Seismic and Thermal Noise Peaks from Blade Internal Modes in an ETM/ITM Quadruple Pendulum.

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1. Introduction.

The seismic isolation expected from the ETM/ITM quadruple pendulum in the region around 10 Hz has been estimated using the MATLAB model of the quad suspension. (See for example "Advanced LIGO Suspension System Conceptual Design", T010103-03-D). The overall isolation achieved with the quad pendulum in conjunction with the two-stage active isolation platform should allow us to meet the target figure of $10^{-19}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$ at 10 Hz. Above this frequency the noise level is expected to fall as a strong function of frequency (~$1/f^7$ from the quad pendulum) and in general lie well below the target Advanced LIGO sensitivity curve. However this fall-off does not continue indefinitely. In particular the vertical transfer function of the stages supported by blades will flatten out at higher frequencies due to the finite masses of the blades. In addition the blades have internal modes which will be excited by both residual seismic noise and thermal excitation. We require to estimate the peak heights of these resonances to establish whether the peaks could compromise the overall sensitivity if they remain undamped, and if so, to estimate how much damping is required to reduce the peaks to lie below the sensitivity curve.

NAR has previously investigated the expected internal mode peak heights excited by seismic noise, and presented the results in a document circulated to the SUS group in April 2004, which can be accessed at
http://www.ligo.caltech.edu/~ctorrie/QUAD_ETM/blade_internal_noise.xls
Information from that document is repeated here, with more explanation in the section on how the design was arrived at. The analysis is extended to include consideration of thermal excitation of the peaks.

2. Blade design.

The following dimensions were chosen for the three sets of blades for the quad controls prototype.

i) top blades: length 48.0 cm, width 9.50 cm, thickness 4.3 mm, f = 2.33 Hz, int f = 70 Hz, stress 980 MPa
ii) middle blades: length 42.0 cm, width 5.90 cm thickness 4.6 mm f = 2.48 Hz, int f = 98 Hz, stress = 990 MPa
iii) bottom blades: length 37.0 cm, width 4.90 cm, thickness 4.2 mm, f = 1.81 Hz, int f = 116 Hz, stress = 980 MPa

The frequencies and stress levels are calculated from the blade dimensions and alpha value (shape factor) using a set of blade equations based on work by the Virgo group, and can be found for example in Calum Torrie's PhD thesis. The stress value corresponds to the maximum stress in the loaded blade. The "int f" is the estimated first internal mode frequency using a simple scaling extrapolation from previously measured blades.

Some background notes on how this design was arrived at are given below.

1) Assume alpha = 1.35 as a working value, as per discussion CIT and NAR had with Mike Plissi late March 2004. This value was chosen after evaluating the results of measurements on various blades used in GEO.
2) Assume we can use a slightly higher stress: - set upper value at 1000 MPa for all blades. (Previously we had considered using 800 MPa for those blades lowest in chain, and 850 and 900 MPa for the middle and top blades respectively). Justify for two reasons. a) Longer heat treatment can improve strength and b) we might move to maraging 300 in later prototypes.
   Further to the above note, the current material of choice is maraging 250 and we are unlikely to change this - see T040108-00-K,Blade process specification, Greenhalgh et al.
3) Put in realistic masses (sapphire with flats and ears 39.6kg, SF2 as penultimate mass also with flats and ears, 38.4kg), top two masses at 22kg each.
   Note added here. As a result of the downselect to silica, the blades may change slightly due to slightly different mass, but this should not be a significant effect. The analysis should however be repeated when the silica suspension design is developed (see conclusions section).
4) Keep the length and width of upper two sets approx. as in conceptual design, T010103-03-D, aim to gain some improvement in isolation by increasing length of lowest set.
5) Keep the blade internal mode frequencies reasonably separate (at least by 15 Hz) to avoid chance of overlap.

Justin Greenhalgh has carried out FEA analysis on blades with these dimensions, and the results for frequencies ("mass on spring" and internal mode frequencies) agree well with the numbers given above. See T040061-01-K "Transmissibility of a revised set of blades", J Greenhalgh, April 2004.

3. Seismic excitation of peaks.

I have taken Justin's FEA analysis on transmissibility of the blades, calculated assuming Q of 10^4 for maraging steel, and combined this with an assumed input noise level and transmissibility of the final stage to estimate the residual seismic noise at the first internal modes of the blades. The results are shown below. The peak height transmissibility (column 3) is the combined vertical transmissibility of the three blade stages as measured at the frequencies of the internal modes. To calculate the total transmissibility from the top of the suspension to the test mass, the values in column 3 are multiplied by those in column 4 (the vertical transmissibility of the final stage on its silica suspension). The longitudinal noise at the test mass (column 7) is calculated by multiplying the entries in columns 3, 4, 5 and 6, where the residual noise level on the seismic platform (column 6) is taken from the Seismic Design Requirements Document (E990303-03-D). The target sensitivity per test mass (column 9) is given by target sensitivity (column 8) *4000. The final column is the ratio column9/column7.

1) Blade	2) freq of first int. mode (from FEA)	peak height transmissibility	transmissibility	X-coupling factor	Seismic platform residual	long. noise	target	target	factor below
	fm (Hz)	(transmissibility	of final stage	(vert. to long.)	vert. noise	at test mass	sensitivity*	sensitivity	target s'tivity
		of blade stages at resonance)	(fo/fm)^2		(m/rtHz)	(m/rtHz)	h (1/rtHz)	per test mass	
								(m/rtHz)	
top	69.4	6.45E-03	7.73E-03	1.00E-03	3.00E-14	1.49E-21	4.00E-24	1.60E-20	1.07E+01
middle	96.6	9.32E-03	3.99E-03	1.00E-03	3.00E-14	1.11E-21	3.00E-24	1.20E-20	1.08E+01
bottom	113.6	5.23E-03	2.88E-03	1.00E-03	3.00E-14	4.52E-22	2.50E-24	1.00E-20	2.21E+01

The following parameters were assumed in this spreadsheet.

uncoupled vert freq. of final stage, fo (Hz)	6.1
cross-coupling factor	1.00E-03
active platform residual noise (m/rtHz)	3.00E-14

The target sensitivity values are taken from Peter Fritschel's SPIE paper on Advanced LIGO from 2002 (P020016-00-R). That sensitivity curve in that paper was drawn for sapphire assuming no coating loss. The curve will change slightly for silica plus coating – see conclusions section for further discussion.

The right hand column shows the factor by which the seismically excited peak due to one blade lies below the target sensitivity. However we should multiply by √2 to allow for uncorrelated addition of two blades. The noise from the top two sets of blades would then lie only a factor of ~7 below the target sensitivity, and hence some damping may be required. See the conclusions section for more discussion.

4. Thermal excitation of peaks.

We now consider how much thermally excited motion will be present at the internal mode peak frequencies. This analysis follows a method suggested by P Fritschel (ref. e-mail).

For a resonant system of mass *m* and angular resonant frequency *ω*, the rms motion is given by

$$x_{rms}^2 = \left(\frac{kT}{m\omega^2}\right) \qquad (1)$$

The most important blades for thermal noise considerations are the lowest set, nearest to the test mass, since noise associated with the blades further up the chain is better isolated at the test mass. For this blade, we have $m = 0.31$ kg, and $\omega = 2\pi f_b$ where $f_b = 114$ Hz. To calculate the amplitude spectral density we divide by the root of the bandwidth Δf where $\Delta f = f_b /Q$, and hence find, with $Q = 10^4$,

$$x = 1.5 \times 10^{-12} \text{ m}/\sqrt{\text{Hz}}$$

The resulting displacement at the test mass is given multiplying by the vertical transmissibility of that stage* (the transmissibility treating the blade as a rigid body), the vertical transmissibility of the final stage on its silica suspension and the cross-coupling factor between vertical and horizontal, giving

$$x_{\text{from one blade}} = 1.5 \times 10^{-12} \times 3\times10^{-3} \times (6.1/114)^2 \times 10^{-3} = 1.3 \times 10^{-20} \text{ m}/\sqrt{\text{Hz}}$$

We should further multiply by $\sqrt{2}$ to take account of the two blades at the lowest stage, giving

$$x_{\text{test mass}} = 1.8 \times 10^{-20} \text{ m}/\sqrt{\text{Hz}} \text{ at } 114 \text{ Hz}$$

This should be compared with a level 10 times lower than the target sensitivity per test mass at that frequency, namely ~ 1×10^{-21} m/$\sqrt{}$ Hz. The estimated noise is around 18 times too high. Since noise level is proportional to $\sqrt{Q}$ we would require to damp to a residual Q_r given by $(10^4/Q_r)^{1/2} = 18$, or $Q_r \sim 30$.

Calum Torrie has performed some lab tests of damping a set of blades of the above specifications, using eddy current damping, achieving a Q ~ 25 (ref to follow), which would be adequate according to the above estimation.

We should consider the blades at one stage above. The thermal noise from those blades will be similar to the set below (similar mass and similar frequency), but the transmissibility should be reduced by one further stage of vertical isolation due to that stage itself, which for the middle blades should flatten out at around 10^{-2}. This would appear to be adequate to reduce the peaks below the target level. However the vertical isolation of the lowest blade stage at the resonant frequency of the middle set of blades (around 97 Hz) will not be as good as its lowest value, since the transmissibility will be rising towards its first resonant peak at 114 Hz, and hence some damping might be required.

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*The vertical transmissibility of a blade/wire/mass stage was calculated following the equation derived in Matt Husman's PhD thesis, where the mass and moment of inertia of the blade are taken into account.

5. Conclusions and Future Work.

From the numbers presented above, we can conclude that the blade most likely to need damping is the lowest blade from thermal noise arguments, and provision for including damping here has been made in the controls prototype design. Other blades may also need damping. However before we can definitively conclude which blades require damping, and by how much, more design work and experimental measurements are required, as follows.

1) We are still gathering information on the actual performance parameters of the blades manufactured for the controls prototype, and these may not be the same as assumed above. In addition, information on the as-measured spring constants and internal mode frequencies will be used to feed back into possible design changes we may want to make to optimize overall performance.

2) With the decision to use silica rather than sapphire for the test masses, the blade design may change slightly going from the controls prototype (where the design was chosen as if for sapphire) to noise prototype and final design, over and above any design changes which result from the experimental tests mentioned in 1).

3) When the blades are in situ in the suspension the observed Qs may not be as high as 10^4 (assumed above) due to coupling or clamping effects. We may be able to measure the in situ Qs by suitably exciting the suspension and observing the resulting transfer function.

4) We note that the target sensitivity used in the estimates above should be updated to take into account the choice of silica rather than sapphire as the test mass material, and better knowledge of the likely coating losses. In fact the sensitivity curve for internal thermal noise of silica with coating loss included, using "nominal" numbers for the parameters, yields numbers very similar to those quoted in the table above for sapphire. See the document "Performance comparison of fused silica and sapphire mirrors", 7/20/2004, at
http://emvogil-3.mit.edu/%7Epf/downselect/substrate_comparison.htm
In this document it can be seen that the silica and sapphire performance overlap around 100 Hz, where the blade resonant frequencies lie.
However we should continue to check on these target numbers as further work is done on measuring internal Qs of silica pieces and investigating coating losses. Also – we should really compare to the total expected noise at these frequencies rather than simply the internal + coating noise. The quantum noise will add to this noise.

5) We need to consider whether eddy current damping of the blade internal resonance has a sufficiently adverse effect on the quality factor of the first resonance of the blade/wire system such that the low frequency thermal noise performance is degraded. Under those circumstances we might need to reconsider its use. We may get an idea of the effect of

the eddy current damping on that resonance from measurements on the controls prototype.

6) We should consider the possibility of excess noise being introduced due to the interaction of the damping magnets with environmental magnetic fields or magnetic fields due to suspension control systems.

In conclusion, the estimates of peak heights for thermally and seismically excited motions of the blades, and the necessity and degree of damping, should be revisited once we have more information as indicated above. If the considerations in points 5) and 6) above suggest that when using enough damping to take any peak below the expected sensitivity level by a factor of 10 such damping has an adverse effect on noise performance in other ways, we may refer the matter to SYS to consider whether we can accept a larger peak.
