LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

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Mass-Bending Calculator

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Mass-bending (A. Grant)

1 Introduction

In the document LIGO-E020811-00-D, the authors take two approaches to the problem of finding the deflection of the ends of a mass carrying cantilever blades (in section 2.3). The deflection is due to the forces which the blade clamps supporting the loaded blades exert on the mass.

In the case of long crossed blades, these forces are the reaction "R" of the inner edge of the blade clamp acting downward, and the upward tension "T" of its fixing bolts. Although R is greater than T by the load on the blade, W, the nett effect is an upward deflection.

In the first approach, to find the deflection of the end, they find the upward deflection due to T using a cantilever beam formula in "Machinery's Handbook". This formula takes account of the distance of the force from the free end of the cantilever. They then subtract the downward deflection due to R to get the total deflection.

Their other method is to treat the forces as a couple consisting of T with (R - W), and a single downward force W. They calculate the upward deflection due to the couple using a formula also to be found in "Machinery's Handbook", but which applies to a couple acting at the end of the beam, not part way in from the end. Subtraction of the deflection due to W gives a total deflection which differs slightly from their first answer due to neglect of the distance of the centre of the couple from the free end.

2 Force - Force Model

The deflection, d, of a horizontal beam due to a vertical force, F, is given by...

$$d = \frac{F L^2}{6 E H} (2 L + 3 B) \tag{1}$$

where L is the distance of the force from the clamped end,
B is the distance of the force from the free end,
E is Young's Modulus,
H is the second moment of area.

This may be re-cast in the form...

$$d = \frac{F L^3}{3 E H} \left(1 + \frac{3 B}{2 L}\right) \tag{2}$$

where $\frac{F L^3}{3 E H}$ is the deflection at the force, F, and $(1 + \frac{3 B}{2 L})$ is a correction factor which takes account of the distance, B, of the force from the end.

3 Force - Moment Model

In the case of the couple [T with (R - W)], the deflection, d_c, at the centre of the couple is given by...

$$d_c = \frac{M L^2}{2 E H} \tag{3}$$

where L is the distance of the centre of the couple from the clamped end,
M is the moment of the couple,
E is Young's Modulus,
H is the second moment of area.

It can be shown that to take account of the distance, B, of the centre of the couple from the end of the beam, this formula has to be multiplied by the correction factor $(1 + \frac{2B}{L})$, giving the formula...

$$d = \frac{M L^2}{2 E H} \left(1 + \frac{2 B}{L}\right) \tag{4}$$

for the deflection due to a couple applied at a distance B from the free end of the beam.

4 Mass Bending Calculator

In this program, we first calculate the deflections uncorrected for the distance 'B' using equation 2 without its correction factor and equation 3. The appropriate correction factors are then applied, and since the distances of the forces and the moment from the end have been accounted for, the two methods now agree.

Title: **Mass-Bending Calculator (Force Model)**
Doc No: **T050031-00-K**

Author: Alastair Grant, Institute for Gravitational Research, University of Glasgow
Compiled by: M.Perreur-Lloyd, IGR, University of Glasgow
C.Cantley, IGR, University of Glasgow

Notes: This calculator serves as an update to the models in Dan Mason's SURF Report Document E020811. Both models in the original paper have been reviewed and the second model (Force-Moment model) has been supplemented with an additional calculation (See Force-Moment Tab and NOTES Tab at the bottom of the page)

Data:- Supplementary Notes

Blade

Length	c (m)	0.480	
Length of clamp	a (m)	0.040	
Load	W (N)	598.41	

Mass

E steel	E alu
1.93E+11	6.90E+10

Young's Modulus	E (Pa)	1.93E+11
Length of flexing section	l (m)	0.300
Breadth	b (m)	0.130
Thickness	d (m)	0.0167
Second moment of area	H (m^4)	5.04558E-08

Forces on blade

Reaction at edge of clamp	R (N)	13763.43
Total tension of bolts	T (N)	13165.02

Calculation of end deflection from sum of deflections of T and R (REF: LIGO-E020811)

Formula for deflection due to force at end, y:

$y = F*L^3/(3EH)$

Contribution of T-

Effective length, L (m)	0.28
Uncorrected upward deflection (m)	9.892E-03

Contribution of R-

Effective length, L (m)	0.260
Uncorrected downward deflection(m)	8.281E-03

Total uncorrected upward deflection (m) 1.612E-03 m

Correction for distance of force from end-

Distance of T from end, B (m)	0.02
Correction factor (1+(3B/2L))	1.107142857
Upward deflection due to 'T' (m)	1.10E-02
Distance of R from end, B (m)	0.040
Correction factor (1+(3B/2L))	1.230769231
Downward deflection due to 'R' (m)	1.02E-02

Total corrected upward deflection (m) 7.61E-04 m 0.76 mm

Title: **Mass-Bending Calculator (Force-Moment Model)**
Doc No: **T050031-00-K**

Author: Alastair Grant, IGR, University of Glasgow
Compiled by: M.Perreur-Lloyd, IGR, University of Glasgow
C.Cantley, IGR, University of Glasgow

Notes: This calculator serves as an update to the models in Dan Mason's SURF Report Document E020811. The model below has been supplemented with an additional calculation [REF: Section 3. Force-Moment Model in the preceeding document]

Data:-

Blade

Length	c (m)	0.480
Length of clamp	a (m)	0.040
Load	W (N)	598.41

Mass

Young's Modulus	E (Pa)	1.93E+11
Length of flexing section	l (m)	0.300
Breadth	b (m)	0.130
Thickness	d (m)	0.0167
Second moment of area	H (m^4)	5.04558E-08

Supplementary Notes

E steel	E alu
1.93E+11	6.90E+10

Forces on blade

Reaction at edge of clamp	R (N)	13763.43
Total tension of bolts	T (N)	13165.02

Calculation of end deflection from couple (T and R-W) and force W (REF: LIGO-E020811)

Formula for deflection due to couple at end,y :

$y = M*L^2/(2EH)$

Value of couple	M (N-m)	263.3004
Effective length	L (m)	0.27
Uncorrected upward deflection	y (m)	9.86E-04

Formula for deflection due to force at end, z :

$z = F*L^3/(3EH)$

Value of force	W (N)	598.41
Effective length	L (m)	0.260
Uncorrected downward deflection	z (m)	3.60E-04

Total uncorrected upward deflection (m) 6.26E-04

Correction factors:

Correction for distance of couple from end

Distance from end, B (m)	0.03
Correction factor (1+(2B/L))	1.22222222
Upward deflection due to couple (m)	1.20E-03

Correction for distance of force W from end

Distance from end, B (m)	4.00E-02
Correction factor (1+(3B/2L))	1.23E+00
Downward deflection due to force (m)	4.43E-04

Total corrected upward deflection (m) 7.61E-04 m 0.76 mm

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Diagram [as per description in section 1. Introduction]: