LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

LIGO-T040232-00-D *Advanced LIGO* 13 Dec 2004

Beamsplitter First Elastic Mode Frequency versus Dimensions (Advanced LIGO)

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1 Introduction

There are many factors that need to be considered in choosing appropriate dimensions for the beamsplitter for advanced LIGO (e.g. manufacturability, coating strain induced distortion, physical limits to fitting in the optical layout, clipping/diffraction loss, vibration isolation performance, etc.). This memo addresses the effect of beamsplitter diameter and thickness on the minimum elastic mode frequency and sets a minimum allowable frequency.

2 Modes for the Baseline Dimensions

The frequencies of the first 10 modes of the beamsplitter were calculated with the I-DEAS finite element code[1] and with the testmass5.c code[2,3], where the beamsplitter is approximated as a right circular cylinder. The baseline BS dimensions[4] are 60 mm thickness and 350 mm diameter. A more realistic model of the beamsplitter with a wedge angle of 1.3 degrees[5] and with flats of 95 mm in length[6] for mounting ears to weld to silica fibers was also analyzed with I-DEAS. For the wedged BS model, the minimum thickness was set to 60 mm (consistent with the manner of specifying thickness and wedge angle for the initial LIGO beamsplitter[7]). In all cases the fused silica properties reported in the appendix of Ryan Lawrence's thesis[8] have been used. The mass of the baseline BS is 12.66 kgm. The mass of the wedged BS with flats is 13.39 kgm.

The results of the modal analysis are compared in Table 1. Observations:

- the lowest frequency mode is the "saddle" mode (sometimes referred to within LIGO as the "butterfly" mode) at 2.7 kHz
- the I-DEAS and testmass5.c results compare well. The maximum error in the frequency for the first 10 modes is < 0.7% and in all cases the finite element result is higher (as would be expected in general due to artificial stiffening for a non-converged mesh).
- the effect of the wedge and flats is to increase the first mode by 5.0%.

[1] I-DEAS, version 9, SDRC, running on sargas.ligo.caltech.edu

[2] Kent Blackburn, Internal Modes of Testmasses, L9700042-00, 1/27/1997. Kent's code is a c-based, extended version of effmass.f originated by A. Gillespie and based on J.R. Hutchinson's paper on solid cylinder elastic modes.

[3] J.R. Hutchinson, Solid Cylinder Elastic Modes, J Appl Mech, 47, 901, Dec 1980.

[4] H. Armandula, et. al., COC Conceptual Design Document, LIGO-T000098-02, 6/20/2004.

[5] consistent with the wedge angle defined in D. Coyne, Optical Layout for Advanced LIGO, LIGO-T010076-01, 7/1/2001.

[6] The 95 mm length was set as the maximum for the testmasses based on clipping loss; it is not optimized for the beamsplitter. The testmass reference is P. Fritschel, Optical loss due to suspension mounting flats (Advanced LIGO), LIGO-T040008-00, 1/23/2004.

[7] see Beamsplitter Substrate, LIGO-D960789-B

[8] R. Lawrence, Active Wavefront Correction in Laser Interferometric Gravitational Wave Detectors, LIGO-P030001, 2/2003.

3 Lowest Mode Frequency with variation in BS Dimension

Which mode shape has the lowest frequency can vary depending upon the aspect ratio and properties of the material. Assuming that for dimensions similar to the baseline, the lowest mode shape is the saddle mode (or the [2,1,1] mode in the testmass5.c terminology), testmass5.c was used to calculate the variation in the lowest frequency with dimensions. The frequency was calculated for a set of 36 diameter and thickness combinations, which range between a diameter of 300 and 600 mm and a thickness of 40 and 90 mm (where the mass is not held constant). Contour plots of the results are shown in Figures 1 and 2.

Figure 1: Effect of Beamplitter thickness and diameter on the lowest elastic mode frequency

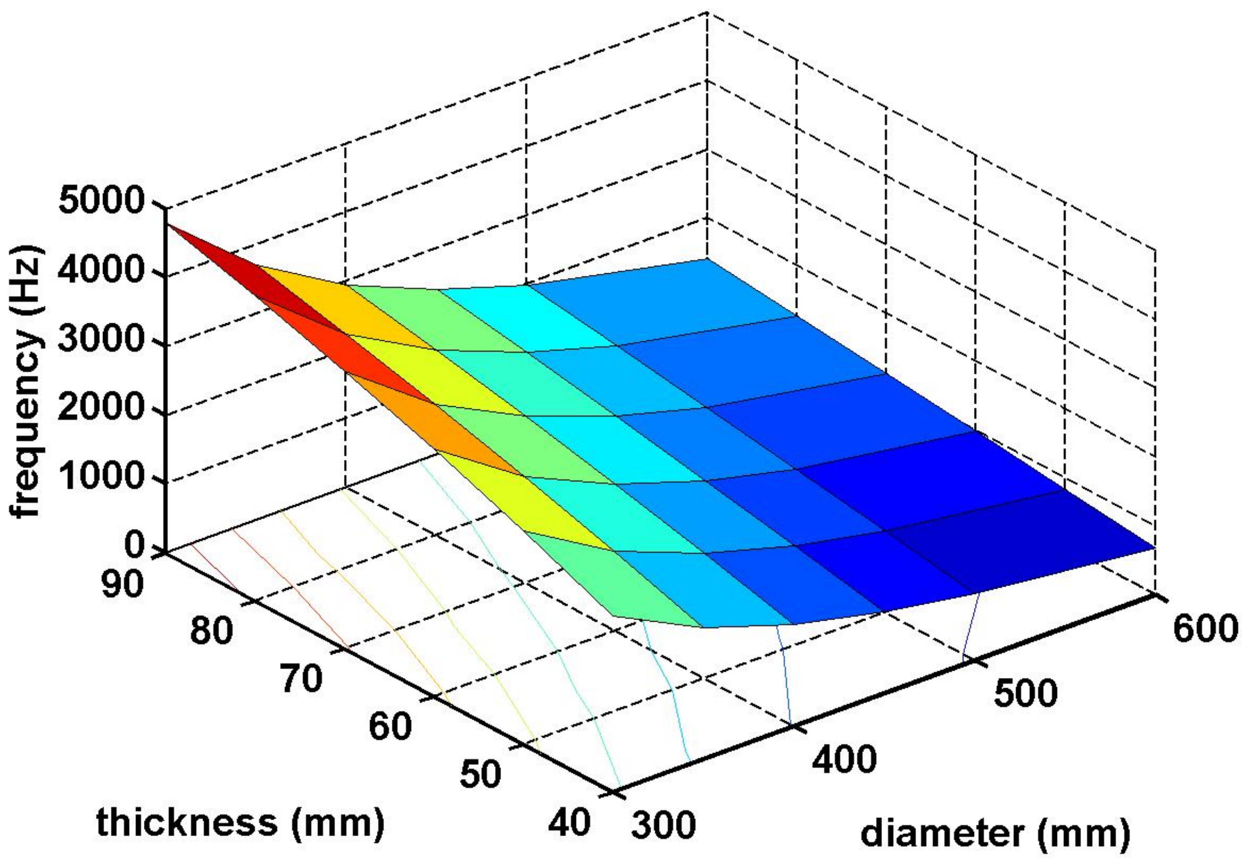
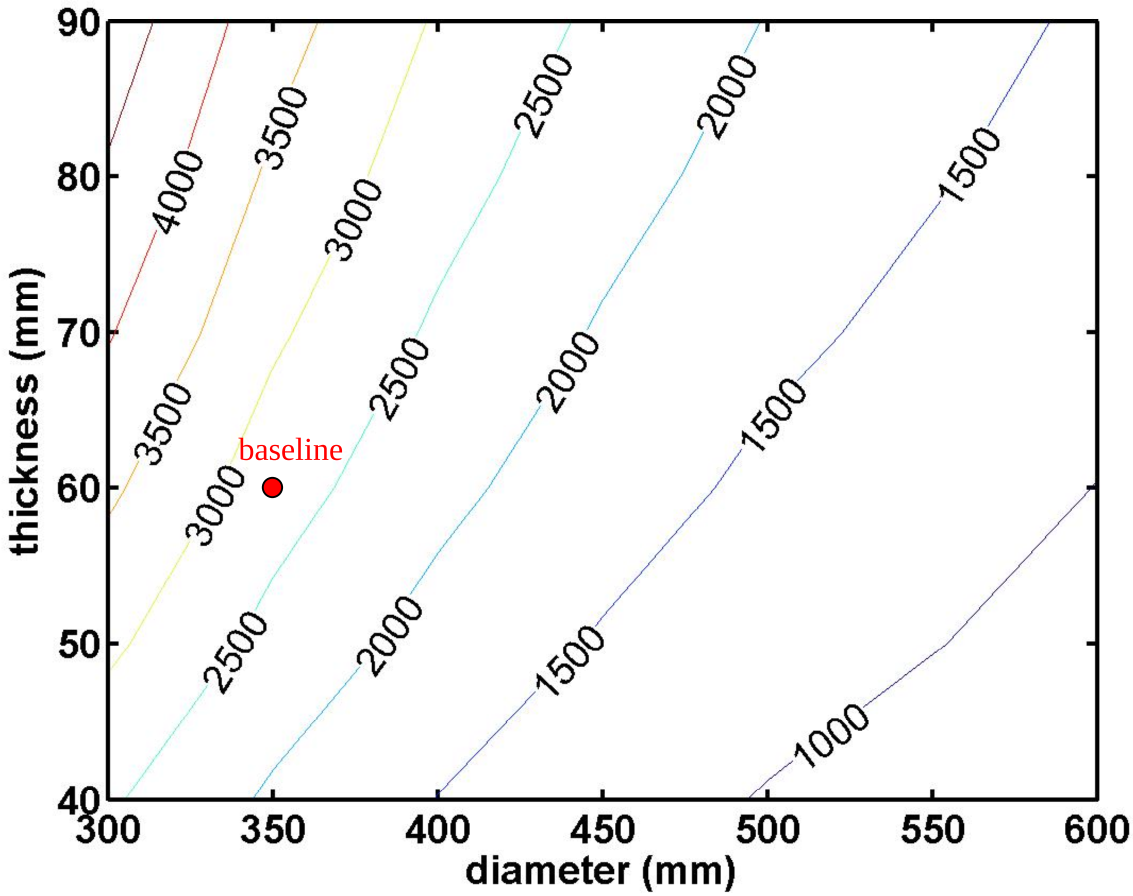


Figure 2: Effect of Beamplitter thickness and diameter on the lowest elastic mode frequency

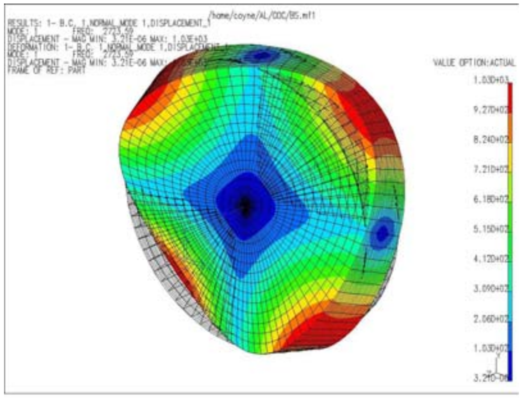
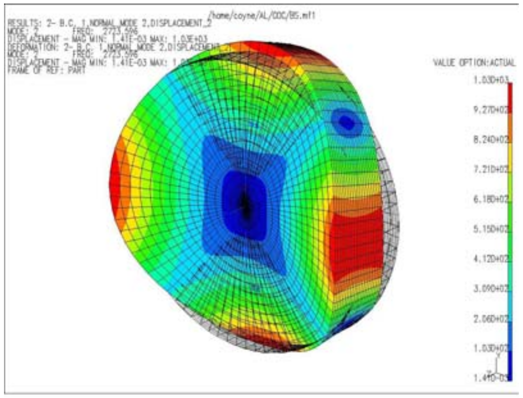


4 Minimum Elastic Mode Frequency

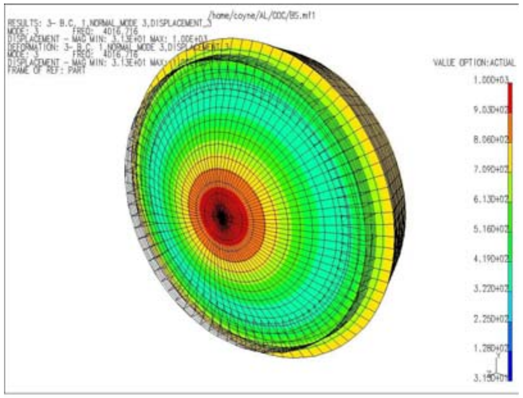
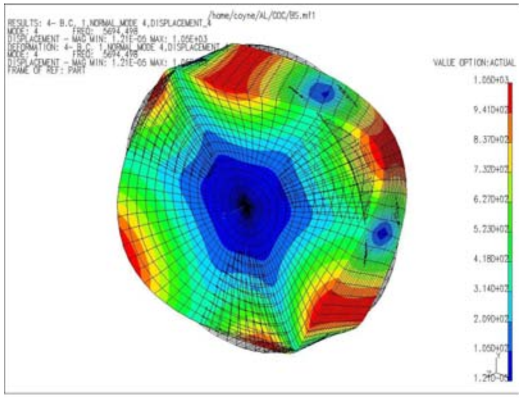
Beamsplitter elastic modes will cause a peak in the length sensing response of the optical system. Due to dynamic range considerations, this response peak will need to be notch filtered. In order that the phase introduced by the notch filter does not adversely affect the length control system, the minimum elastic mode frequency should be well above the upper unity gain frequency (bandwidth) of the length control servo system. The length control servo system bandwidth[9] will be about 100 Hz. Consequently the lowest BS elastic mode should be greater than ~10 times the bandwidth, or > 1000 Hz. From Figure 2, to achieve a 1 kHz first frequency the beamsplitter would need to be about 500 mm diameter and 41 mm thick, or 600 mm diameter and 60 mm thick. An I-DEAS finite element model of a right circular cylinder, 41 mm thick and 500 mm diameter , confirms that the first mode is the saddle mode, with a frequency of 982 Hz. With an approximate 5% increase due to the wedge and flats, the resultant first frequency is ~1000 Hz.

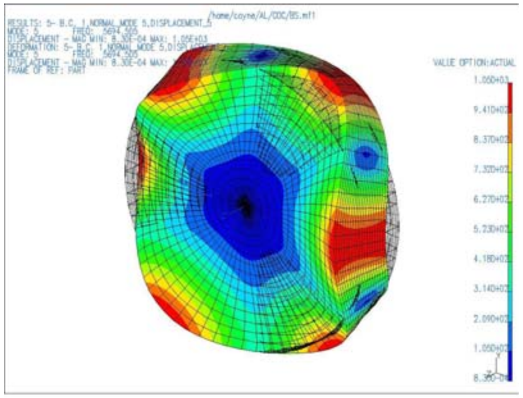
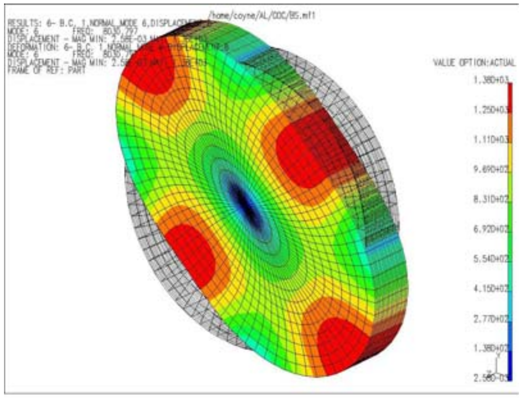
[9] According to P. Fritschel, 12/10/2004.

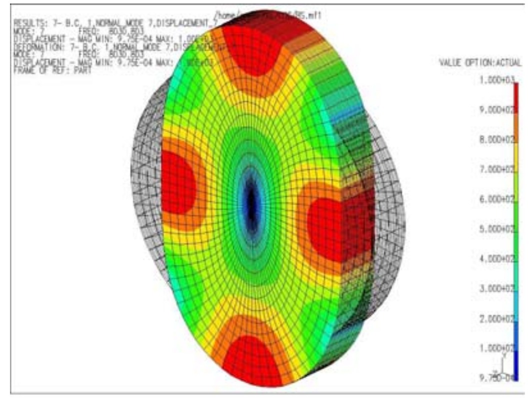
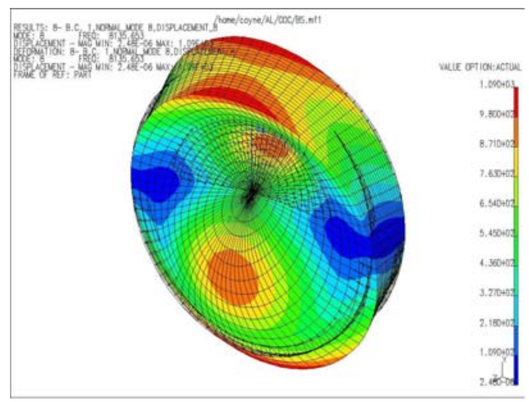
Table 1: Baseline BS modal frequencies

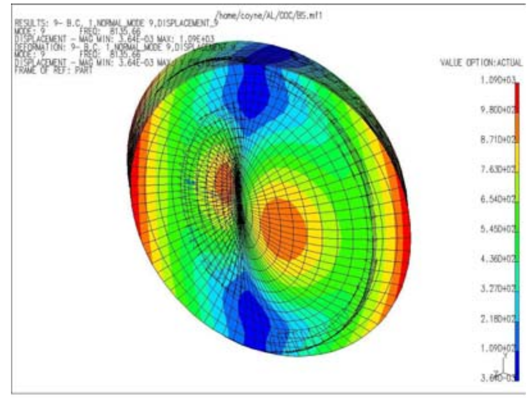
Mode No.	I-DEAS Frequency (Hz)		testmass_v5.c		I-DEAS Mode Shape
	Right circular cylinder	wedged, with side flats	mode shape number[10]	frequency (Hz)	(right circular cylinder model)
1	2724	2861	[2,1,1]	2722	
2	2724	2916	[2,1,1]	2722	

[10] The three integers are [circumferential mode number, parity mode number, mode order number] where the mode order number is for a given circumferential mode and parity.

Mode No.	I-DEAS Frequency (Hz)		testmass_v5.c		I-DEAS Mode Shape (right circular cylinder model)
	Right circular cylinder	wedged, with side flats	mode shape number[10]	frequency (Hz)	
3	4017	4250	[0,1,1]	4017	
4	5694	5918	[3,1,1]	5673	

Mode No.	I-DEAS Frequency (Hz)		testmass_v5.c		I-DEAS Mode Shape (right circular cylinder model)
	Right circular cylinder	wedged, with side flats	mode shape number[10]	frequency (Hz)	
5	5695	6037	[3,1,1]	5673	
6	8031	8039	[2,0,1]	8018	

Mode No.	I-DEAS Frequency (Hz)		testmass_v5.c		I-DEAS Mode Shape
	Right circular cylinder	wedged, with side flats	mode shape number[10]	frequency (Hz)	(right circular cylinder model)
7	8031	8085	[2,0,1]	8018	
8	8136	8455	[1,1,1]	8104	

Mode No.	I-DEAS Frequency (Hz)		testmass_v5.c		I-DEAS Mode Shape
	Right circular cylinder	wedged, with side flats	mode shape number[10]	frequency (Hz)	(right circular cylinder model)
9	8136	8545	[1,1,1]	8104	
10	9022	8936 (different mode)	[4,1,1]	8960