LIGO-T050138-00-D

Oscillator Phase Noise Coupling

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Abstract:
The LIGO detectors use a variation of the Pound-Drever-Hall reflection locking technique to sense and control the longitudinal degrees of freedom. This involves modulation of the light beam with a radio frequency signal. Phase noise originating from the local oscillator which drives the modulation can contaminate the gravitational-wave readout. This project involves directly measuring the oscillator phase noise coupling to the anti-symmetric port where gravity-wave signals are anticipated to exist. Additionally, a new method of performing the signal demodulation is attempted by deriving the local oscillator signal from the two-omega signal which is available at the anti-symmetric port. The new scheme provides lower coupling in a frequency band between 150-900 Hz, reaching a maximum improvement of a factor of ~5 compared to the original scheme. The two schemes are indistinguishable at higher frequencies. This provides evidence for oscillator phase noise to output amplitude noise conversion.

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Introduction

The primary goal of the LIGO interferometers is to provide direct experimental evidence of gravitational waves. In order to achieve this, it is necessary for all avoidable sources of noise to be minimized. The design sensitivity assumes a certain amount of seismic noise at low frequencies and radiation shot noise at higher frequencies. In particular, all noise from the control scheme should be at a level well below the more "fundamental" noise sources. A required step in removing sources of noise is identifying and understanding these sources. In particular this paper was concerned with phase noise in the local oscillator and how it affects the gravitational-wave readout.

The large baseline LIGO interferometers are currently approaching their design sensitivity. Three interferometers make up the LIGO fleet, two of which have four kilometer long arms located at Hanford, Washington and Livingston, Louisiana and an additional two kilometer interferometer at the Hanford site. They are expected to be the first direct experimental test of gravitational waves. Gravitational waves are predicted by Albert Einstein's General Theory of Relativity. There exists strong indirect evidence for the existence of gravitational radiation,[1] in fact, Joseph Taylor and Russell Hulse shared the Nobel Prize in Physics in 1993 for the discovery of the binary pulsar system which led to this confirmation.

Like most forms of radiation, the amplitude of the waves created by a point source diminishes as the distance between source and observer increases. Astrophysical sources like coalescing binary neutron stars as far away as the Virgo cluster are expected to produce waves of strain amplitude 10^{-21} when they reach earth. The effect of gravitational waves on the detector is to change the distance between test masses, and will be read by optical phase changes in an advanced version of a Michelson interferometer.

Pound-Drever-Hall reflection locking

The maximum sensitivity of a non-resonant Michelson interferometer to gravitational waves is possible when the time light takes to propagate to the end of the arm and back is half the period of the gravitational wave.[2] To achieve this for a gravitational wave with 200Hz frequency, the length of the interferometer would need to be nearly 400km. This is larger than feasible for an Earth based detector. Because of this, LIGO has been designed with optical resonators in the arms to allow light to make several round trips before detection. The form of the resonators is a Fabry-Perot cavity in each arm. In order for the cavities to be useful, they must be kept on resonance, which requires that the length of the cavity stays near an integer multiple of half wavelengths of the laser light. This can be achieved using a feedback loop system developed by Drever and Hall to lock a Fabry-Perot cavity on resonance.[3] Because the resonance condition of a cavity treats laser frequency and cavity length on equal footing, it is possible to vary either one to resonate with the other. This is used widely as a means to stabilize the frequency of a laser (as in LIGO), but the main use which is more important to the current subject is to control the cavity length to satisfy the resonance condition. In fact the authors of the 1983 paper revealing this technique even mention resonant gravitational-wave detectors as a possible application of their technique. This technique uses phase modulated laser light to construct an error signal to be used to lock the cavity.

A radio frequency (rf) signal is applied to an electro-optic modulator (EOM), also known as a Pockels cell, which creates the phase modulation. The maximum deviation of the phase introduced by the Pockels cell is called the modulation depth. A typical arrangement is shown in Figure 1.

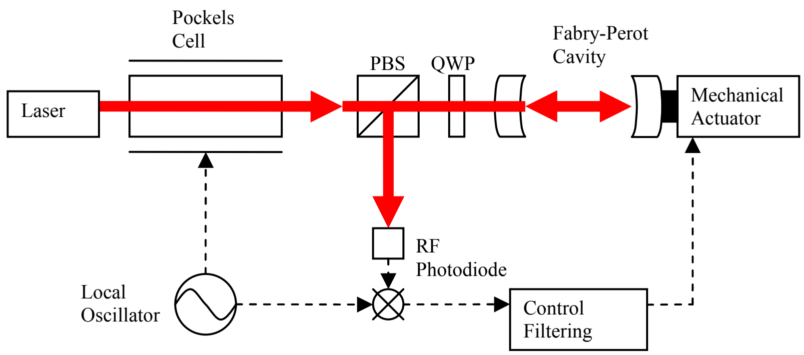

Figure 1: Pound-Drever-Hall reflection locking

The effect of the Pockels cell is to add rf sideband components to the laser beam. The beam can be used to interrogate whether the cavity is satisfying resonance and can be used to create a signal to control the cavity. The phase of the carrier has a large dependence on the mirror positions near resonance, while the sidebands are kept far from resonance and have roughly constant phase. The beat note between sidebands and carrier produced at the photodiode can be used to measure the phase of the carrier light (relative to the sidebands) and hence measure if the cavity is resonating. Demodulation of the beat note provides a control signal which can be fed back to a mechanical actuator to hold the cavity on resonance. A more detailed discussion of the reflection locking is given in appendix B.

The mirrors act as test masses that can be influenced by a passing gravitational wave. This wave will perturb the length of the cavity and begin to push it off resonance. The control system will consider this action as an input and correct for this perturbation. It is then a matter of linear control systems theory to read the signal which was required to control the mirror and extract the gravitational wave “input” from that. The Pound-Drever-Hall technique can be generalized from this specific case to include several coupled cavities and is used extensively at LIGO for the detection and control of several longitudinal degrees of freedom.[4]

Phase Noise

The effects of phase noise are quite similar to the effects of phase modulation of a signal. Recall that the phase modulation of the laser light had the effect of placing sidebands symmetrically spaced from the carrier. Consider an oscillating signal with a time varying term which alters the phase with respect to a perfect harmonic signal: $e^{i[\Omega t + d\varphi(t)]}$. This phase altering term can be expressed as a sum of sinusoidal waves of different frequencies and amplitudes, the amplitudes take the role of the modulation depth. The effect of each term would be to place sidebands separated from the carrier by the noise frequency. This similarity between phase noise and phase modulation will come in handy when measuring the oscillator phase noise coupling.

The exact way that phase noise affects the output is not well understood in LIGO. Initial models using lower order laser profile modes has provided results that do not agree with experimental tests.[5]

Demodulation schemes

This paper also introduces a new technique to derive the local oscillator and demodulate the detected light in the hope to better understand the coupling of oscillator phase noise in the LIGO interferometers. This scheme uses the two-omega signal which can be measured by photodetectors at the anti-symmetric port. A voltage controlled oscillator is used in a phase locked loop circuit to re-derive the local oscillator reference from the two-omega signal. The so-called two-omega locking technique was expected to provide a better reference and cancel some phase noise, thereby reducing the coupling.

Experimental setup

The two-omega locking scheme

The two-omega locking scheme involved a procedure to derive the local oscillator using the beat note of the two sidebands available at the anti-symmetric port. The Michelson is locked onto a dark fringe for the carrier but not the sidebands due to the Schnupp asymmetry. This causes the sidebands to always be present and thus so is the two-omega signal. This signal can be passed through a frequency divider to recreate a signal at the modulation frequency. Two possible steps can be taken at this point, the newly created signal can be directly used for the demodulation or, to reduce noise, one may perform a phase locked loop which locks a voltage controlled oscillator to the modulation frequency.

The two-omega locking scheme is an alternate technique than the one currently used in LIGO which drives the demodulation directly from the signal which drives the Pockels cell. Figure 2 shows a schematic of each scheme.

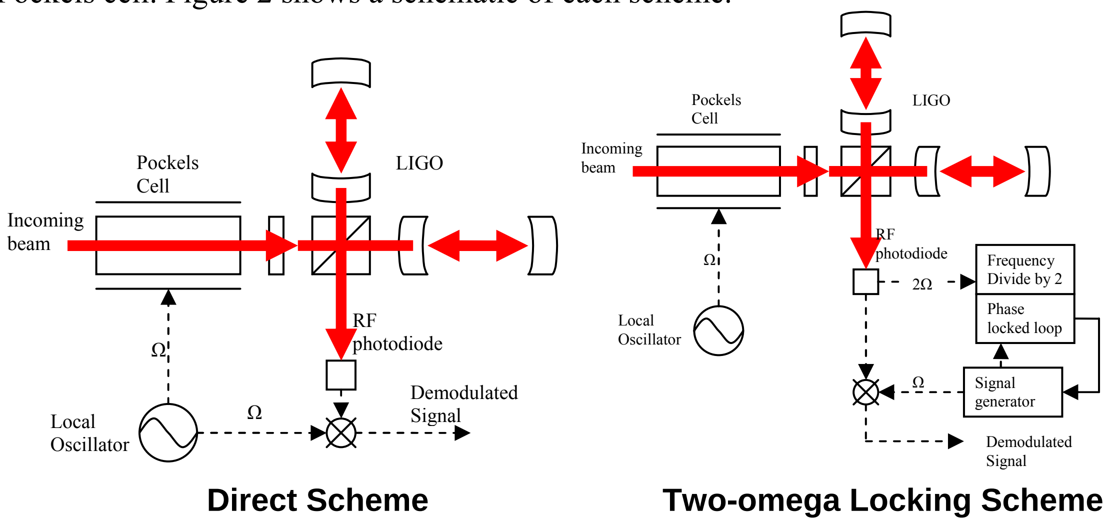

Figure 2: Demodulation techniques

The motivation for trying a new scheme for the demodulation has to do with the way the gravitational wave signal is extracted at the anti-symmetric port. The gravitational wave signal will exist as amplitude modulation (AM) occurring at the fundamental frequency (Ω) due to differential changes in arm cavity length. The specific channel is called AS_Q which stands for the quadrature component of the signal at the anti-symmetric port. Phase noise should not contaminate an AM reading. This assumes that the phase of the signal one attempts to extract and the phase of the local oscillator signal remain synchronized after the signal has passed through the optical system. This, of course is not always a valid assumption, especially in a system as complex as LIGO. When measuring the AM, one wants the demodulation reference to remain in phase with the signal. Supposing the signal phase varies with respect to the local oscillator, the demodulated signal will have FM (frequency modulation) components present, which includes phase noise. This is because the modulation phase and the FM noise are no longer orthogonal. This effect can be mitigated if the local oscillator varies in the same way as the main signal. The demodulation phase will "track" the signal phase even if it is somewhat noisy. This should keep the FM orthogonal from the demodulation and out of the final output signal.

To make phase noise coupling measurements possible, the hardware necessary to re-derive the local oscillator signal had to be designed and built. The phase locked loop circuitry consisted of an analog to digital stage for each the two omega input and the voltage controlled oscillator inputs. The two-omega signal is fed through a flip flop to perform a frequency divide by two. The divided two omega and the oscillator signals are then compared at a digital phase detector and the output was filtered and fed to a differential amplifier. Then, the signal is amplified and made available for controlling the frequency of the voltage controlled oscillator. There also exists a stage where excitation can be injected into a summing junction for open loop transfer function measurements. The control system bandwidth was tuned to be 21 kHz. A schematic is included in appendix A as well as open loop transfer function measurement results.

Transfer function measurements

Phase noise coupling, in this paper, is quantified as a transfer function in units of meters displacement noise (at the anti-symmetric port) divided by radians of modulation. Transfer function measurements were performed on the LIGO 2 kilometer interferometer in Hanford. Transfer functions can be determined experimentally by doing a swept sine. A signal consisting of a sine wave at a single frequency and known amplitude is injected into the modulation, and then the amplitude of the signal at the same frequency is measured at the anti-symmetric port. The ratio of the output amplitude to the input amplitude as well as the phase difference makes up one data point of a Bode Plot. This is repeated as one sweeps the frequency and records points for each frequency.

LIGO is set up with a network which allows measurements to be made with software commands. One needs to physically attach an excitation channel to the hardware where the injection will take place and define which channel will be measured. Phase modulation is not possible with the oscillator which runs the modulation distribution nominally at LIGO. The modulation distribution was instead driven by an IFR 2023A frequency synthesizer capable of phase modulation, and the excitation channel was fed into the modulation input of the frequency synthesizer.

Transfer function calibration

The measured transfer function must be calibrated so that the units represent relevant observable quantities. For a swept sine measurement this must be done for both the excitation and measurement channels.

The calibration for the anti-symmetric port channel (called H2:LSC-DARM_ERR) was obtained by using a Matlab script. This calibrates the signal measured at the anti-symmetric port to meters of cavity length displacement. The calibration is shown in figure 3.

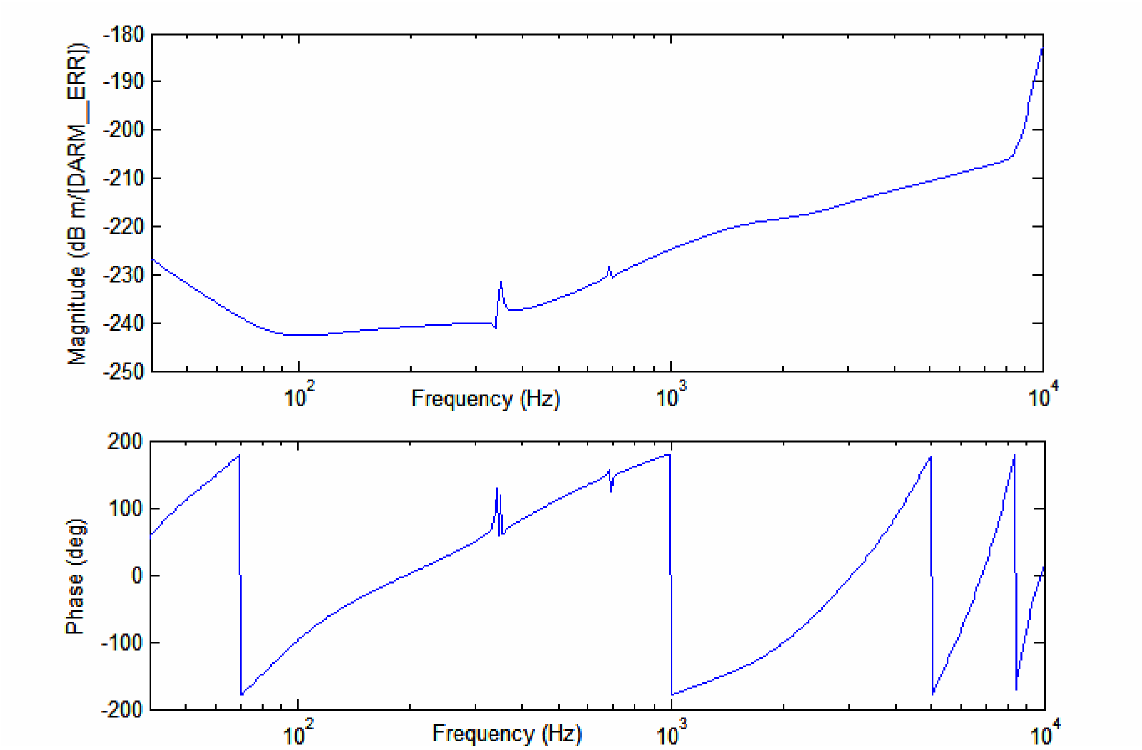

Figure 3: DARM_ERR displacement calibration

Additionally the excitation channel (H2:LSC-MOD_3_EXC) had to be calibrated. To do this, one must determine the transfer function from the modulation input to the phase modulation depth. This was done by a swept sine. The input being the IFR external modulation port and the output was the signal after being demodulated.

There also existed a delay in the excitation channel which was measured and accounted for in the calibration.

The phase modulated output of the frequency synthesizer consists of a carrier and two sidebands. The amplitude of each can be used to determine the phase modulation depth. The relationship of the phase modulation depth (Γ) and the ratio of the sideband amplitude to the carrier amplitude (A_{sb}/A_c) is given as:

$$\Gamma = 2\frac{A_{sb}}{A_c} \qquad (1)$$

This relationship is an approximation which is valid while the modulation depth is small. The amplitudes of the signal components were measured with a radio-frequency spectrum analyzer. Data were taken for a single frequency (1 kHz).

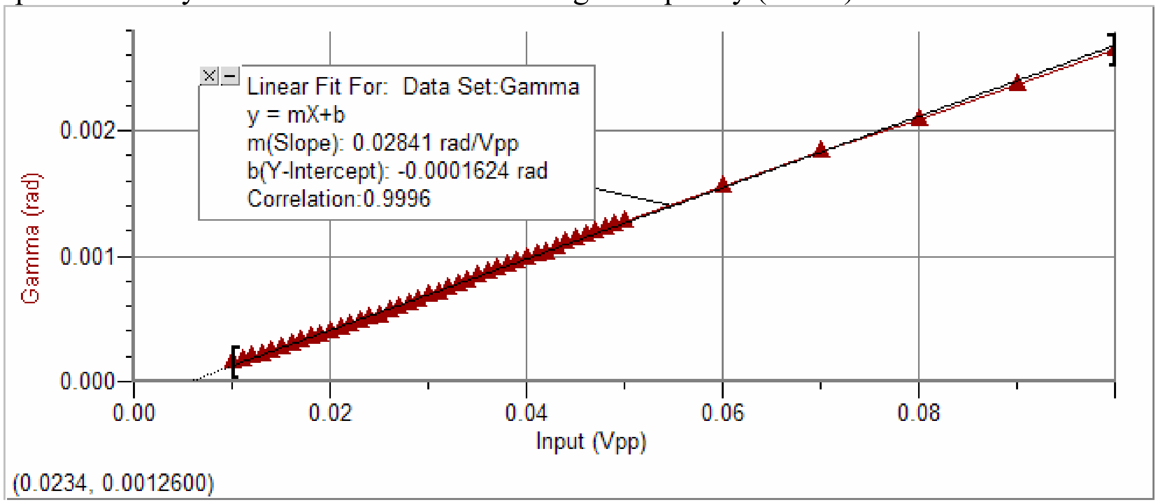

Figure 4: Input modulation amplitude (Volts peak-to-peak) versus modulation depth (rad)

The slope of the linear fit curve defines a conversion constant used to relate input modulation signal amplitude to modulation depth. At 1 kHz and a modulation deviation setting of 0.1 radians, the calibration constant is: 0.0284±0.0002 rad/V_{pp}. The modulation transfer function was then rescaled so that it intersected with the measured value at 1 kHz. This ensures that all other measured points are in the correct place relative to the single measured point. The combined calibration is shown in figure 5.

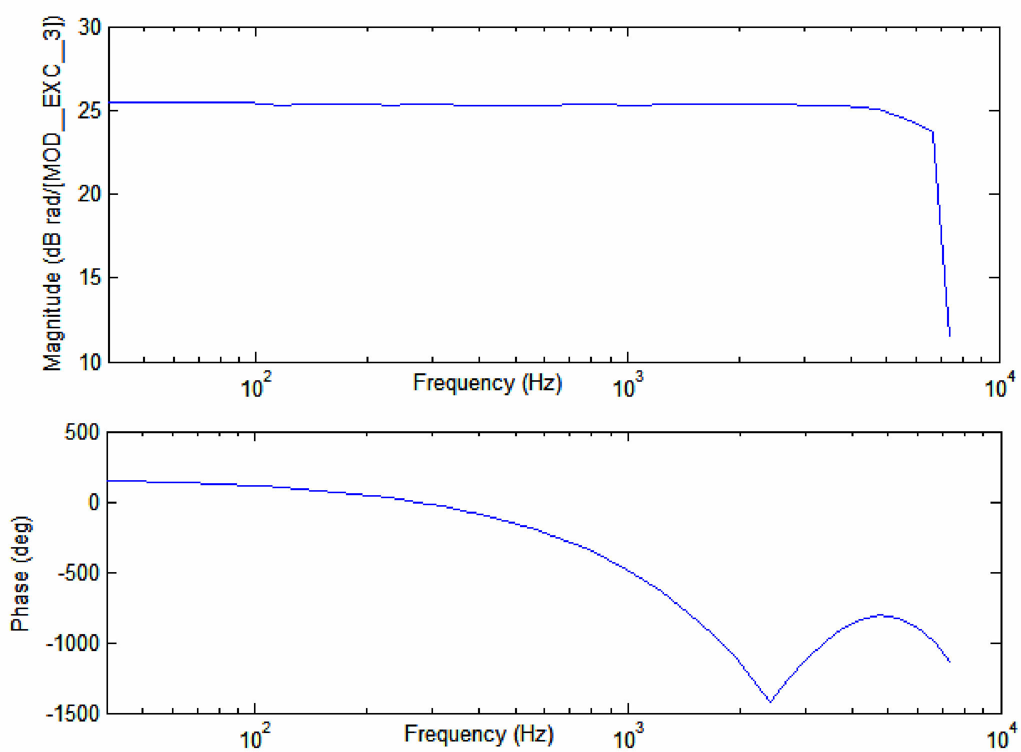

Figure 5: MOD_3_EXC modulation depth calibration

It is then possible to apply these calibrations to the excitation and measurement channels. This gives transfer functions in as m/rad.

Transfer function test

When the modulation distribution was being driven by a frequency synthesizer (IFR) it was less stable than the crystal oscillator normally in place. The displacement noise of the detector increased as a result of this.

The phase noise of the IFR was measured by feeding a signal into a mixer with a reference signal which has much lower phase noise. The mixer output was then read on a spectrum analyzer and the power spectrum represents the phase noise of the signal generator. Car had to be taken to make sure that the reference is 90° out of phase with the signal being interrogated. The phase noise spectrum of the frequency synthsiser is shown in figure 6.

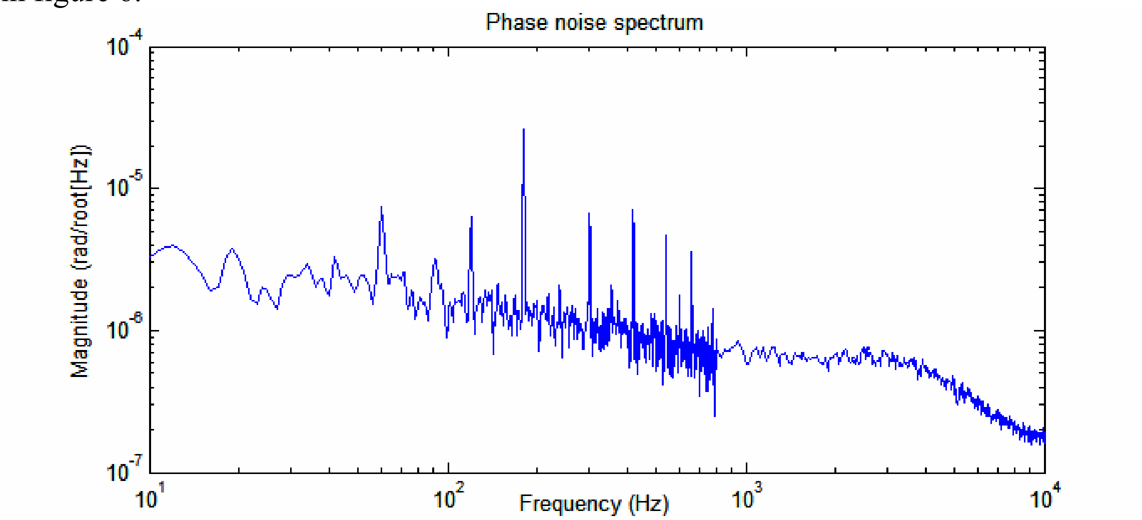

Figure 6: IFR2023A Phase noise

This spectrum can then be multiplied by the measured transfer function of the oscillator phase noise coupling to obtain a prediction for the displacement noise contribution from oscillator phase noise. In figure 7, this is compared to two measurements of the total displacement noise of the detector, one while the IFR is driving the modulation distribution, the other when the less noisy crystal oscillator is driving the modulation.

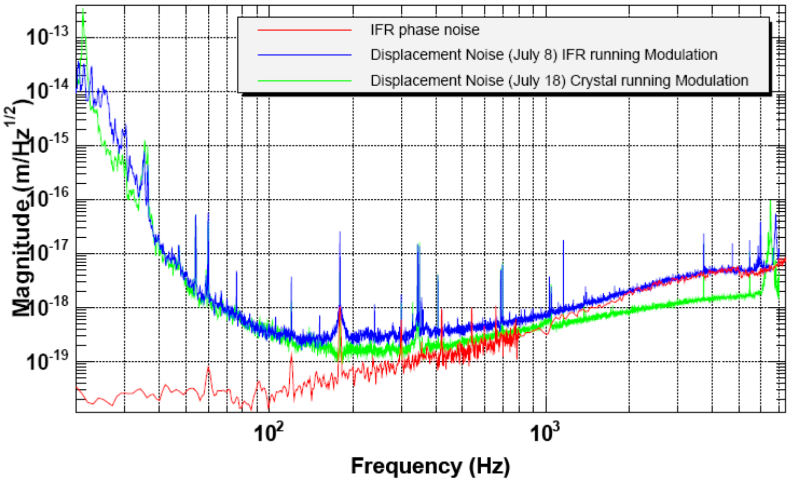

Figure 7: Displacement noise prediction

Figure 7 depicts the displacement noise as a result of oscillator phase noise in the IFR frequency synthesizer. It shows that at higher frequencies the oscillator phase noise of the IFR frequency synthesizer becomes dominating. The displacement spectrum change is correctly predicted by our measurement.

Results

The main result of this paper is a comparison of the oscillator phase noise coupling of two demodulation schemes: the direct scheme and the new two-omega locking scheme. Transfer function measurements were taken for both demodulation schemes described. The results are shown in figure 8.

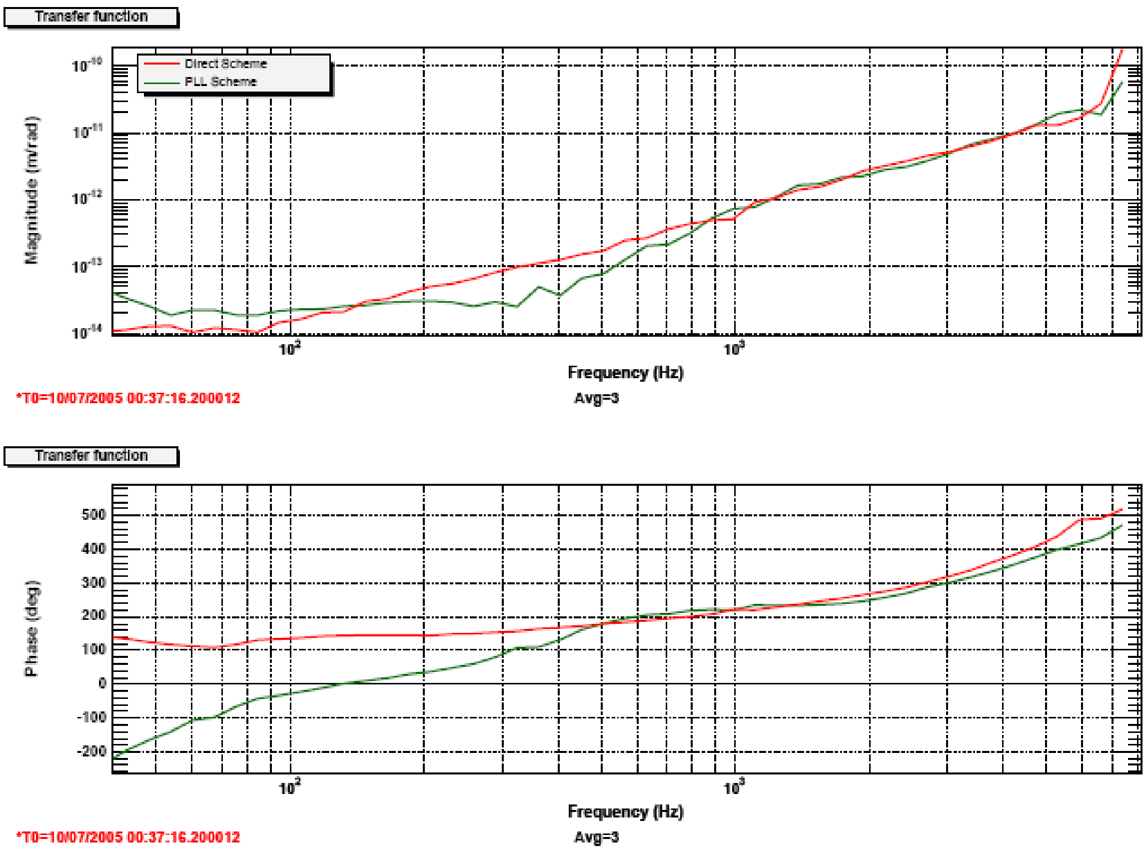

Figure 8: Oscillator Phase Noise Coupling

The two-omega locking scheme (given in green) provides an advantage in a frequency band between ~150-900 Hz where the coupling is lower than the direct scheme (red). Above 1 KHz, the two schemes are indistinguishable. Below 100 Hz, there is greater coupling than the original scheme.

The new scheme provides a maximum improvement of a factor of roughly 5 in the general region of current best sensitivity of the detector. The scheme was developed as a means to decrease coupling at higher frequencies. The data show that this is not the case and the oscillator phase noise coupling at frequencies above 1 kHz is dominated by a different mechanism.

Conclusions

A conclusion that can be drawn from our measurement is that the phase noise does not stay "intact" as it propagates through the LIGO detector. Most likely, the interferometer is converting the phase noise of the oscillator into amplitude noise which is indistinguishable from the real signal at the anti-symmetric port. These noise terms cannot be reduced by a better local oscillator reference. This is true for frequencies above 1 kHz where the two schemes have little difference. At frequencies where the two-omega scheme has lower coupling, the improved reference canceled phase noise coupling as was hoped. At frequencies below 150 Hz where the new scheme has even higher oscillator phase noise coupling than before, there are sources of noise coupling that the direct local oscillator signal was immune to, while the two-omega locked signal is not.

A possible way to determine where in the detector oscillator phase noise is converted into AM noise would be to do similar transfer function measurements at different points in the detector. One could measure the coupling of the oscillator phase noise to the light measured at several stages of the detector, such as before and after the input mode cleaner, or reflected from the recycling cavity.

Appendix A

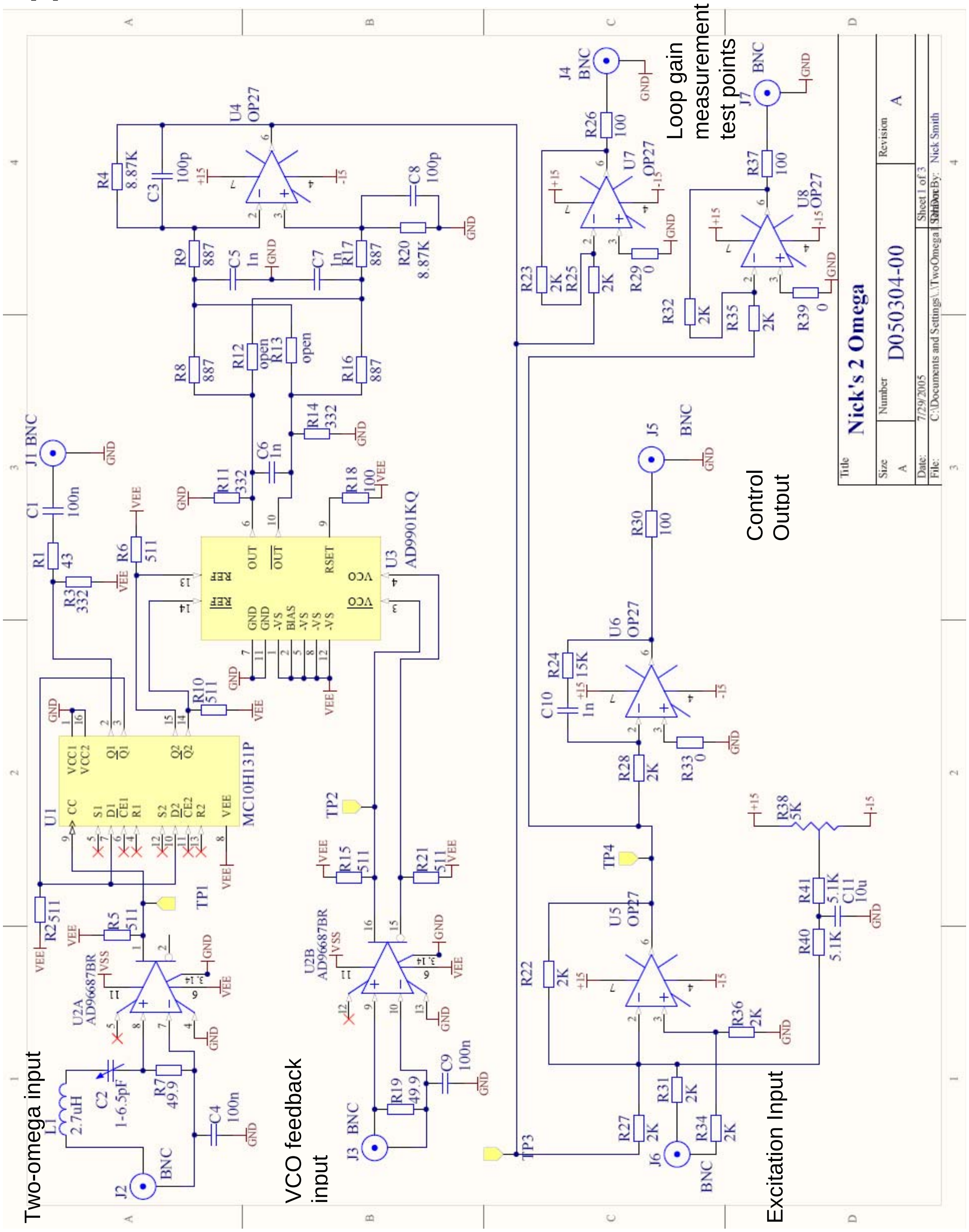

Figure 9: Two-omega phase locked loop circuit schematic

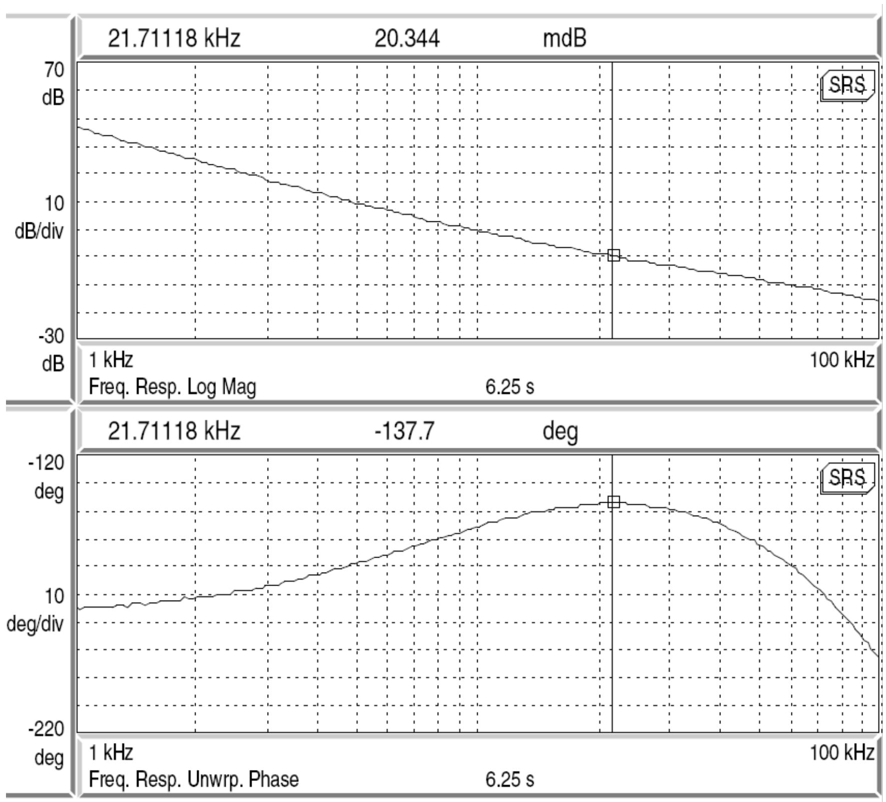

Figure 10: Phase locked loop open loop transfer function.
Cursor set on unity gain frequency

Appendix B

The effect of phase modulation on the incoming laser light is to add two sidebands (to a first order approximation) which occur on either side of the original frequency, which is referred to as the carrier. This can be seen mathematically by adding a cosine term to the phase of the laser light. It is useful to consider the electric field of the laser light as a phasor.

$$E_{laser} = e^{[i\omega t + i\Gamma\cos(\Omega t)]} \qquad (2)$$

In this equation, ω is the angular frequency of the light which is about 10^{15} rad/s, Γ is a parameter known as the modulation depth, and Ω is the modulation frequency (29.5MHz/2π for the 2K interferometer), also the carrier amplitude has been normalized to unity. The sum in the exponent can be represented as a product and one of the exponentials can be expanded to its first two terms.

$$\begin{aligned} E_{laser} &= e^{i\omega t} \cdot e^{i\Gamma\cos(\Omega t)} \\ &\approx e^{i\omega t} \cdot [1 + i\Gamma\cos(\Omega t)] \\ &= e^{i\omega t} \cdot [1 + \frac{i\Gamma}{2} e^{i\Omega t} + \frac{i\Gamma}{2} e^{-i\Omega t}] \\ &= e^{i\omega t} + \frac{i\Gamma}{2} e^{i(\omega+\Omega)t} + \frac{i\Gamma}{2} e^{i(\omega-\Omega)t} \end{aligned}$$

Of course in reality energy is conserved and the modulation process does not contribute any energy to the laser beam and so the amplitude of the carrier must diminish, but the approximation is valid as long as Γ is small. The true amplitude of the carrier is $J_0(\Gamma)$ and of both sidebands are $J_1(\Gamma)$, where J_n is a Bessel function of the nth kind.

The laser light must now be used to probe the status of the cavity. The effect that the cavity has on light which reflects off the near mirror is an essential part to understanding the method for locking the carrier into resonance with the cavity. This can be achieved by choosing the parameters of the cavity in order to make the carrier near resonant and the sidebands far from resonance. The ideal picture puts the sidebands directly between resonances. This can be achieved by choosing a modulation frequency which is half of the free spectral range of the cavity. If the sidebands are far from resonance, they can act as a phase reference which then can be compared to the phase of the carrier. The transfer function for light reflected from a cavity is:

$$\begin{aligned}\frac{E_{ref}}{E_{in}} &= -r_1 + \frac{r_2 t_1^2 e^{-i\omega 2L/c}}{1 - r_1 r_2 e^{-i\omega 2L/c}} \\ &= \frac{r_2 e^{-i\omega 2L/c} - r_1}{1 - r_1 r_2 e^{-i\omega 2L/c}}\end{aligned} \tag{3}$$

Here, r_1 and r_2 are the amplitude reflectivities of the front and rear mirror, respectively, t_1 is the front mirror amplitude transmittance, ω is the angular frequency of the light, L is the length of the cavity and c is the speed of light.

We are interested in the phase of the reflected light near resonance, in the case of the carrier, and far from resonance in the case of the sidebands.

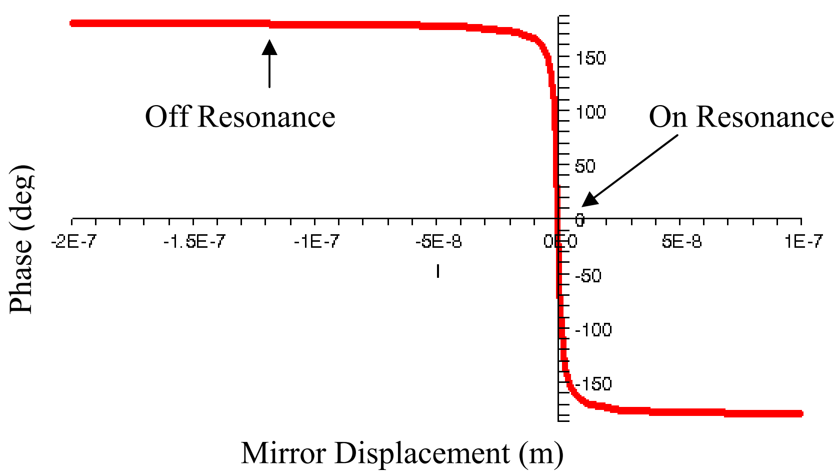

Figure 11: Resonance dependence of beam phase

As shown in figure 11, the phase of the carrier changes rapidly as the cavity moves through resonance. The photodetector, which measures the light power, provides a signal when the carrier and sidebands beat against each other and generate AM. When one demodulates the signal from the photodetector with the local oscillator frequency, a signal is produced which is shown in figure 12.

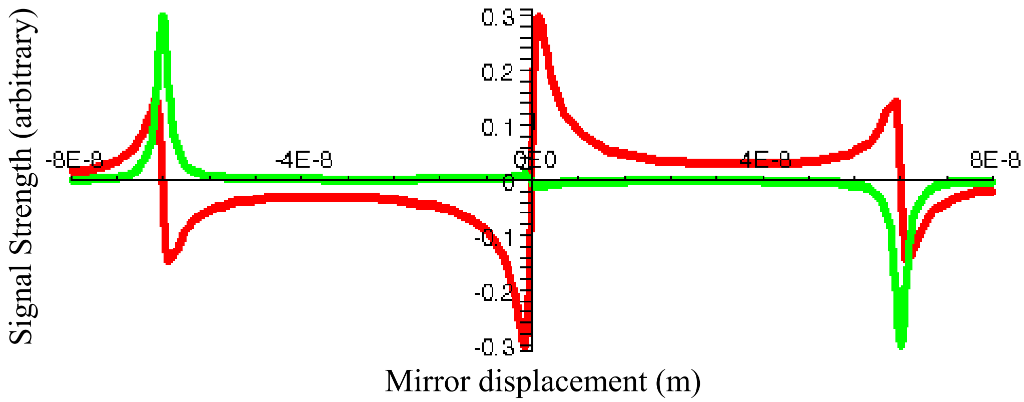

Figure 3: Demodulated Signal. Red: In-phase. Green: Quadrature phase.
Numerical values used: front mirror power reflection: 3%, rear mirror reflection: 100%, cavity length: 4km, laser wavelength: 10^{-6} m, modulation frequency: 25MHz, modulation depth (Γ): 0.3.

Close to resonance the in-phase demodulated signal acts as an error signal which indicates how far from resonance the cavity length has drifted, and includes a positive or negative sign indicating which direction it has moved. Sending the error signal through a servo control system to a mechanical actuator provides a closed feedback loop which will keep the cavity on resonance.

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[1]Taylor, J. H.; Weisberg, J. M. "A new test of general relativity - Gravitational radiation and the binary pulsar PSR 1913+16" Astrophysical Journal, Part 1, vol. 253, Feb. 15, 1982, p. 908-920.
[2] Peter Saulson *Fundamentals of Interferometric Gravitational Wave Detectors* p. 85 World Scientific Publishing, 1994
[3] R. W. P. Drever, J. L. Hall, F. V. Kowalski, J. Hough, G. M. Ford, A. J. Munley, and H. Ward, "Laser phase and frequency stabilization using an optical resonator," Appl. Phys. B 31, 97-105 (1983)
[4] P. Fritschel, R. Bork, G. Gonzáles, N. Mavalvala, D. Ouimette, H. Rong, D. Sigg, M. Zucker, "Readout and control of a power-recycled interferometric gravitational-wave antenna," APPLIED OPTICS, Vol. 40 No. 28 (1 October 2001)
[5] Daniel Sigg, personal communication