82 zw3=zb2-SQRT(l2**2-(n2-n3)**2)
83 zcg3=zw3-d2
84 zb3=zcg3-d3
85 !*
86 ! Test Mass Wires/Ribbons
87 ! zw4 is the height coordinate TM wire attachment to the TM
88 ! zcg4 is the height coordinate of the c.g. of the TM
89 n4=0.1585
90 n5=0.1585
91 l3=0.6020
92 r3=0.00022
93 d4=0.001
94 s1=0.015
95 zw4=zb3-SQRT(l3**2-(n4-n5)**2)
96 zcg4=zw4-d4
97
98 !*****
99 !*      MASS PARAMETERS
100 !*****
```

```
101 !* N.B.: The moments of inertia below (Inx, Iny, Inz) are values without
102 !           the blade springs included.
103 !   Top Mass
104 mn=22.1
105 Inx=0.0670
106 Iny=0.4289
107 Inz=0.4309
108 !*
109 !   UI Mass
110 m1=21.8
111 I1x=0.0581
112 I1y=0.5005
113 I1z=0.4988
114 !*
115 !   PM
116 m2=38.4
117 I2x=0.2765
118 I2y=0.2752
119 I2z=0.4704
120 !*
121 !   TM
122 m3=39.61
123 I3x=0.2692
124 I3y=0.2679
125 I3z=0.4556
126
127 !*****
128 !*           MATERIAL PROPERTIES
129 !*****
130 maryoung=1.76e11
131 marpoiss=0.3
132 mardens=7800
133 wireyoung=2.0e11
134 wirepoiss=0.3
135 wiredens=7800
136
137 !MPTEMP,,,,,,,,
138 !MPTEMP,1,0
139 !MPDATA,EX,1,,maryoung
140 !MPDATA,PRXY,1,,marpoiss
141 !MPDATA,DENS,1,,mardens
142 !MPTEMP,,,,,,,,
143 !MPTEMP,1,0
144 !MPDATA,EX,2,,wireyoung
145 !MPDATA,PRXY,2,,wirepoiss
146 !MPTEMP,,,,,,,,
147 !MPTEMP,1,0
148 !MPDATA,DENS,2,,wiredens
149 MP,EX,1,maryoung
150 MP,EY,1,maryoung
```

```
151 MP,EZ,1,maryoung
152 MP,PRXY,1,marpoiss
153 MP,DENS,1,mardens
154 MP,EX,2,wireyoung
155 MP,EY,2,wireyoung
156 MP,EZ,2,wireyoung
157 MP,PRXY,2,wirepoiss
158 MP,DENS,2,wiredens
159
160 !*****
161 !*      GENERATE GEOMETRY
162 !*****
163 !*      BLADES
164 !*****
165 ! Top Blade, Right
166 K,1,0,nn0,0,
167 K,2,-bnb*cos(cnb)/2,nn0-bnb*sin(cnb)/2,0,
168 K,3,bnb*cos(cnb)/2,nn0+bnb*sin(cnb)/2,0,
169 K,4,lnb*sin(cnb),nn0-lnb*cos(cnb),0,
170 K,5,lnb*sin(cnb)-anb*cos(cnb)/2,nn0-lnb*cos(cnb)-anb*sin(cnb)/2,0,
171 K,6,lnb*sin(cnb)+anb*cos(cnb)/2,nn0-lnb*cos(cnb)+anb*sin(cnb)/2,0,
172 LSTR, 1, 4 ! line 1
173 LSTR, 4, 6 ! line 2
174 LSTR, 6, 3 ! line 3
175 LSTR, 3, 1 ! line 4
176 LSTR, 4, 5 ! line 5
177 LSTR, 5, 2 ! line 6
178 LSTR, 2, 1 ! line 7
179 AL,1,2,3,4 ! area 1
180 AL,1,5,6,7 ! area 2
181 !*
182 ! Top Blade, Left
183 K,7,0,-nn0,0,
184 K,8,bnb*cos(cnb)/2,-nn0+bnb*sin(cnb)/2,0,
185 K,9,-bnb*cos(cnb)/2,-nn0-bnb*sin(cnb)/2,0,
186 K,10,-lnb*sin(cnb),-nn0+lnb*cos(cnb),0,
187 K,11,-lnb*sin(cnb)+anb*cos(cnb)/2,-nn0+lnb*cos(cnb)+anb*sin(cnb)/2,0,
188 K,12,-lnb*sin(cnb)-anb*cos(cnb)/2,-nn0+lnb*cos(cnb)-anb*sin(cnb)/2,0,
189 LSTR, 7,10 ! line 8
190 LSTR,10,12 ! line 9
191 LSTR, 9,12 ! line 10
192 LSTR, 7, 9 ! line 11
193 LSTR,10,11 ! line 12
194 LSTR, 8,11 ! line 13
195 LSTR, 7, 8 ! line 14
196 AL,8,9,10,11 ! area 3
197 AL,8,12,13,14 ! area 4
198 !*
199 ! Middle Blade, Right
200 K,13,0,n0,zb1,
```

```
201 K,14,-blb*cos(c1b)/2,n0-blb*sin(c1b)/2,zb1,
202 K,15,blb*cos(c1b)/2,n0+blb*sin(c1b)/2,zb1,
203 K,16,11b*sin(c1b),n0-11b*cos(c1b),zb1,
204 K,17,11b*sin(c1b)-alb*cos(c1b)/2,n0-11b*cos(c1b)-alb*sin(c1b)/2,zb1,
205 K,18,11b*sin(c1b)+alb*cos(c1b)/2,n0-11b*cos(c1b)+alb*sin(c1b)/2,zb1,
206 LSTR, 13,16 ! line 15
207 LSTR, 16,18 ! line 16
208 LSTR, 18,15 ! line 17
209 LSTR, 15,13 ! line 18
210 LSTR, 16,17 ! line 19
211 LSTR, 17,14 ! line 20
212 LSTR, 14,13 ! line 21
213 AL,15,16,17,18 ! area 5
214 AL,15,19,20,21 ! area 6
215 !*
216 ! Middle Blade, Left
217 K,19,0,-n0,zb1,
218 K,20,blb*cos(c1b)/2,-n0+blb*sin(c1b)/2,zb1,
219 K,21,-blb*cos(c1b)/2,-n0-blb*sin(c1b)/2,zb1,
220 K,22,-11b*sin(c1b),-n0+11b*cos(c1b),zb1,
221 K,23,-11b*sin(c1b)+alb*cos(c1b)/2,-n0+11b*cos(c1b)+alb*sin(c1b)/2,zb1,
222 K,24,-11b*sin(c1b)-alb*cos(c1b)/2,-n0+11b*cos(c1b)-alb*sin(c1b)/2,zb1,
223 LSTR, 19,22 ! line 22
224 LSTR, 22,24 ! line 23
225 LSTR, 21,24 ! line 24
226 LSTR, 19,21 ! line 25
227 LSTR, 22,23 ! line 26
228 LSTR, 20,23 ! line 27
229 LSTR, 19,20 ! line 28
230 AL,22,23,24,25 ! area 7
231 AL,22,26,27,28 ! area 8
232 !*
233 ! Bottom Blade, Right
234 K,25,0,n2,zb2,
235 K,26,-b2b*cos(c2b)/2,n2-b2b*sin(c2b)/2,zb2,
236 K,27,b2b*cos(c2b)/2,n2+b2b*sin(c2b)/2,zb2,
237 K,28,12b*sin(c2b),n2-12b*cos(c2b),zb2,
238 K,29,12b*sin(c2b)-a2b*cos(c2b)/2,n2-12b*cos(c2b)-a2b*sin(c2b)/2,zb2,
239 K,30,12b*sin(c2b)+a2b*cos(c2b)/2,n2-12b*cos(c2b)+a2b*sin(c2b)/2,zb2,
240 LSTR, 25,28 ! line 29
241 LSTR, 28,30 ! line 30
242 LSTR, 30,27 ! line 31
243 LSTR, 27,25 ! line 32
244 LSTR, 28,29 ! line 33
245 LSTR, 29,26 ! line 34
246 LSTR, 26,25 ! line 35
247 AL,29,30,31,32 ! area 9
248 AL,29,33,34,35 ! area 10
249 !*
250 ! Bottom Blade, Left
```

```
251 K,31,0,-n2,zb2,
252 K,32,b2b*cos(c2b)/2,-n2+b2b*sin(c2b)/2,zb2,
253 K,33,-b2b*cos(c2b)/2,-n2-b2b*sin(c2b)/2,zb2,
254 K,34,-l2b*sin(c2b),-n2+l2b*cos(c2b),zb2,
255 K,35,-l2b*sin(c2b)+a2b*cos(c2b)/2,-n2+l2b*cos(c2b)+a2b*sin(c2b)/2,zb2,
256 K,36,-l2b*sin(c2b)-a2b*cos(c2b)/2,-n2+l2b*cos(c2b)-a2b*sin(c2b)/2,zb2,
257 LSTR, 31,34 ! line 36
258 LSTR, 34,36 ! line 37
259 LSTR, 33,36 ! line 38
260 LSTR, 31,33 ! line 39
261 LSTR, 34,35 ! line 40
262 LSTR, 32,35 ! line 41
263 LSTR, 31,32 ! line 42
264 AL,36,37,38,39 ! area 11
265 AL,36,40,41,42 ! area 12
266 !*****
267 !*           MASSES
268 !*****
269 !*
270 ! Top Mass
271 K,100,0,0,zcg1,
272 K,101,0,nn1,zw1,
273 K,102,0,-nn1,zw1,
274 K,103,su,n0,zb1,
275 K,104,-su,n0,zb1,
276 K,105,su,-n0,zb1,
277 K,106,-su,-n0,zb1,
278
279 !*
280 ! Upper Intermediate (UI) Mass
281 K,200,0,0,zcg2,
282 K,201,su,n1,zw2,
283 K,202,-su,n1,zw2,
284 K,203,su,-n1,zw2,
285 K,204,-su,-n1,zw2,
286 K,205,si,n2,zb2,
287 K,206,-si,n2,zb2,
288 K,207,si,-n2,zb2,
289 K,208,-si,-n2,zb2,
290
291 !*
292 ! Penultimate Mass (PM)
293 K,300,0,0,zcg3,
294 K,301,si,n3,zw3,
295 K,302,-si,n3,zw3,
296 K,303,si,-n3,zw3,
297 K,304,-si,-n3,zw3,
298 K,305,s1,n4,zb3,
299 K,306,-s1,n4,zb3,
300 K,307,s1,-n4,zb3,
```

```
301 K,308,-s1,-n4,zb3,
302
303 !*
304 ! Test Mass (TM)
305 K,400,0,0,zcg4,
306 K,401,s1,n5,zw4,
307 K,402,-s1,n5,zw4,
308 K,403,s1,-n5,zw4,
309 K,404,-s1,-n5,zw4,
310
311 !*****
312 !*           WIRES
313 !*****
314 !*
315 ! Top Wires
316 LSTR,1,101      ! line 43
317 LSTR,7,102     ! line 44
318 !*
319 ! UI Wires
320 LSTR,103,201   ! line 45
321 LSTR,104,202   ! line 46
322 LSTR,105,203   ! line 47
323 LSTR,106,204   ! line 48
324 !*
325 ! PM Wires
326 LSTR,205,301   ! line 49
327 LSTR,206,302   ! line 50
328 LSTR,207,303   ! line 51
329 LSTR,208,304   ! line 52
330 !*
331 ! TM Wires
332 LSTR,305,401   ! line 53
333 LSTR,306,402   ! line 54
334 LSTR,307,403   ! line 55
335 LSTR,308,404   ! line 56
336
337 !*****
338 !*           RIGID LINKS
339 !*****
340 !*
341 ! Top Mass
342 LSTR,100,101   ! line 57, top wire 1
343 LSTR,100,102   ! LINE 58, top wire 2
344 LSTR, 15,103   ! LINE 59, middle blade to wire 1
345 LSTR, 14,104   ! LINE 60, middle blade to wire 2
346 LSTR, 20,105   ! LINE 61, middle blade to wire 3
347 LSTR, 21,106   ! LINE 62, middle blade to wire 4
348 LSTR,100, 16   ! LINE 63, c.g. to blade root 1
349 LSTR,100, 17   ! LINE 64, c.g. to blade root 2
350 LSTR,100, 18   ! LINE 65, c.g. to blade root 3
```

```
351 LSTR,100, 22 ! LINE 66, c.g. to blade root 4
352 LSTR,100, 23 ! LINE 67, c.g. to blade root 5
353 LSTR,100, 24 ! LINE 68, c.g. to blade root 6
354 !*
355 ! UI Mass
356 LSTR,200,201 ! line 69, UI wire 1
357 LSTR,200,202 ! LINE 70, UI wire 2
358 LSTR,200,203 ! LINE 71, UI wire 3
359 LSTR,200,204 ! LINE 72, UI wire 4
360 LSTR, 27,205 ! LINE 73, bottom blade to wire 1
361 LSTR, 26,206 ! LINE 74, bottom blade to wire 2
362 LSTR, 32,207 ! LINE 75, bottom blade to wire 3
363 LSTR, 33,208 ! LINE 76, bottom blade to wire 4
364 LSTR,200, 28 ! LINE 77, c.g. to blade root 1
365 LSTR,200, 29 ! LINE 78, c.g. to blade root 2
366 LSTR,200, 30 ! LINE 79, c.g. to blade root 3
367 LSTR,200, 34 ! LINE 80, c.g. to blade root 4
368 LSTR,200, 35 ! LINE 81, c.g. to blade root 5
369 LSTR,200, 36 ! LINE 82, c.g. to blade root 6
370 !*
371 ! PM
372 LSTR,300,301 ! LINE 83, c.g. to PM wire 1
373 LSTR,300,302 ! LINE 84, c.g. to PM wire 2
374 LSTR,300,303 ! LINE 85, c.g. to PM wire 3
375 LSTR,300,304 ! LINE 86, c.g. to PM wire 4
376 LSTR,300,305 ! LINE 87, c.g. to TM wire 1
377 LSTR,300,306 ! LINE 88, c.g. to TM wire 2
378 LSTR,300,307 ! LINE 89, c.g. to TM wire 3
379 LSTR,300,308 ! LINE 90, c.g. to TM wire 4
380 !*
381 ! TM
382 LSTR,400,401 ! LINE 91, c.g. to TM wire 1
383 LSTR,400,402 ! LINE 92, c.g. to TM wire 2
384 LSTR,400,403 ! LINE 93, c.g. to TM wire 3
385 LSTR,400,404 ! LINE 94, c.g. to TM wire 4
386 !*
387 ! Plot Areas
388 aplot
389
390 !*****
391 !* MESH GEOMETRY
392 !*****
393 !*
394 !*****
395 ! Blade Springs
396 !*****
397 !ET,1,SHELL63,0,0,1,,0,0
398 !ET,1,SHELL43,,2
399 !ET,1,SHELL181,1
400 !ET,1,SHELL93
```



```
401 ET,1,SHELL63
402 TYPE,1      ! shell elements
403 MAT,1      ! maraging steel
404 !          Top Blades
405 R,1,hnb          !for the blade
406 REAL,1
407 ESIZE,anb/4,0
408 amesh,1,4
409 !          Middle Blades
410 R,2,h1b          !for the blade
411 REAL,2
412 ESIZE,a1b/4,0
413 amesh,5,8
414 !          Bottom Blades
415 R,3,h2b          !for the blade
416 REAL,3
417 ESIZE,a2b/4,0
418 amesh,9,12
419
420 !*****
421 !          Suspension Masses
422 !*****
423 ET,2,MASS21,0,0,0
424 TYPE,2      ! discrete mass elements
425 !          Top Mass
426 R,4,mn,mn,mn,Inx,Iny,Inz
427 REAL,4
428 KMESH,100
429 !          UI Mass
430 R,5,m1,m1,m1,I1x,I1y,I1z
431 REAL,5
432 KMESH,200
433 !          PM
434 R,6,m2,m2,m2,I2x,I2y,I2z
435 REAL,6
436 KMESH,300
437 !          TM
438 R,7,m3,m3,m3,I3x,I3y,I3z
439 REAL,7
440 KMESH,400
441
442 ! Wire Clamps
443 !R,8,mc,mc,mc,Icx,Icy,Icz
444 !REAL,8
445 !kmesh,?
446
447 !*****
448 !          Wires
449 !*****
450 ET,3,BEAM4,,0,,,0
```

```
451 TYPE,3      ! beam elements
452 MAT,2
453 !      Top Wires
454 wAn=3.14159*rn**2
455 wIn=(3.14159*rn**4)/4
456 wJn=(3.14159*rn**4)/2
457 R,9,wAn,wIn,wIn,rn,rn,0
458 RMORE,0,wJn
459 REAL,9
460 ESIZE,0.005
461 LMESH,43,44
462 !      UI Wires
463 wA1=3.14159*r1**2
464 wI1=(3.14159*r1**4)/4
465 wJ1=(3.14159*r1**4)/2
466 R,10,wA1,wI1,wI1,r1,r1,0
467 RMORE,0,wJ1
468 REAL,10
469 ESIZE,0.005
470 LMESH,45,48
471 !      PM Wires
472 wA2=3.14159*r2**2
473 wI2=(3.14159*r2**4)/4
474 wJ2=(3.14159*r2**4)/2
475 R,11,wA2,wI2,wI2,r2,r2,0
476 RMORE,0,wJ2
477 REAL,11
478 ESIZE,0.005
479 LMESH,49,52
480 !      TM Wires
481 wA3=3.14159*r3**2
482 wI3=(3.14159*r3**4)/4
483 wJ3=(3.14159*r3**4)/2
484 R,12,wA3,wI3,wI3,r3,r3,0
485 RMORE,0,wJ3
486 REAL,12
487 ESIZE,0.005
488 LMESH,53,56
489
490 !*****
491 !      Rigid Links
492 !*****
493 ET,4,MPC184,1,0
494 TYPE,4      ! rigid body constraint elements
495 ESIZE,,1
496 LMESH,57,94
497
498 NUMMRG,NODE,0.00001
499
500 !*****
```

```
501 !*      BOUNDARY CONDITIONS
502 !*****
503 !*
504 ! Clamp the upper blade roots
505 DL, 2, ,ALL,0
506 DL, 5, ,ALL,0
507 DL, 9, ,ALL,0
508 DL,12, ,ALL,0
509 EPLOT
510
511 FINISH
512
513 !*****
514 !   Static Preload Analysis
515 !*****
516 /SOL
517 ANTYPE,STATIC
518 PSTRES,ON
519 G=9.84
520 ACEL,0,0,G
521 SOLVE
522 FINISH
523 !/POST1
524 !PLDIST,0
525 !FINISH
526 !*****
527 !   Eigenvalue Analysis
528 !*****
529 /SOL
530 ANTYPE,MODAL
531 MSAVE,0
532 !*
533 MODOPT,LANB,60
534 EQSLV,JCG
535 !MXPAND,0, , ,0
536 LUMPM,OFF
537 PSTRES,ON
538 !*
539 /STATUS,SOLU
540 SOLVE
541 FINISH
542 /POST1
543 /output,etmsus,out,,
544 !*Status,argx
545 SET,LIST
546 /output
547 *get,freq1,mode,1,freq
548 *get,freq2,mode,2,freq
549 *get,freq3,mode,3,freq
```