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An apparatus for measuring non-Gaussian noise in silica-sapphire bonds

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ABSTRACT

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KEYWORDS

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1 Purpose

The purpose of this experiment is to measure the rate of non-Gaussian, or impulsive events that originate in a sapphire-silica bond, and to see if this event rate is higher than that in a silica-silica bond.

2 Apparatus

2.1 Samples

The experiment will require two samples, one test sample and one control, to separate noise in the bonds from noise in the rest of the suspensions. (See Figure 1.) The test sample should be a small sapphire mirror (1/2” diameter), bonded to a small fused-silica ear. For a control, the experiment should also include a fused-silica mirror (the same size as the sapphire one) bonded to a fused-silica ear. Both samples should be mounted identically, from the same seismic isolation system, and measured with identical readout systems. Events due to external noise will occur in both samples simultaneously and can be ignored.

The two readout systems should be identical, but there is still the possibility that one set of electronics will generate more false events than the other. This problem can be overcome by running the experiment for a while, then swapping readout systems (electronics, optics, and all), and then running the experiment again for the same amount of time under the same conditions. The weights could also be swapped for the same reason.

Alternately, one system could be used to test both samples at different times.

If, then, the sapphire-silica sample consistently shows a higher rate of events than the control sample, one may conclude that these excess events probably originate in the sapphire-silica bond.

2.2 Transducers

There are two possible transducers that would be suitable for this experiment:

1. Fibers whose vibration amplitude is measured optically, or

2. PZT crystals glued (epoxied?) onto the mirrors and sensed electrically.

2.3 Readout system

Braginsky used an optical readout system when he did a similar experiment with tungsten fibers, and we could just use his design. Alternately, we could use a capacitor, with the fiber vibrating at the edge of the plates. The second system would probably be simpler, and less prone to seismic noise since the capacitor could be mounted on the same seismic isolation system as the sample. I haven’t decided which is better yet, because I haven’t thought about it long enough.

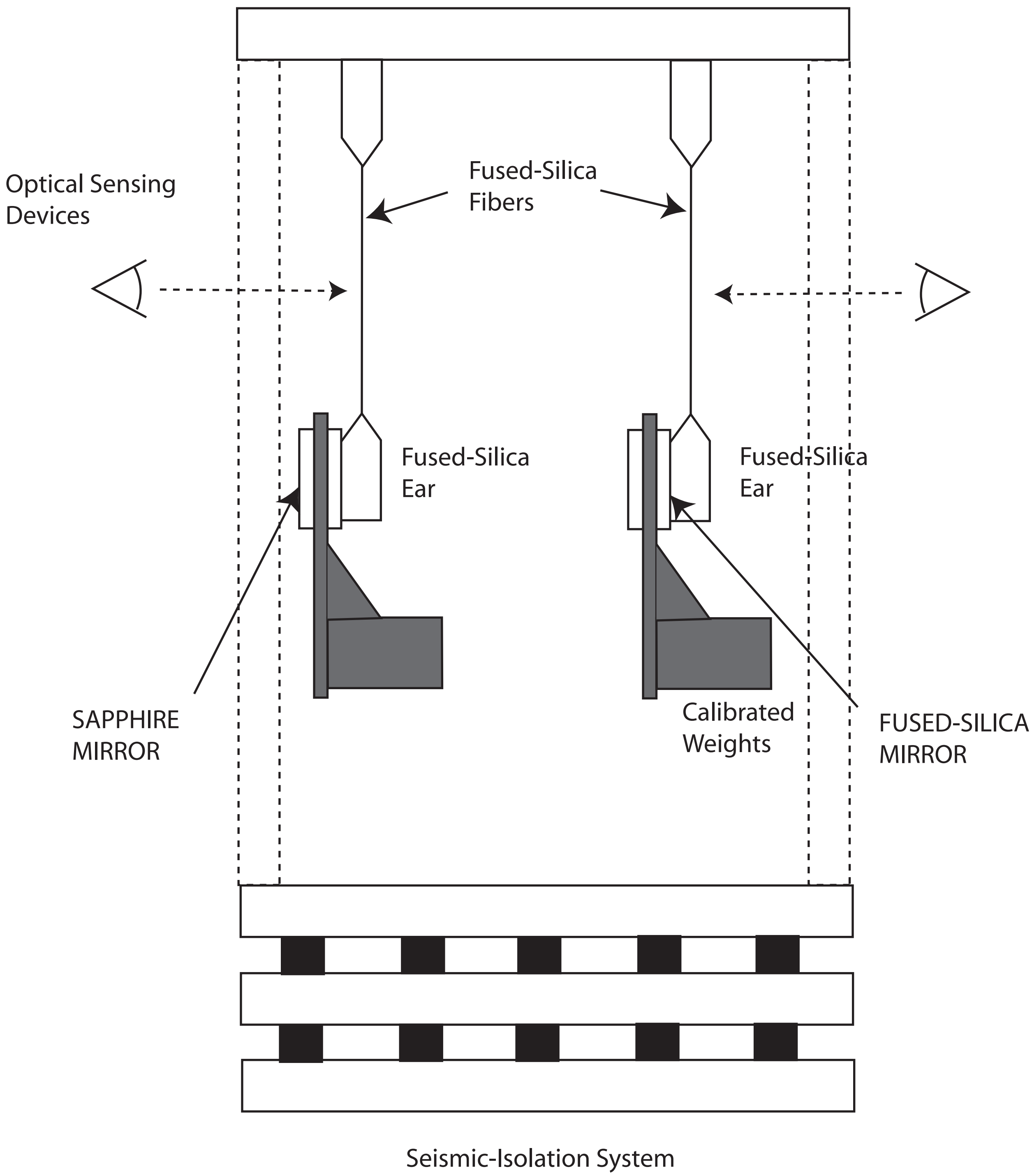

Figure 1: Basic design of the apparatus.

2.4 Isolation

The apparatus must be contained in vacuum to acoustically isolate it from the rest of the environment, and it must be mounted on a seismic isolation stack to filter out ground noise. For acoustic isolation, a simple roughing vacuum should be sufficient. Seismic isolation will be quantitatively considered below.

2.5 Data acquisition and analysis

The most straightforward way to do this, if fibers are used, is probably with two separate lock-in detectors, each referenced to the resonant frequency of the fiber being measured. (The two fibers will be nominally identical, but their resonant frequencies will probably differ slightly.) Because data will be recorded over long periods of time (days to weeks?), signals from both samples should be recorded using a computer and stored on a hard drive. Because of the high Q of the fibers and the frequency downconversion of the lock-in, data will not need to be sampled very often, probably no more than once per second. In any case, data should not be sampled more often than once per lock-in time constant, and once every three to ten time constants is probably more appropriate.

As of this writing, I'm not sure of the best way to measure and record a signal from PZTs. Note to self: Think about this and put a scheme in a revised version.

3 Quantitative considerations

3.1 Fundamental thermal noise limit

If we use a fiber as a transducer, we may approximate it as a simple harmonic oscillator with a natural resonant frequency (for the n-th mode) of

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\lambda}},$$

an effective mass

$$\mu_n = \frac{\lambda L}{n^2},$$

and loss angle ϕ. Here, λ is the linear mass density, L is the length of the fiber, and T is the tension. The rms thermal noise level in the fundamental mode is then

$$x_{Th}(f) = \sqrt{\frac{1}{8\pi^3}\frac{k_B T}{\mu_1 f}\frac{f_1^2\phi}{(f_1^2 - f^2)^2 + f_1^4\phi^2}}.$$

On resonance, this becomes

$$x_{Th}(f_0) = \sqrt{\frac{1}{2\pi^3}\frac{k_B T}{\mu_1 f_1^3}}Q,$$

where $Q = 1/\phi$.

3.2 Resolution

Any non-Gaussian events will dump additional energy into the resonant mode of the fiber and thus increase its amplitude above the thermal noise floor. If the instrument is thermal noise limited, it can resolve the baseline energy in the fundamental mode of $k_B T \approx 1/40 eV$. For a signal-to-noise ratio of unity, then, the instrument will be able to resolve any event that dumps more than $1/40eV$ into the fundamental mode of the fiber.

How does this compare with a single dislocation slip, for example? One would expect structural events within a solid to involve much more energy than a single $k_B T$, otherwise the solid would melt.

The resolution of this instrument ought to be good enough, then, to resolve individual molecular bond rearrangements in a silica-sapphire interface, provided the coupling to the fiber is reasonably good. Right, now, I don't have a good way to estimate this coupling, but I expect it to be fairly high, given the geometry of the system. With the bond isolated at the bottom of the fiber, there is nowhere for the energy released in an event to go but up, through the fiber.

3.3 Seismic isolation requirements

The rms seismic noise is

$$x_S(f) = x_0(f)\sqrt{\frac{f_1^2(1+\phi^2)}{(f_1^2 - f^2)^2 + f_1^4\phi^2}},$$

where $x_0(f)$ is the amplitude of the anchor point. On resonance, this becomes

$$x_S(f_1) = x_0(f_1)Q,$$

and the thermal noise is

$$x_{Th}(f_1) = \sqrt{\frac{1}{2\pi^3}\frac{k_B T}{\mu_1 f_1^3}Q},$$

where $Q = 1/\phi$. Getting the seismic noise below the thermal noise requires the anchor-point motion to be less than

$$x_0(f_1) < \sqrt{\frac{1}{2\pi^3}\frac{k_B T}{\mu_1 f_1^3}\frac{1}{Q}}.$$

3.4 Fiber dimensions

For this, let's assume we have a fixed length for the fiber L and a fixed amount of weight we can hang from it M. From this, let's calculate what diameter we would need to get a decent resonant frequency. Recall that the fundamental mode frequency is

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\lambda}}.$$

The tension is just the weight of the mass M, the 'calibrated weights' shown in Figure 1.

$$T = Mg$$

The linear mass density is

$$\lambda = \frac{\rho \pi d^2}{4},$$

where ρ is the density of fused silica, and d is the diameter of the fiber. This gives the required diameter as a function of all the other variables.

$$d = \frac{1}{f_1 L}\sqrt{\frac{Mg}{\pi \rho}}$$

3.5 Numbers

Assume the following numbers, just as a starting point.

$$\begin{aligned} L &= 10cm \\ f_1 &= 1kHz \\ M &= 1kg \\ \rho &= 2.2 \times 10^3 kg/m^3 \\ Q &= 10^8 \end{aligned}$$

3.5.1 Fiber diameter

This means the fiber would have to have a diameter of

$$d = 0.38mm$$

which doesn't sound unreasonable. It ought to be relatively easy to grow, and it ought to be more than adequate to support a $1kg$ mass.

3.5.2 Seismic attenuation

Calculating this anchor-point requirement and comparing it to the expected ground noise will tell us how much seismic isolation we need to do this experiment. For a metal wire, we can expect something like this:

$$\begin{aligned} f_0 &= 1.5kHz \\ \mu &= 4.1 \times 10^{-7} kg \\ Q &= 3 \times 10^5 \end{aligned}$$

(I used Braginsky's values of f_0 and μ, from his tungsten-wire experiment, and a typical Q value for wires.) Resolving the thermal noise in a wire, then, requires that the anchor point have no more than

$$x_0(1kHz) < 9.4 \times 10^{-17} m/\sqrt{Hz}.$$

How does this compare with expected ground motion? Typical ground motion is

$$x_g(f) \approx 10^{-9} \frac{m}{\sqrt{Hz}} \left(\frac{10Hz}{f} \right)^2$$

above about $10Hz$. This gives

$$x_g(1kHz) \approx 1 \times 10^{-13} \frac{m}{\sqrt{Hz}},$$

so the seismic isolation system would need to provide an attenuation of at least $10^3 = 60dB$ at this frequency.

3.5.3 Readout sensitivity

The on-resonance thermal noise, using these numbers, is

$$x_{Th}(1kHz) = 2.8 \times 10^{-11} \frac{m}{\sqrt{Hz}},$$

which should be easy to resolve with an optical readout system, or even a capacitive transducer.

APPENDIX 1 APPENDIXTITLE

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APPENDIX 2 ANOTHER APPENDIX

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