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When do the pulsar upper limits become interesting?

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1 Introduction

The title is a loaded question.

The purpose of this document is to motivate the statement in the S2 known pulsar paper [1] that "distortions of [$\epsilon = 4.5 \times 10^{-6}$] would be permitted within at least one exotic theory of neutron star structure" and provide a reference. The reference is temporary—we are working on a journal article based on these notes and more—but there is a need for something to quote right now before the S2 paper is submitted.

LIGO's S2 upper limits on the ellipticities of compact objects are orders of magnitude above the maximum ellipticities predicted for standard neutron stars. However, there are now reasons to suspect that much of a typical neutron star is not made of neutrons. There are many theoretical models of what the other matter might be, and most of them allow for a large part of the star to be more or less solid and thus support an ellipticity that could radiate gravitational waves. For some reason no one has calculated the maximum ellipticities until now. We have a decent calculation for one exotic theory and are working on the rest.

The very wildest of the matter models (solid strange quark stars) predicts maximum ellipticities at about the level of the best S2 upper limits. This is not a model that theorists are inclined to believe these days. But wearing our observer hats we say: "So what? Fashions in theory come and go. It's one thing for a theorist to say he doesn't like it on such-and-such principle. It's a very different thing for us to confront the theory with an observation." This was one of the reasons we advocated very strongly that the S2 paper be submitted to PRL.

The other exotic matter models are much more interesting to theorists. The maximum ellipticities are much smaller than for strange quark stars, but some of them are larger than for conventional neutron stars.

2 Solid strange quark stars

This is an idea that was originally proposed in the 1970s, soon abandoned, and has fallen in and out of fashion several times since. The latest incarnation is being pushed mainly by R. X. Xu at Peking University in a series of papers starting in 2003 [2].

The drive behind the current incarnation is some phenomenology which seems to us (as theorists) fairly suspicious. In Ref. [2], Xu argues that the quasi-periodic oscillations (QPOs) observed in low-mass x-ray binaries could be explained by t-modes (torsional oscillations) of solid strange stars if the shear modulus of the strange matter is about 4×10^{32} erg/cm^3. Most x-ray astronomers think that the QPO frequencies are very close to the rotation frequencies, although they change too quickly to be exactly equal, so this is not an attractive model from that point of view. However it is still worth thinking about what happens if the shear modulus is really that large.

Xu then searches for some mechanism to give solid strange matter such a shear modulus, and comes up with the idea that the quarks are clustering in groups of 18 or so with a van der Waals interaction. As far as we know, there is nothing in low-energy QCD to strictly rule this out, but it seems to us *a priori* unlikely. The QCD theorists haven't exactly jumped on this paper yet either.

Having said that, this theory is still at the outer limits of what is publishable these days, so we should take it seriously enough to evaluate when LIGO observations can begin to confront it. We choose the word "confront" with care: An observation of no signal from a particular pulsar does not rule out any theory of exotic matter, since one can always say that this particular star happens to be much flatter than it could be.

3 Maximum ellipticity

The simplest version of the calculation of maximum ellipticity starts from the calculation of the maximum quadrupole moment in the paper by Ushomirsky, Cutler, and Bildsten [3], known to its authors as Crust Monster.

The main result of Crust Monster is Eq. (69), which says that for a conventional neutron star where nothing but the crust is solid, the maximum $m = 2$ quadrupole is

$$Q_{22,\mathrm{max}} = 1.2 \times 10^{38}\ \mathrm{g\ cm}^2 \left(\frac{\sigma_{\mathrm{max}}}{10^{-2}}\right) \left(\frac{R}{10\ \mathrm{km}}\right)^{6.26} \left(\frac{1.4\ M_\odot}{M}\right)^{1.2}. \tag{1}$$

Here we have left out the dependence on the chemical composition of the crust, except as it appears in the funny power laws. As discussed above Eq. (69) in Crust Monster, the "raw" scaling would be R^7/M^2. The definition of ellipticity used in the S1 paper [4] is

$$\epsilon = \sqrt{\frac{5}{96\pi}} Q_{22}/I_{zz}, \tag{2}$$

as can be shown by playing around with standard multipole definitions [5] and recalling that Crust Monster uses a real perturbation. Inserting the empirical formula [6]

$$I_{zz} = 9.2 \times 10^{44}\ \mathrm{g\ cm}^2 \left(\frac{M}{1.4\ M_\odot}\right) \left(\frac{R}{10\ \mathrm{km}}\right)^2 \left[1 + 0.7 \left(\frac{M}{1.4\ M_\odot}\right) \left(\frac{10\ \mathrm{km}}{R}\right)\right], \tag{3}$$

we obtain a translation of Crust Monster into ellipticity:

$$\epsilon_{\mathrm{max}} = 1.7 \times 10^{-8} \left(\frac{\sigma_{\mathrm{max}}}{10^{-2}}\right) \left(\frac{1.4\ M_\odot}{M}\right)^{2.2} \left(\frac{R}{10\ \mathrm{km}}\right)^{4.26} \left[1 + 0.7 \left(\frac{M}{1.4\ M_\odot}\right) \left(\frac{10\ \mathrm{km}}{R}\right)\right]^{-1}. \tag{4}$$

For our fiducial set of numbers ϵ_{max} is almost exactly 1×10^{-8}.

Now simply alter this result (4) using Xu's numbers [2]. The quadrupole moment (1) goes roughly as a volume integral of the shear modulus. Xu's shear modulus (9) is 400 times the Crust Monster number. (Actually they don't quote a fiducial number, but you can get it from μ/p). The volume of the solid star is roughly $R/(3d)$ times the volume of a crust of thickness $d =$ 1–2 km, so you get a multiplier of about 600–1200 and roughly

$$6 \times 10^{-6} < \epsilon_{\mathrm{max}} < 1.2 \times 10^{-5}. \tag{5}$$

This is the number Ben came up with early this summer and used in arguments to the Periodic Sources Analysis Group that the S2 upper limits were about to reach interesting territory, and the number that Ben talked about at the August LSC meeting. (That talk is available as LIGO-G040318-00-Z.)

This isn't quite right yet, because we need to account for the structure of the star. First go back to Crust Monster and substitute Eq. (67) into Eq. (64). Note that their assumption that $\bar{U} \ll 1$

means that the gravitational constant $g(r)$ scales like r^{-2}, which is saying that all the mass of the star is inside r. This is appropriate for a conventional neutron star since the crust is a very small fraction of the total mass, but not for a strange star which is almost uniform density. For uniform density g scales like r, so $\bar{U} = 3$. This changes the constant γ mentioned above Eq. (68) from 97.97 to 24.74, which reduces $Q_{22,\max}$ by 4.0.

It also gets rid of the funny power laws in the Crust Monster result. For uniform density the integral in Eq. (64) is the same as Eq. (68) with $\bar{U}$ changed to 3 and $g = GMr/R^3$. Doing the integral we find that $I = \mu R^6/(3GM)$, which is more than the crust case by a factor R/d. (Ben forgot a factor of 3 in his first result above.) Putting in the scalings, which no longer have complicated equation of state factors, the quadrupole is now

$$Q_{22} = 1.8 \times 10^{41} \text{ g cm}^2 \left(\frac{\sigma_{\max}}{10^{-2}}\right) \left(\frac{\mu}{4 \times 10^{32} \text{ erg/cm}^3}\right) \left(\frac{R}{10 \text{ km}}\right)^6 \left(\frac{1.4 \, M_\odot}{M}\right). \tag{6}$$

Another structure effect is that Bejger and Haensel [6] find a different empirical relation for the moments of inertia of strange stars,

$$I_{zz} = 1.7 \times 10^{45} \text{ g cm}^2 \left(\frac{M}{1.4 \, M_\odot}\right) \left(\frac{R}{10 \text{ km}}\right)^2 \left[1 + 0.14 \left(\frac{M}{1.4 \, M_\odot}\right) \left(\frac{10 \text{ km}}{R}\right)\right]. \tag{7}$$

This is greater than for neutron stars by about 10% for our fiducial numbers. (Remember that a uniform density star has more of its mass at higher radii than an inhomogeneous star.)

The resulting maximum ellipticity is

$$\epsilon_{\max} = 1.4 \times 10^{-5} \left(\frac{\sigma_{\max}}{10^{-2}}\right) \left(\frac{1.4 \, M_\odot}{M}\right)^2 \left(\frac{R}{10 \text{ km}}\right)^4 \left[1 + 0.14 \left(\frac{M}{1.4 \, M_\odot}\right) \left(\frac{10 \text{ km}}{R}\right)\right]^{-1} \tag{8}$$

for Xu's shear modulus. This has actually gone slightly up, again because of the R/d estimate–in fact the d that matters here is smaller than the whole thickness of the crust, because the quadrupole moment is dominated by the inner part of the crust with its high shear modulus and that is thinner than d by a bit.

This is a big number, but note the strong dependence on R. Actually we should put in Xu's scalings [2] for his phenomenological fit to the shear modulus,

$$\mu = 4 \times 10^{32} \text{ erg/cm}^3 \left(\frac{10}{x}\right)^2 \left(\frac{R}{10 \text{ km}}\right)^2 \left(\frac{\rho}{10^{15} \text{ g/cm}^3}\right) \left(\frac{f}{10^3 \text{ Hz}}\right)^2. \tag{9}$$

Here the x's are eigenvalues of t-modes and f is the frequency the oscillation is observed at. Leaving out the x and f scalings since the little errors in the fiducial values for each cancel out, we have

$$\epsilon_{\max} = 1.0 \times 10^{-5} \left(\frac{\sigma_{\max}}{10^{-2}}\right) \left(\frac{1.4 \, M_\odot}{M}\right)^3 \left(\frac{R}{10 \text{ km}}\right)^3 \left[1 + 0.14 \left(\frac{M}{1.4 \, M_\odot}\right) \left(\frac{10 \text{ km}}{R}\right)\right]^{-1} \tag{10}$$

where we have used $\rho = 6.7 \times 10^{14}$ g/cm^3 for our fiducial M and R. Note that 10 km is about the maximum radius predicted for a strange star; they could go down to 7 km which brings $\epsilon_{\max}$ down to 3×10^{-6}, which is still in the ballpark.

How justified is the Crust Monster calculation? What are the assumptions? That gravity is Newtonian, which is probably fine up to a redshift factor of 1.3 or so. That the background star is spherical, which is violated at about the 10% level at most. That the star is in hydrostatic equilibrium, which should be very close. That all the strain is in the $\ell = m = 2$ multipole, which is fine for an upper limit but who knows in real life? That the breaking strain is 10^{-2}, which is pretty high for terrestrial materials (steel gets within a factor of two or so) but makes sense if the low-mass x-ray binary spin frequencies are due to the Bildsten-Wagoner mechanism.

One more substantial thing we have to think about is the boundary condition on the surface of the star. Crust Monster did the integral by parts, assuming liquid above and below the crust so that the traction vanishes. When we put the bottom of the solid region at the center of the star, the traction still vanishes by symmetry and finiteness etc. At the surface, without an ocean on top it might not, but the contribution to the integral from that region should be very small so we should do fine with this calculation.

Also we could worry about the Cowling approximation, i.e. the neglect of the perturbation of the gravitational potential. Crust Monster says that dropping that approximation increases Q_{22} by 25–200%, but that's for a thin crust not a solid star. We could do it with their Eq. (73) and get the potential perturbation (the part they couldn't do) using the method of Lindblom, Mendell, and Owen [7].

None of these factors is large enough to really change the implications for LIGO, though.

4 Implications for LIGO

LIGO isn't there yet. The best upper limit from the S2 paper [1] is $\epsilon < 4.5 \times 10^{-6}$, but that's for a pulsar where there is a limit from radio observations of the spindown that's orders of magnitude lower. Of the pulsars in the S2 paper [1] with "spinups" due to acceleration by a globular cluster, the best limit is $\epsilon < 4.3 \times 10^{-5}$. This is above the maximum solid strange star prediction.

When will LIGO upper limits get into the realm where they will start to confront the solid strange star theory? The gravitational waves from the pulsars mentioned above would be at 406 Hz and 655 Hz respectively. A glance at the publicly available noise curves (LIGO-G030379-00-E) shows that the noise (as measured by $\sqrt{S_h(f)}$) in the Livingston interferometer was a factor of 10 above design. The amount of useful data was 300 hours for L1 and 900 hours for H1, which mostly compensated for H1's noise being worse by a factor of 2 since the upper limit improves roughly as the square of the observation time. If the duty cycle is of order 50% for a year this would improve the upper limit by about 5.5.

Extrapolating to a year of data at design sensitivity, then, LIGO would see upper limits around 1×10^{-6} for the best S2 pulsar with no spindown upper limit and 1×10^{-7} for the best one period. The latter result will not be in competition with the spindown upper limit, but the former will be well into the regime where the theory of solid strange quark stars predicts something might exist. If we see something, it's champagne all around. If not, we are not ruling out the theory—one could always say that any N pulsars just happen to be much flatter than they could be—but we are confronting it, and it is worth thinking about when N gets large enough to say that observations are eroding the credibility of the theory.

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