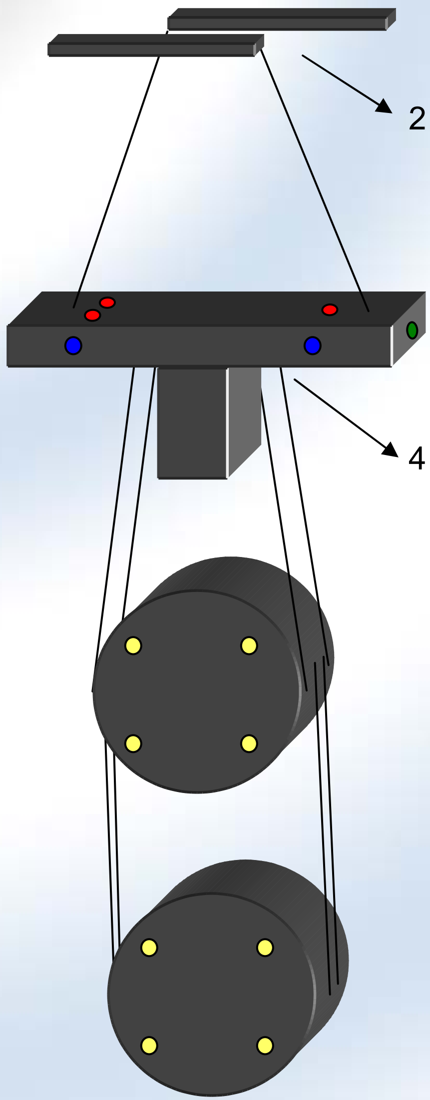
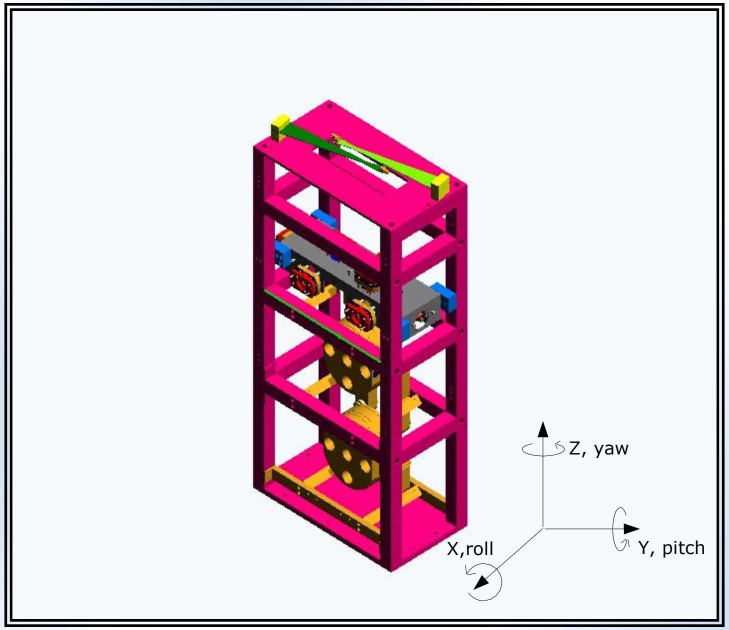
MODAL CONTROL and ESTIMATOR

LIGO

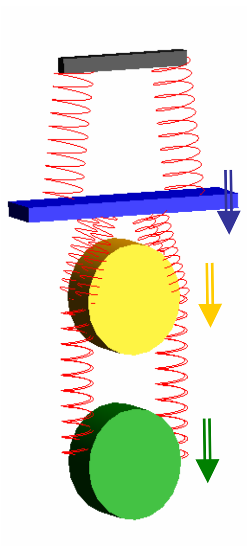
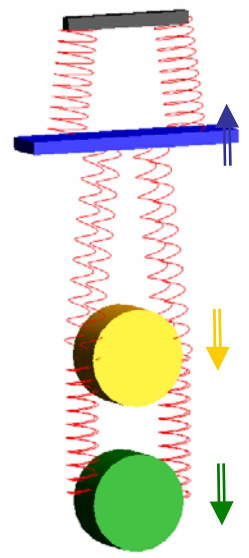
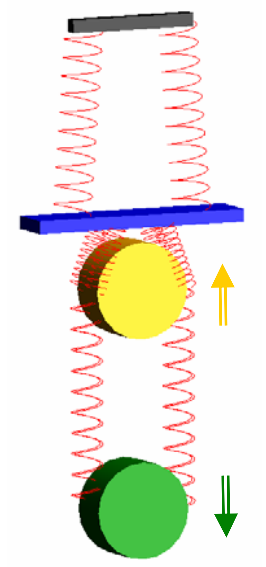
TRIPLE PENDULUM





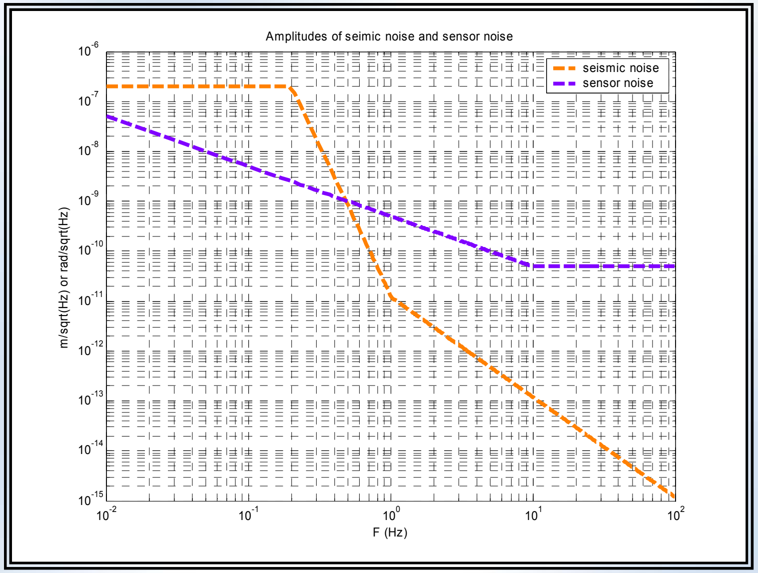
TRIPLE PENDULUM

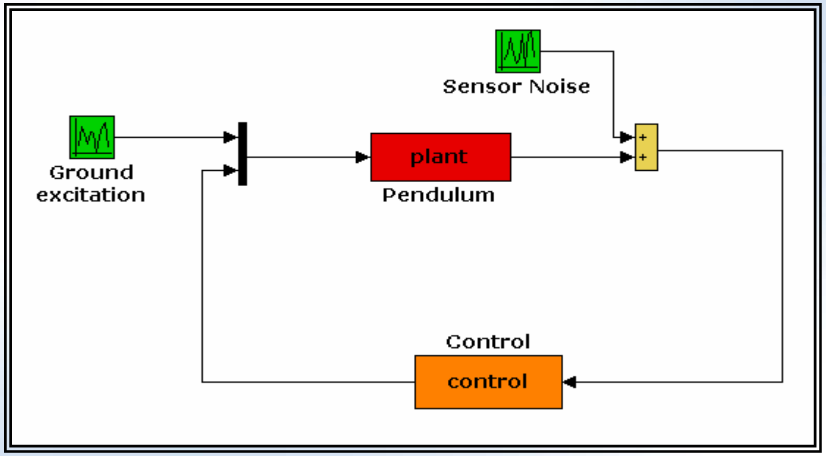
- Vertical direction

Mode1 at 1.18Hz	Mode2 at 4.25Hz	Mode3 at 46Hz
		

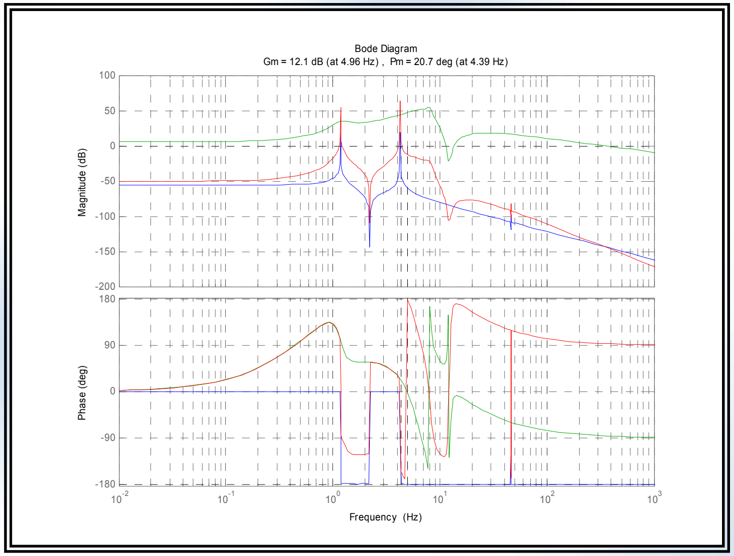
INTRODUCTION

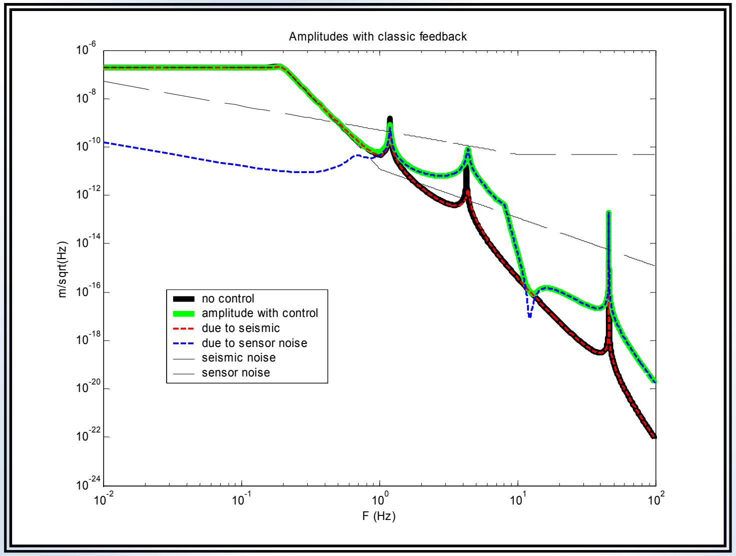
- Resonances to damp : 0-10Hz
- 2 sources of noise :
  - Seismic noise
  - Re-injected sensor noise
- Challenge : damp the resonances & reduce the sensor noise re-injection

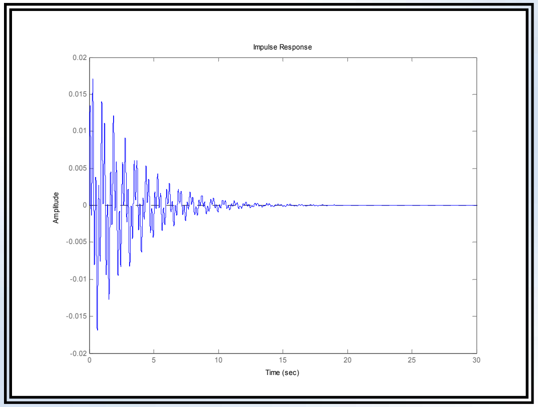




CLASSIC FEEDBACK CONTROL







- Motion of mass 1 is measured, filtered, and re-injected as a force into mass 1.
- Drawbacks of the classic filtering feedback method :
  - Time consuming and complex work to design filters
  - Not flexible
  - Performances are not very good

MODAL CONTROL : MATHEMATICS

- Goal : write the equations of dynamic in a new basis so that the equations are uncoupled

$$M\ddot{x} + Kx = 0$$

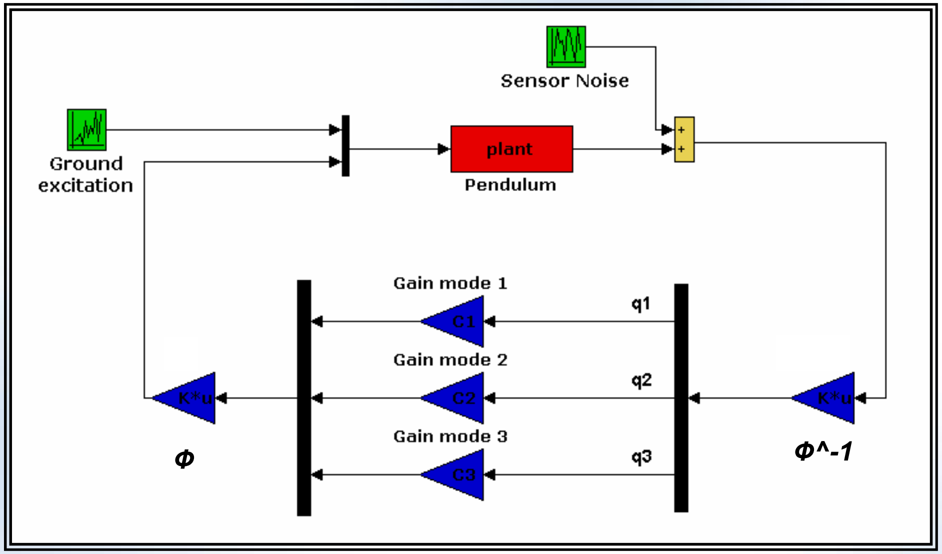
$$x = Xe^{i\omega t}$$

$$M^{-1}KX = X\omega^2$$ Where ω^2 are the eigenvalues and X are the eigenvectors of inv(M)*K.

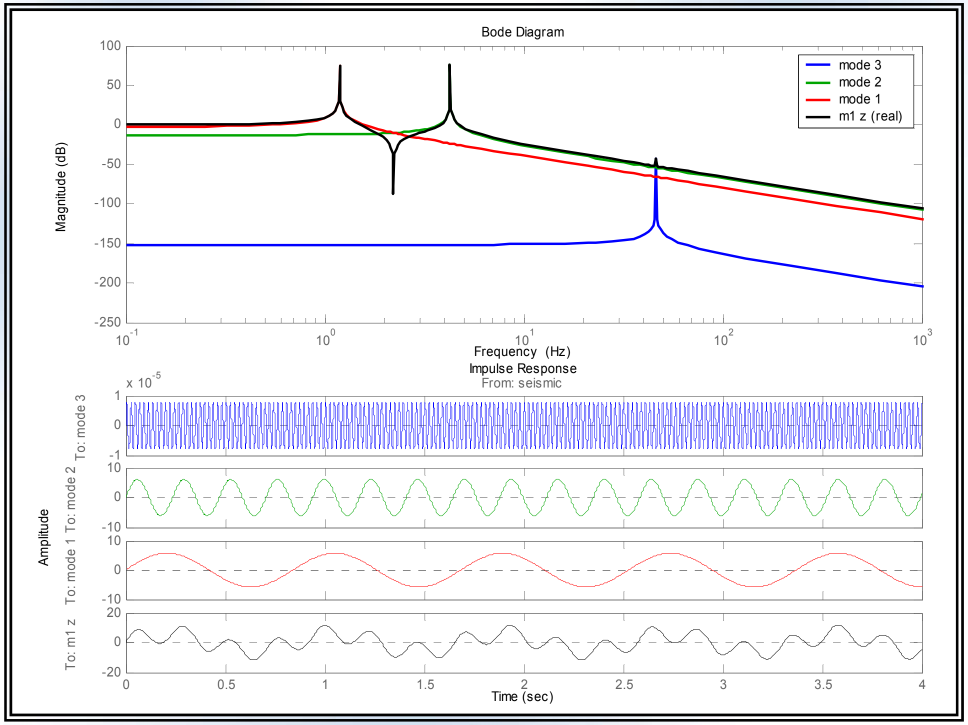
In the new basis Φ formed by the vectors X :

$$x = \phi.q$$

$$\ddot{q} + \phi^{-1}M^{-1}K\phi.q = \phi^{-1}F$$ Matrix is now diagonal, equations are **decoupled**

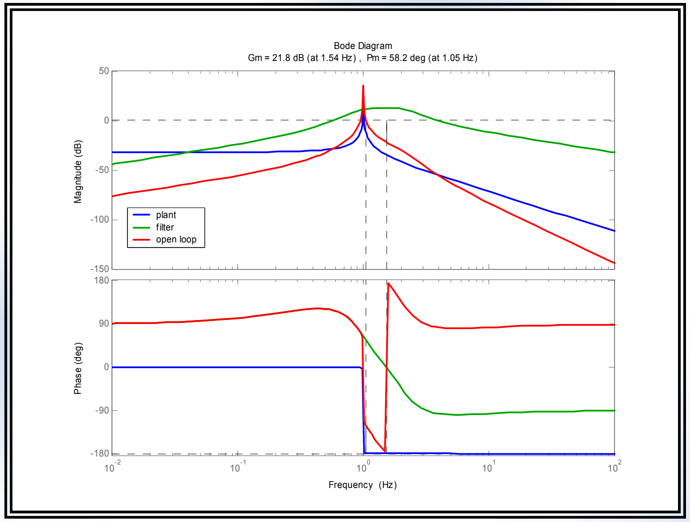


MODAL CONTROL : DECOMPOSITION

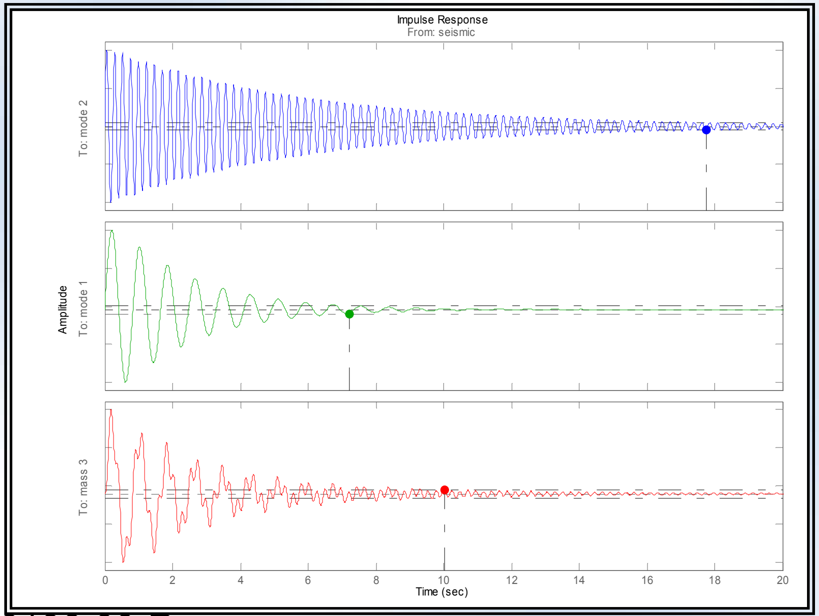


- We can design a control filter for each mode very easily because the transfer function is very simple
  - In practice, we use a parameterized filter (the shape is the same for every filter but the frequencies of poles/zero change depending on the frequency of the mode)
  - Makes the filter design very simple : do the design once and use it for all modes

MODAL CONTROL : CHOICE OF THE GAINS



- Simple filter parameterized with the mode frequency

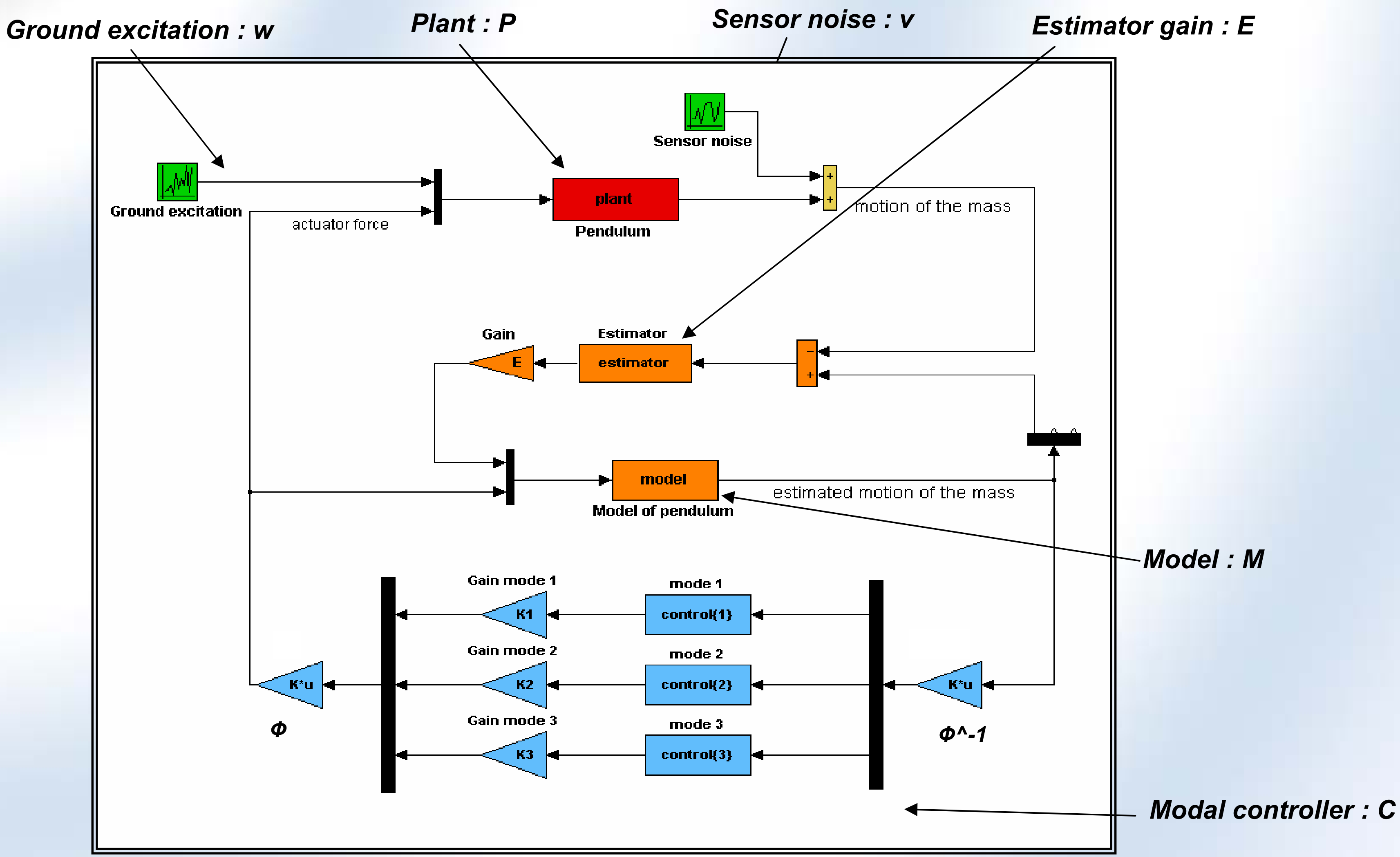


- Modal controller gains :
  - K1=80
  - K2=3
  - K3=0

MODAL CONTROL : CONCLUSION

- How does modal control help ?
  - Equations decoupled => easy to choose gain and filter for each mode
  - Lowest modes are easy to damp and filter => good damping
  - Highest modes can have lower gains => reduce noise transmission
- But
  - It needs as many measurement as DoF (needs to know the full state), this is not possible with the triple pendulum => estimator

ESTIMATOR LAYOUT



ESTIMATOR INTRODUCTION

- The estimator reconstructs the full state of the system
- It compares the estimated output with the real measurement to converge
- It can also filter the noisy measurements

$$x = \left[\frac{C.M.P + E.M.P - P}{C.M + E.M - 1 - E.M.C.P}\right].w + \left[\frac{E.M.C.P}{C.M + E.M - 1 - E.M.C.P}\right].v$$

Closed loop

$$E \to \infty$$

$$x = \left[\frac{P}{1 - C.P}\right].w + \left[\frac{C.P}{1 - C.P}\right].v$$

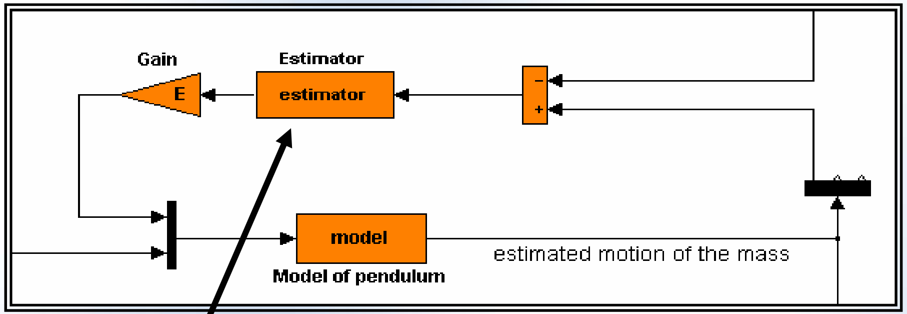
Equation of loop with no estimator

$$E \to 0$$

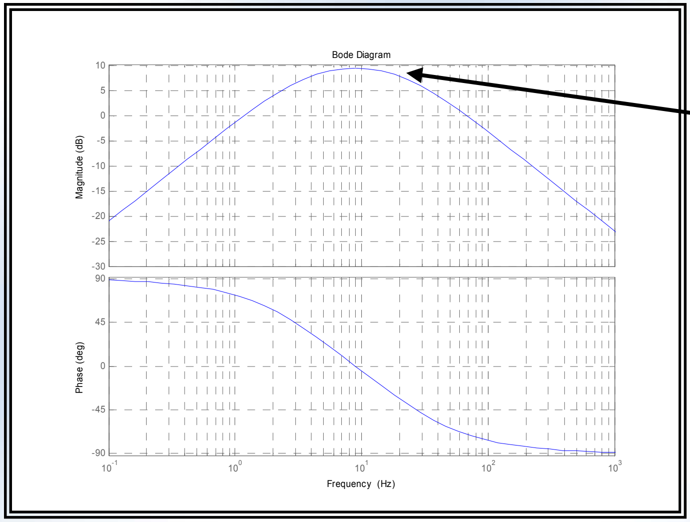
$$x \to [P].w + [0].v$$

no control

ESTIMATOR : FEEDBACK CONTROL

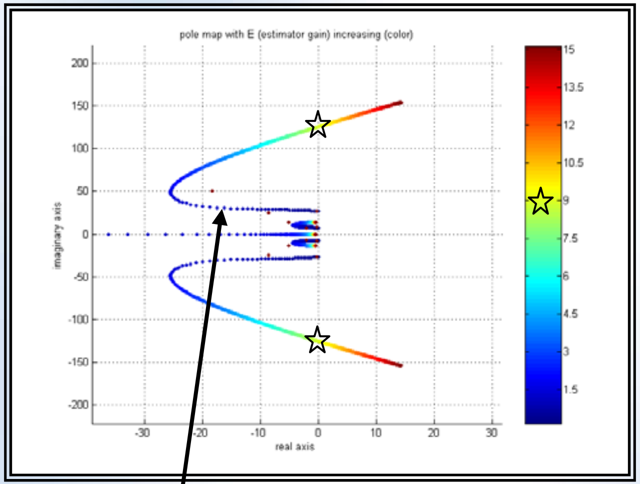


- Priority is stability
- Need to design a controller
  - Filtered feedback
  - MIMO (LQR, …)
- Design of a filter and choice of the gain
- Choice of a very simple filter shape to optimize stability and reduce HF noise

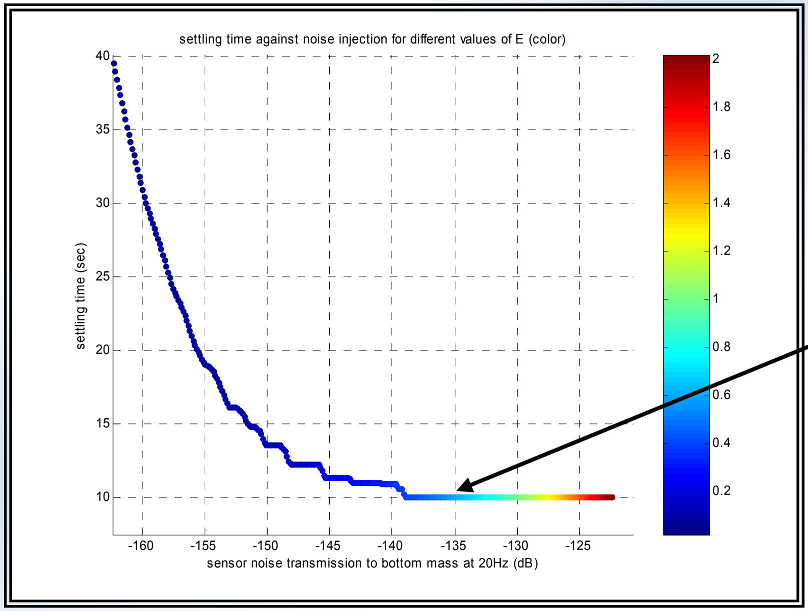


High gain at resonances

CHOICE OF ESTIMATOR FEEDBACK GAIN



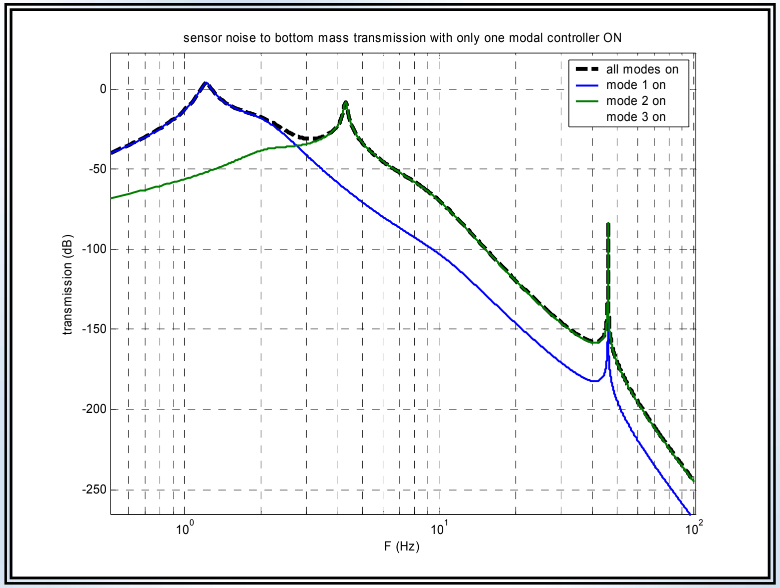
- Stability
  - Pole map of the closed loop
  - The color represents the estimator gain
  - System is unstable if the real part of the poles>=0
- Damping/Noise
  - X is sensor noise transmission at 20Hz (in dB)
  - Y is settling time (in sec)
  - Color is the estimator gain



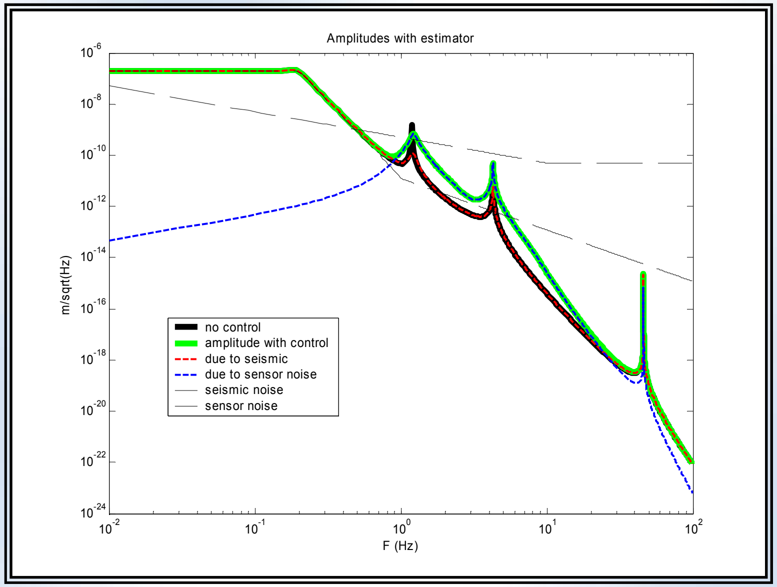
- We choose E=0.8

INFLUENCE OF EACH MODE ON SENSOR NOISE TRANSMISSION

- Transfer function between sensor noise and bottom mass motion, each modal controller turned on one by one
  - Shows the influence of each modal controller on the sensor noise injection
  - As expected, the lowest mode doesn't transmit a lot of noise
  - All the noise is carried by the 2nd mode
- Tells you how to improve the controller
  - Increase lower mode gain to keep a good damping
  - Decrease highest mode gain to lower noise injection

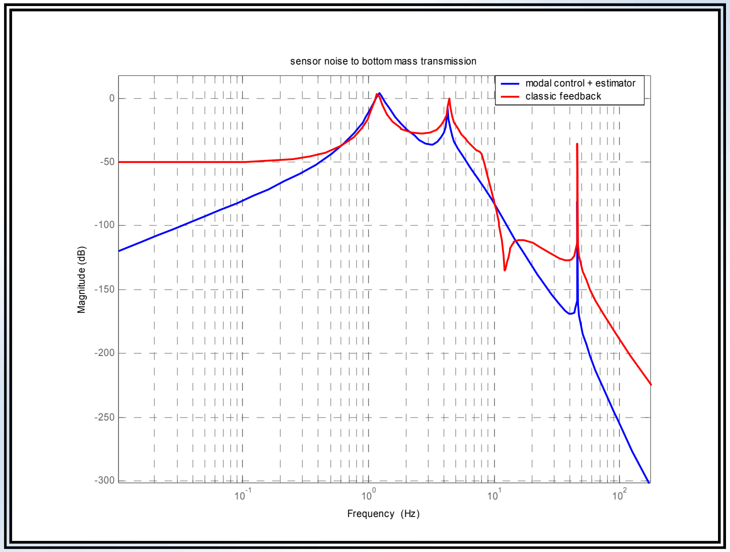


RESULTS



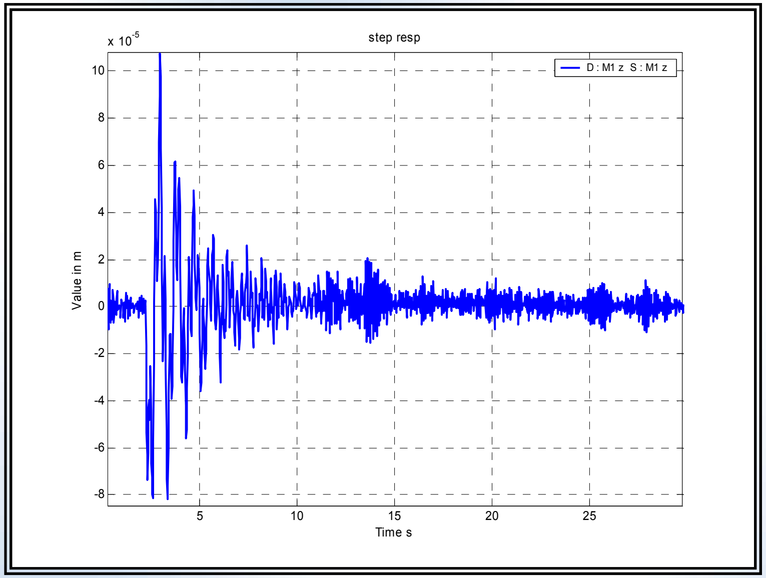
- Very good noise reduction with a simple filter shape
- More efficient than classic feedback for noise filtering

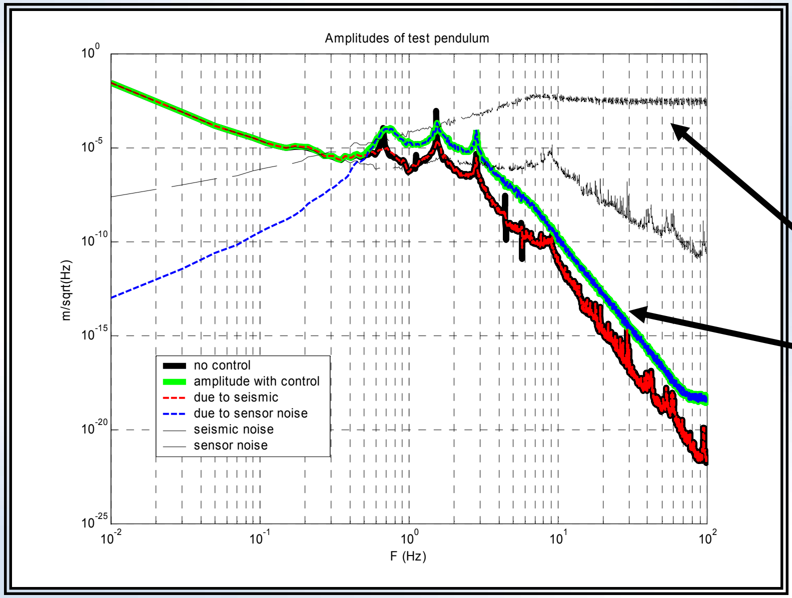
- Conclusion
  - Easy to design
  - Flexible
  - Good performances
  - Easy to improve



EXPERIMENTS

- Damping test on LASTI triple pendulum
  - successful
- Noise test is difficult in LASTI
  - Need to inject artificial sensor noise
  - Relative sensors limit the experimentation
  - => optical cavity between 2 triple pendulums



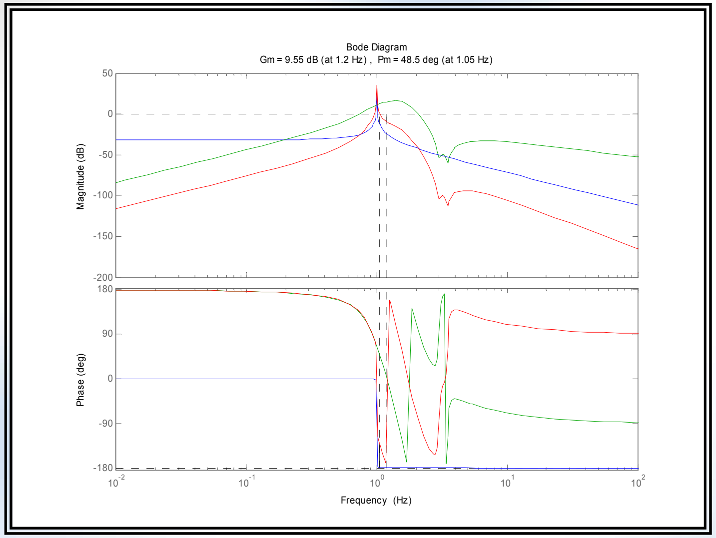


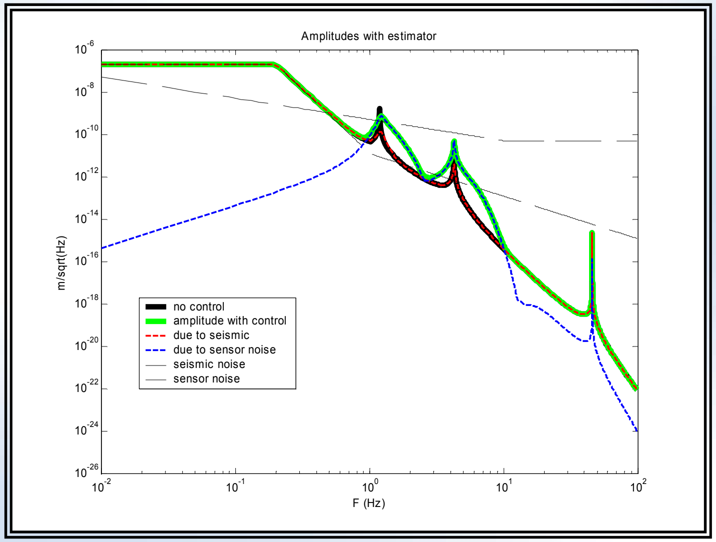
Artificial sensor noise is injected to compensate for big seismic noise

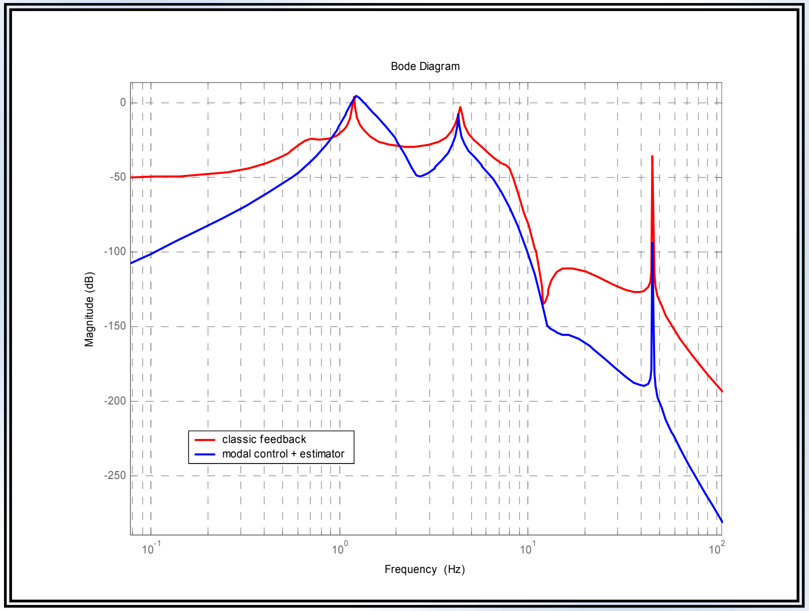
Expected noise on the bottom mass (X direction)

BETTER PERFORMANCES

- Improving the modal control filters is an easy way to improve performances (example in Z here)
- Work on MIMO estimator in progress, gain of few dB expected

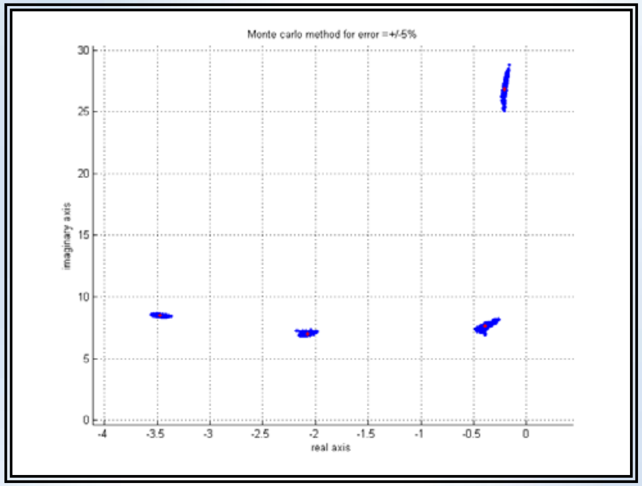




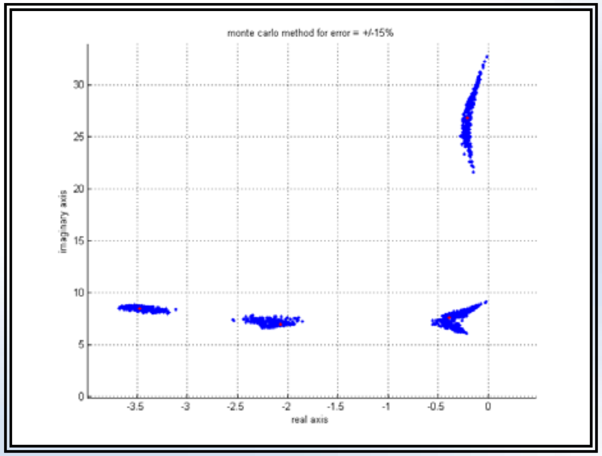


STABILITY WITH MODEL MISMATCH

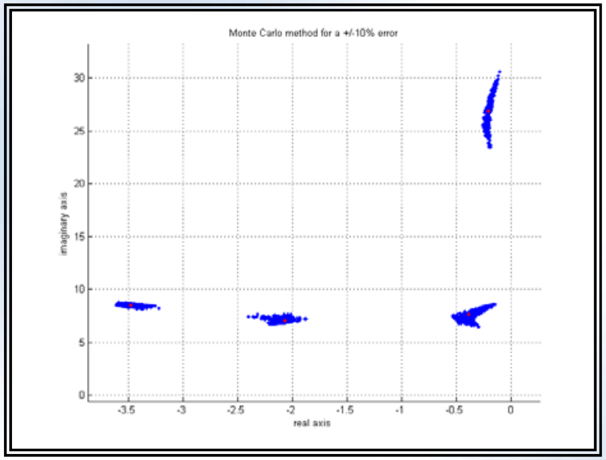
- What happens if the model mismatch ?
  - 1dof : easy to simulate by adding +/- 10% to the resonance (see document)
  - Multi dof : hard to quantify the mismatch, Monte-Carlo on closed loop pole map
- The parameters to know
  - The resonances need to be well known (within 10%)
  - The Q doesn't need to be well known



5% mismatch



15% mismatch



10% mismatch

CONCLUSION

- The modal control has many advantages
  - Easy to design
  - Flexible
  - Good performances in sensor noise re-injection minimization

- The estimator enables us to use modal control by generating unknown states
- The stability is easy to study and bad modeling can be anticipated

- Model could be adjusted to match the plant even better (gradient minimization)