LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
-LIGO-
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Distorted PRM SB fields in Eikonal approximation		
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INTRODUCTION

Early locking of the LIGO I power recycling cavities immediately revealed distinct behaviors characteristic of the inherent degeneracy of these short (compared to the driving beam Rayleigh length) structures. Specifically, the resonant internal field profile could be changed in a strong qualitative way (say from sub driving beam waist Gaussian to broad "donut" cross section) with fractional wavelength equivalent offset tuning of the l^+ servo null. This may be contrasted to the characteristic tuning behavior expected for a high finesse stable locked cavity, where detuning leaves the [mode] shape essentially invariant. It was soon pointed out (S. Whitcomb) and further developed that an elementary "eikonal" model accurately described the situation and was perhaps most appropriate since it would be expected to be a better approximation as exact degeneracy is approached.

Since that epoch there has developed much interest in differential effects in the PRM, and in particular their association with imbalance of the upper/lower (+/-) SB fields. Although this eikonal approach was extended to the full PRM dofs (D. Ottaway private comm., and myself) this was previous to full TCS operation and concomitant data. This note gives a most straightforward extension of eikonal expressions for the full, generally distorted, "LIGO I" like PRM configuration. It is explicitly shown how differential distortion characteristically results in +/- SB imbalance. Further, it is pointed out that even common distortion can result in imbalance if the RF SB wavelength is not exactly tuned to PRM common [macroscopic] length resonance.

Eikonal approximation in PRM

By "eikonal" model (of PRM) is meant neglecting transverse diffraction and ray divergence entirely. In this limit the field (of a given frequency component) inside the cavity at a given transverse position is determined *only by the driving input field at that point and the round trip parallel path phase advance in* **z**. Without loss of generality we make the assumption throughout this note that the input coupling mirror ("RM") is flat; the BS is perfect; and that, optically, there is exact cylindrical symmetry (so that we can simplify transverse position to a single variable, ρ). In this situation the input beam is taken to be a pure Gaussian TEM_{00} mode ("$|0\rangle$") of amplitude waist w_0 at the RM (i.e. its phase front is flat at the RM). The input field of this mode, $\mathbf{E}(\rho)_{|0\rangle}$, would then be perfectly matched to a PRM with perfectly reflective, *flat* ITMs. The reference planes for these [imagined] flat ITMs can be precisely adjusted (servoed) in **z** such that for a particular "carrier" (CR) frequency the mode will be exactly resonant and dark at the **AS** port.

What is the basis of validity for this eikonal approximation? The concept is that an initial $\hat{\mathbf{e}}_{\mathbf{z}}$ geometrical "ray" at ρ in the driving field, during its effective propagation within the PRM, should deviate $\Delta\rho <<$ its initial ρ. By definition $\Delta\rho=0$ for the flat-flat reference PRM. Any deflecting distortion satisfying this condition, must at the same time be capable of significantly altering the round trip propagation [eikonal] phase. An example suffices to show that this holds in our PRM within the range of possible TL distortions. Take the distortion to be a (common) curvature (ROC=R) of the ITMs. Rays are then deflected by an angle $2\rho/R$ on each round trip or $2N\rho/R$ during a mean storage time (N~ 20 for our case). The deviation, $\Delta\rho$, then is $2NL\rho/R$. For *extreme* distortion (R~4000m), and with L_{PRM}~10m, we have $\Delta\rho/\rho \sim 0.1$. On the other hand, the round trip phase variation at $\rho \sim w_0$ is $4\pi\, h(w_0)/\lambda = 2\pi\, w_0^2/R\lambda > \pi/2$.

Expressions for the eikonal field solutions.

In the LIGO I interferometer configuration the CR and SB fields behave qualitatively differently. Here we solve for the SB fields while *fixing* the CR according to its known stable modal excitation, servoed to exact resonance. The reference PRM, with an exactly resonant and dark field $\mathbf{E}(\rho)_{|0\rangle}$, is perturbed in two ways. First, ideal arm cavities are added with input reference planes coincident with the PRM "ITM" reference planes. In the idealization (arm cavities of exactly resonant mode $|0\rangle$, and arbitrarily high finesse) this has no influence on the SB fields (they are "anti-resonant" in these arms) and causes precisely a π phase shift for the CR field. However the servoed PRM shifts its reference "ITM" ends exactly to compensate for this π phase shift. Second, an [cylindrically symmetric] OPD perturbation, $h(\rho)$ is introduced distinctly in each branch. Round trip through the PRM the CR field acquires an amplitude factor due to this perturbation of:

$$\langle 0|\exp(4\pi i h(\rho)/\lambda)|0\rangle$$

This may always be written as a constant mean phase shift and an orthogonal modal part:

$$\langle 0|\exp(4\pi i h(\rho)/\lambda)|0\rangle \approx e^{i\phi}\left[1 + i\langle 0|\eta(\rho)|0\rangle\right] = e^{i\phi}$$

Of course there will generally be matrix elements to HTMs:

$$i\langle n|\eta(\rho)|0\rangle \neq 0$$

But it is exactly these terms, which are cancelled by the action of the resonant arms (see the appendix for a modal review of this phenomenon). Thus the *only* effect of the distortion perturbations on the CR field are mean phase shifts, $e^{i\phi}$. Again, the servo action is then assumed to shift the ITM reference planes to compensate for these phases.

With the microscopic phases then *fixed* in this idealized CR sevoed configuration, the concomitant SB $\pm$ fields (which are driven in the same mode $|0\rangle$ as the CR) are (at a reference plane for the RM, coupling $t=\sqrt{1-r^2}$):

$$\mathbf{E}^{\pm}(\rho)=\mathbf{E}_{|0\rangle}(\rho)t/\left\{1-re^{\pm i\sigma}e^{i\bar{\eta}(\rho)}Cos(\Delta\eta(\rho)\pm S)\right\}$$

where S is the Shnupp phase = $\Omega_{SB}\ell^{-}/c$, and $\bar{\eta}(\rho),\Delta\eta(\rho)$ are respectively the common and differential pure orthogonal *microscopic* distortions. The *macroscopic* branch length difference, ℓ^{-}, is entirely independent of the servoed microscopic lengths. Similarly σ represents the *macroscopic* common cavity length phase for the RF frequency $\pm\Omega_{SB}$.

This expression for the SB fields, $\mathbf{E}^{\pm}(\rho)$, with respect to the CR servoed interferometer contains the complete content of the eikonal model of the PRM.

Examples of the SB fields under distortion: SB imbalances.

If there are no distortions we have:

$$\mathbf{E}^{\pm}(\rho)=\mathbf{E}_{|0\rangle}(\rho)t/\left\{1-re^{\pm i\sigma}Cos(\pm S)\right\}$$

and the SB+/- fields are manifestly balanced (within a complex conjugation). Next consider a pure common distortion, $\Delta\eta(\rho)=0$. Even in this case there will still generally be imbalance between the SBs, if the *macroscopic*, RF, wavelength is not tuned to exact PRM common resonance:

$$\mathbf{E}^{\pm}(\rho)=\mathbf{E}_{|0\rangle}(\rho)t/\left\{1-re^{\pm i\sigma}e^{i\bar{\eta}(\rho)}Cos(\pm S)\right\}$$

There remains an asymmetric interference between the [constant] *macroscopic* phase $\pm i\sigma$, and the *microscopic* distortion phase $i\bar{\eta}(\rho)$. The LIGO I configuration has been macroscopically designed so that $\sigma\approx 0$. The slight practical deviations from this are taken to be small, so that this source of imbalance is presumed negligible.

On the other hand, differential distortions (and here we take the pure case, $\bar{\eta}(\rho)=0$ for simplicity) cause an intrinsic imbalance since the interference is then between the deliberate *macroscopic* Schnupp phase, $\pm S$, and the *microscopic* differential distortion phase, $\Delta\eta(\rho)$:

$$\mathbf{E}^{\pm}(\rho) = \mathbf{E}_{|0\rangle}(\rho)t / \left\{1 - re^{\pm i\sigma}Cos(\Delta\eta(\rho) \pm S)\right\}$$

Here, even if σ=0, there is a fundamental, real, imbalance. This effect can be quite large for typical TL distortions and the design S~0.15. As an elementary example take the distortion to be a simple lens in each branch (TL in ITM substrate). Such a lens equivalent to an ITM reflective radius of curvature R would have an OPD h(ρ)= ρ^2/R. Written in orthogonal (Laguerre-Gauss) polynomial components:

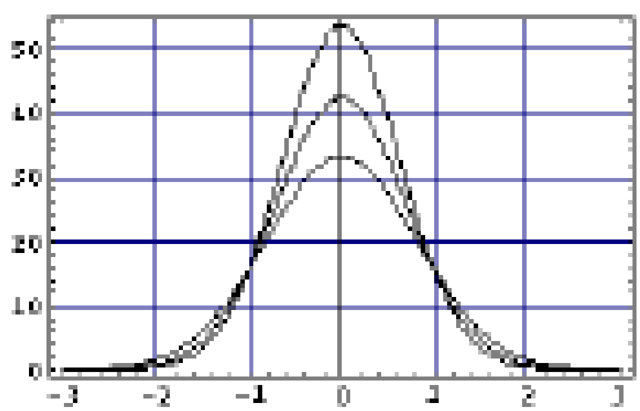
$$h(\rho) = -(1 - 2\rho^2 / w_0^2)w_0^2 / R + w_0^2 / R$$

$$= -(w_0^2 / R)L_1^0(2\rho^2 / w_0^2) + const$$

Where $L_1^0(2\rho^2 / w_0^2)$ is the lowest order orthogonal component of a spherical distortion, so that (there being $\pm h(\rho)$ in each branch):

$$\Delta\eta(\rho) = \pi / \lambda(w_0^2 / R)L_1^0(2\rho^2 / w_0^2)$$

For LIGO I beam waist w_0~3.2 cm, and R=180km (this corresponds to half the thermal lens in an ITM with 5ppm/cm absorption and 1.5 W ifo input beam power) $\left|\mathbf{E}^{\pm}(\rho)\right|^2$ is plotted:



The abscissa is in units of w_0; the ordinate is for $\left|\mathbf{E}_{|0\rangle}(0)\right|^2$ normalized to unity. The middle curve is for zero distortion. The upper curve is For the minus SB (and the lowest curve for the plus SB).

Appendix (Arm cavity influence on CR modal components).

Reference: W. Kells , in Gravitational Waves and Experimental Gravity, Rencontres de Moriond, 2000, p225.

REFERENCES:

See LHO detector e-logs January 2 and 14, 2001
LHO detector e-log February 26, 2004

Also reported in Commissioning meeting of 12/23/02 (with FFT results on Common PRM SB field profiles for LIGO I configuration model)