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Excess Noise Mechanism in LIGO Output Mode Cleaner

Keita Kawabe, Luca Matone and Joseph Betzwieser

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California Institute of Technology
LIGO Project - MS 18-34
1200 E. California Blvd.
Pasadena, CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project - NW17-161
175 Albany St.
Cambridge, MA 02139
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

LIGO Hanford Observatory
P.O.Box 1970
Mail Stop S9-02
Richland, WA 99352
Phone (509) 372-8106
Fax (509) 372-8137

LIGO Livingston Observatory
P.O.Box 940
Livingston, LA 70754
Phone (225) 686-3100
Fax (225) 686-7189

http://www.ligo.caltech.edu/

Abstract

The angular/displacement jitter in the incoming field on the output mode cleaner (OMC) can generate a large amount of excess noise. There are two mechanisms for this, i.e. coupling via the generation of higher-order transmissive mode and via the DC offset in the OMC length. These can cause the coupling of the lateral displacement of the waist position to the ASQ signal as large as 10^{-9}, which agrees well with the measurement. The use of four-mirrored ring cavity instead of triangular one is expected to mitigate the higher-order mode coupling considerably. Combined with minor tweaks (i.e. the g-factor, the finesse and better mode matching), it seems to be possible to obtain two orders of magnitude of reduction. After this, the coupling via the DC offset of the OMC length would become dominant.

Contents

1 Introduction

In May 2004, the OMC was installed on the optical table for the AS port of LIGO Hanford 4k interferometer, in a hope to reject "junk" light (i.e. the field that doesn't contain any gravitational-wave information), which in turn would allow us to have larger laser power without saturating the photo diode in the AS port. However, disappointingly, it has turned out that the photo detector after the OMC always sees larger noise than the one before the OMC. Nobody successfully explained the origin of this excess noise, but it seemed that the noise is related to the jitter in the incoming beam on the OMC. The following is a list of important observations made by various scientists:

- The OMC has a large jitter-to-error coupling.
- The coupling is always larger in horizontal direction than in vertical.
- The coupling seems to be bi-linear.
- Nobody successfully eliminated the linear term, nevertheless.
- Stabilization of the beam jitter didn't decrease the noise.

Apart from the noise and the jitter themselves, various measurements were done including:

- The mode analysis of the input beam on the OMC.
- Use (or abuse) of various things (like aperture and edge) to change the mode before and/or after the OMC.
- Use of phase camera to see the 25MHz and DC field distribution.

Details for these measurements are seen on elogs from April, May and June 2004.

In this report, a simple model is presented that explains many (though not all) of the above mentioned results. This model gives the jitter-to-error coupling which is comparable to the measured value under reasonable assumptions. Before presenting the model, however, some basic properties are shown.

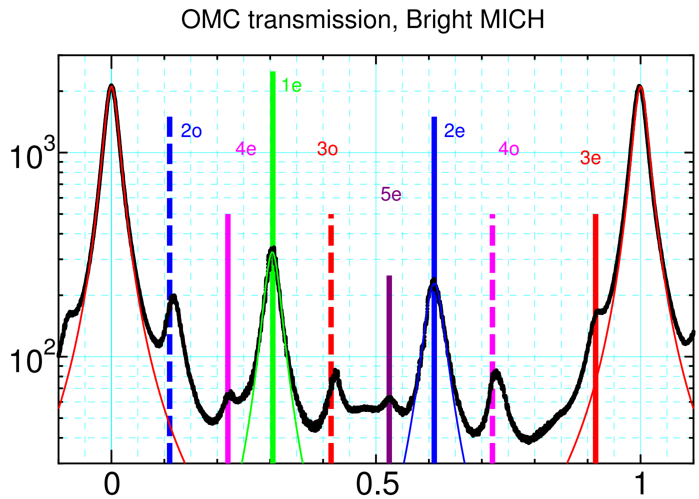

Figure 1: By scanning the OMC length and by studying the transmission peaks for various modes, the transverse mode spacing of the OMC was calculated to be 0.305. Also, from TEM00 transmission, the finesse was determined to be 30. For notational convention for the modes ("4e" "5o" etc.), see the original elog[1].

1.1 Transverse mode spacing and the finesse

By scanning the OMC length using bright Michelson AS field, the mode spacing and the finesse of the OMC were measured to be $x_{\mathrm{t}} = 0.305$ and $\mathcal{F} \sim 30$, respectively [1] (Fig.1). Figure 2 shows the theoretical plot of the amplitude transmission of the OMC and the positions of all of the modes up to 6th order. Actually, since there's finite incident angle on the curved mirror, the OMC has a small astigmatism, and the measured value $x_{\mathrm{t}} = 0.305$ should be thought of as the mean spacing for horizontal and vertical spacing. The nominal values are $x_{\mathrm{t}}^{\mathrm{H}} = 0.309$ (horizontal) and $x_{\mathrm{t}}^{\mathrm{V}} = 0.302$ (vertical), which agrees well with the measured mean value. The nominal original design parameters are found in LIGO-T040018-00-D[2].

1.2 Modal content of "detect" AS field

Figure 3 shows the OMC scanning measurement in "detect" mode[3]. The length of the OMC was scanned using the PZT on the curved mirror, and the transmitted optical power was measured. From this data, one can calculate the power for each of the mode, which is

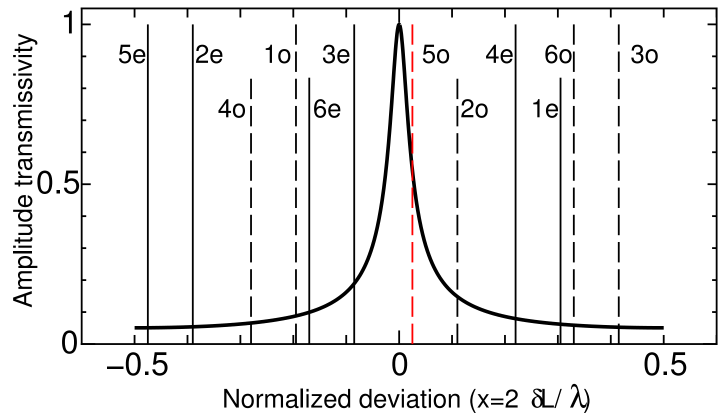

Figure 2: The amplitude transmission curve of a Fabry-Perot cavity with the finesse of 30. The vertical lines show the positions of various transverse modes when the transverse mode spacing is 3.05.

Modes	"0e" (TEM00)	"2e"	"2o"	"4e"	"4o"	"6e"	"6o"
Power/Total	43%	14.1%	8.8%	10.6%	10.1%	6.7%	6.7%

Table 1: Modal content of the AS field of 4k in "dark".

shown in Tab.1. Note that it's impossible to separate degenerate modes (like U_{20} and U_{02}) using this result; for example, the power for "2e" is simply the sum of the power for U_{20} and U_{02}.

1.3 Measured misalignment-ASQ coupling

Various attempts were made to measure the misalignment-ASQ coupling. Basically, a PZT-driven steering mirror was used to dither the incident beam on the OMC, and the resulting ASQ signal was measured. Here I refer to the measurement by Stefan and Nergis[4]: In terms of angle-to-ASQ, the coupling was 2×10^{-9}m/rad for yaw and 3×10^{-10}m/rad for pitch, respectively.

Some care has to be taken, though. By using PZT steerer, both the angle and the transverse displacement of the beam are induced. Since the distance from the PZT to the OMC waist positi on was 342mm[5], the angle jitter of $\Delta\theta$ also induces the displacement jitter of $\Delta a = \Delta\theta \times 342$mm. Since the waist radius $w_0$and the divergence angle θ_0 of the

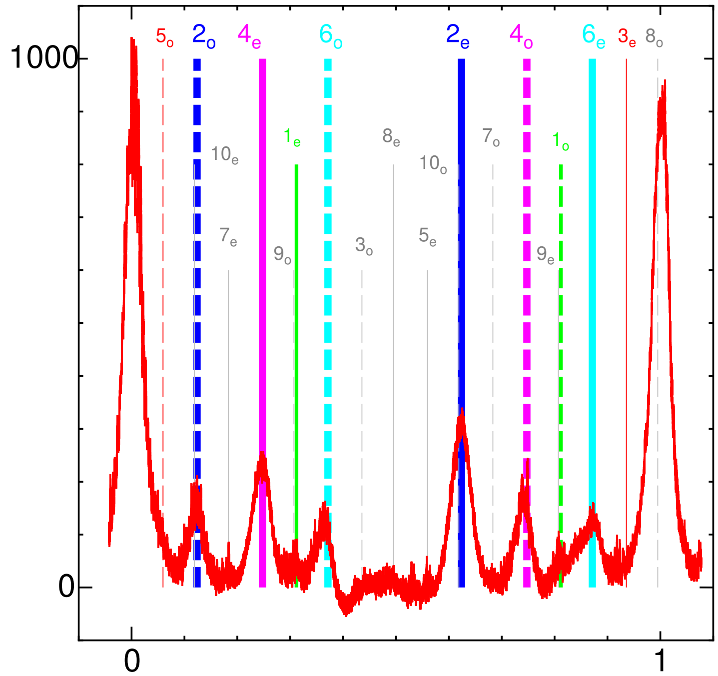

Figure 3: The power transmission against OMC length in "detect". Thick lines show the modal content observed in the data, while the thin lines are just the placeholders for positions of non-existing modes.

OMC eigenmodes are about 0.1mm and 3mrad, respectively, the normalized displacement parameter $\Delta a/w_0$ is always one order of magnitude larger than the angle parameter $\Delta\theta/\theta_0$:

$$\frac{\Delta a}{w_0} \sim 3 \times 10^3 \Delta\theta \gg \frac{\Delta\theta}{\theta_0} \sim 3 \times 10^2 \Delta\theta. \tag{1}$$

Therefore, from now on, we ignore the angular jitter and take only the lateral displacement into account. The measured couplings in terms of dimensionless lateral-displacement-to-ASQ are:

$$\frac{ASQ}{\Delta a_{\mathrm{Yaw}}} \sim 6 \times 10^{-9} \tag{2}$$

$$\frac{ASQ}{\Delta a_{\mathrm{Pitch}}} \sim 9 \times 10^{-10}. \tag{3}$$

2 Model for higher-order mode coupling

Short summary: When one of the higher order modes induced by misalignment is close to the resonance of U_{00}, the ASQ signal is contaminated by the error signal from that higher-order mode. Specifically in our case, "5o" seems to be the bad guy.

This section tries to give basic insights about this mechanism by using a simple example. Please note the over-simplified expressions throughout this section. For example I didn't even write the sideband and the carrier separately. Hopefully this helps for some people to have some intuitive picture of what's happening. If you don't want an intuitive picture, directly go to the next section where a quantitative discussion is given.

2.1 The ASQ signal after the OMC without misalignment

The ASQ signal is generated by the beat between the carrier and the RF sidebands. If the input field has many modal components, the ASQ signal becomes the liner superposition of the signals generated by each of the modal components. Just to have some simple example, let's assume that the input field has only two modal contents:

$$E_{\text{example}} = 0.63 U_{00} + 0.3 U_{40}. \tag{4}$$

If the OMC is roughly locked to the fundamental mode carrier and is transmissive only to the fundamental mode, the ASQ signal is not contaminated by higher order modes. Under such a situation, our model ASQ signal is written by something like the following:

$$ASQ(E_{\text{example}}, x) = 0.63^2 ASQ_0(x) + 0.3^2 ASQ_0(x + 4x_{\text{t}}) \sim 0.63^2 ASQ_0(x) \tag{5}$$

where $x = 2\delta L/\lambda$ is the deviation of the OMC length (normalized by half wavelength) from the resonance for U_{00} carrier, $ASQ_0(x)$ is the "normalized"[1] error signal for U_{00}, and x_{t} is

[1] Error signal "normalized" by the DC amplitude of the carrier and the RF SBs.

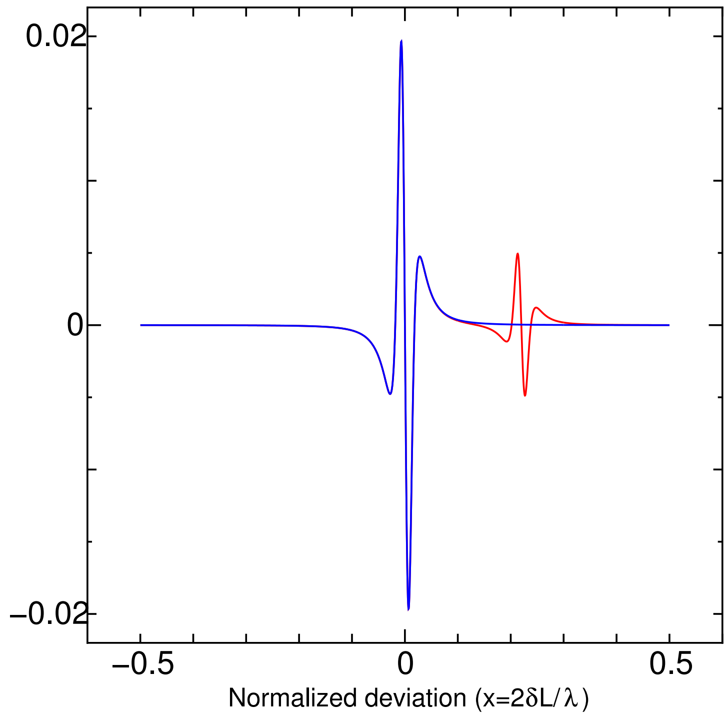

Figure 4: The ASQ error signal with (red) and without (blue) higher order U_{40} term in our example. When the OMC is not transmissive for U_{40}, the error signals are mostly the same.

the transverse mode spacing (Fig.4). [2]

2.2 The ASQ signal WITH misalignment on the OMC

So far, so good. However, as we saw before (Subsec.1.1), the OMC has a transverse mode spacing of $x_{\mathrm{t}} = 0.305$ and the finesse of 30 [1]. Under such a condition, actually the OMC is partly transmissive for 5th-order offaxis modes with an odd symmetry for the horizontal direction, i.e. U_{50}, U_{32} and U_{14} (Fig.2). I just follow the notational convention of myself [3] and label these modes as "5o"; 5 for $n+m=5$ and "o" for $n=$ odd for horizontal direction.

Without any misalignment both in the interferometer itself as well as in the OMC, there should be no "5o" modes (Eq.4), which was verified to some extent by my measurement

[2] Here I- and Q-phase are defined in such a way that the GW signal is maximized in ASQ. Since RF SBs are mostly, but not completely, transmissive, both ASQ and ASI have sensitivity to the deviation x. The OMC length deviation changes both the carrier and the SBs while the GW signal only acts on the carrier, and the change in the SBs would cause the mixing of sensitivity to x in Q- and I-phase. If the SB frequency is infinitely small compared with the width of the cavity, only I-phase would become sensitive to x. However, clearly that's not the case with the OMC. See Sec.9 for details.

(Fig.3). However, the AS field has a set of higher-order fields that couple to "5o" via misalignment, namely "4e", "4o", "6e" and "6o". Approximately, the power of each of "4" modes is 10% of the total, and each of "6" is 6.7%, while the power of the fundamental mode is 40% (just like our example field E_{example}). In our example, there's only one "4e" component, U_{40}, and this would couple to "5o" when there's any misalignment (like the transverse displacement of the beam Δa):

$$Op[E_{\text{example}}] \sim .63 \left(U_{00} - \frac{\Delta a}{w_0} U_{10} \right) + 0.3 \left(U_{40} + 2\frac{\Delta a}{w_0} U_{30} - \sqrt{5}\frac{\Delta a}{w_0} U_{50} \right) \tag{6}$$

where $Op[]$ is the lateral displacement operator and w_0 is the waist radius of the OMC eigenmodes.

Figure 5 shows the "normalized" ASQ error curves for various modes up to 6th order. Each of the modal components generates some error signal in ASQ, but the most important one in our example is U_{50} because of its close resonant position to U_{00}. The ASQ error signal is approximated by

$$ASQ(Op[E_{\text{example}}], x) \sim 0.63^2 ASQ_0(x) + 0.3^2 \times 5 \left(\frac{\Delta a}{w_0} \right)^2 ASQ_0(x + 5x_{\text{t}} + 0.5) \tag{7}$$

where the additional 0.5 in the resonant point comes from the fact that the triangular cavity has 180 degrees phase flip for horizontal asymmetry. Even if the OMC length doesn't have any fluctuation, any beam-position dithering on the OMC generates the ASQ signal which is proportional to the square of the dithering.

2.3 So what?

Does this matter? The answer is yes, which is explained in the next section

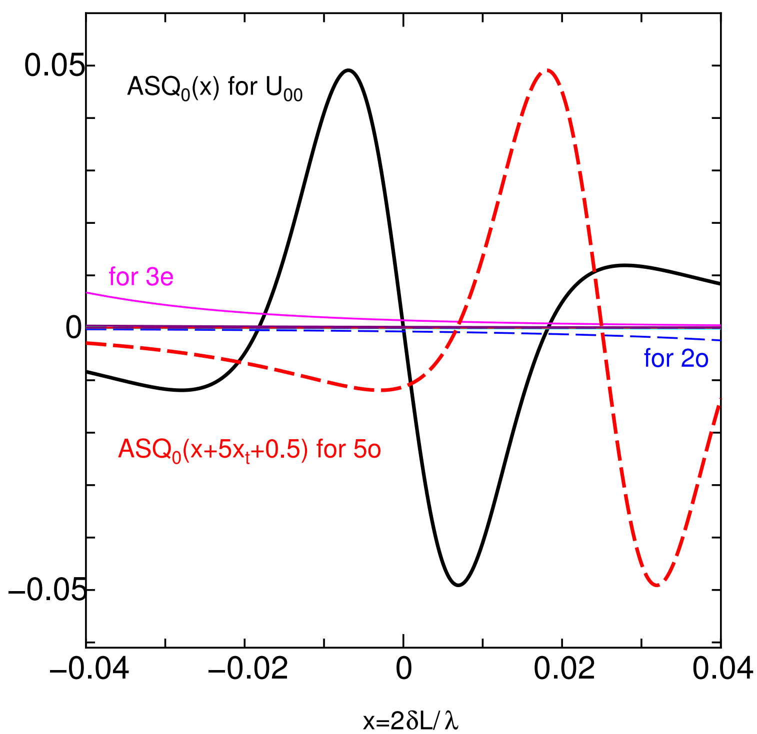

Figure 5: The "normalized" error signal , ASQ_0, at around $x = 0$ for different modes up to 6th order modes. All except "5o" are negligible, but "5o" is comparable to the error signal for U_{00}.

3 Quantitative discussion about the higher-order mode coupling

Let's try to get some number on the coupling here, and see if that is on the same order as the measured one. To see the pure effect of this modal coupling mechanism, we assume the followings:

- The carrier and the RF SBs have exactly the same modal composition.
- There's no imbalances in USB and LSB.
- The OMC is exactly locked to the carrier U_{00}.

We know that these are not the case, and each of these assumptions would affect the result of calculation. However, these assumptions are reasonable in that they don't seem to change the results by an order. The cases where these assumptions are not valid are studied separately in later sections.

3.1 Error signal plot for "detect" AS field

Figure 6 shows the theoretical plot of the error signal using the measured power distribution. As we have seen before, at around $x \sim 0$, the only modal content that counts is TEM00. The slope for this curve at around the center is:

$$\left.\frac{\mathrm{d}ASQ(x)}{dx}\right|_{(x\sim 0)} \sim 4.9[\mathrm{dimless}]. \tag{8}$$

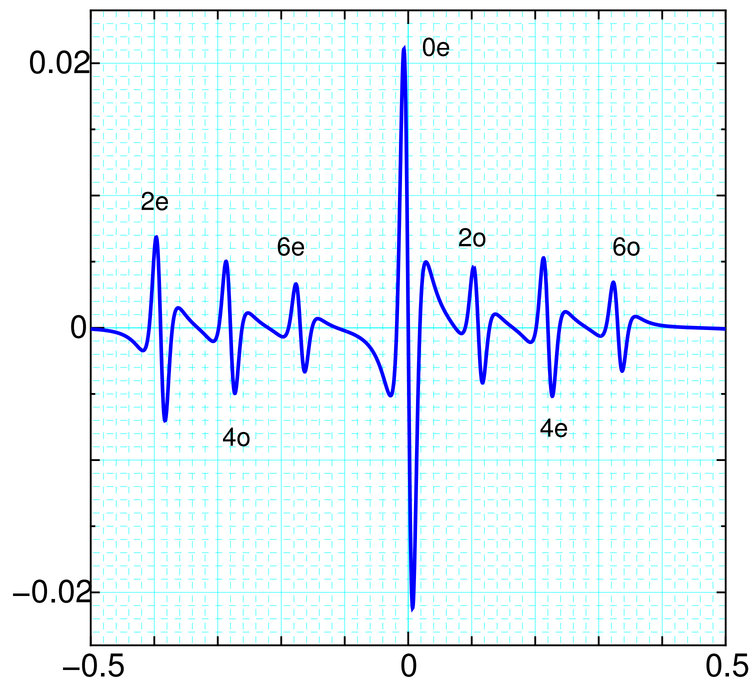

Figure 6: ASQ error signal using the measured modal contents.

3.1.1 Important modes when the OMC is misaligned

We assume that the OMC is exactly locked to U_{00} carrier.[3][4] Under this assumption, we can neglect the error signal generated by U_{00}. Also we can neglect "2" modes, because they don't generate any significant ASQ signal and they don't couple to "5o".

The only important modes here are "4" and "6" ones: They don't generate any measurable ASQ signal themselves, but they would couple to "5o" by the misalignment on the OMC. For yaw-misalignment, "4e" and "6e" couples to "5o" but "4o" and "6o" don't, while for pitch the important guys are "4o" and "6o". So we can simplify the field:

$$E_{\rm yaw} \sim a_{40}U_{40} + a_{22}U_{22} + a_{04}U_{04} + a_{60}U_{60} + a_{42}U_{42} + a_{24}U_{24} + a_{06}U_{06} \tag{9}$$

$$E_{\rm pitch} \sim a_{31}U_{31} + a_{13}U_{13} + a_{51}U_{51} + a_{33}U_{33} + a_{15}U_{15}. \tag{10}$$

[3]See Sec.5 for the quantitative discussion about the coupling via the offset in the OMC locking point.

[4]As for the direct coupling from the OMC length fluctuation to the ASQ, Peter F.'s measurement[6] showed that the effect was insignificant.

From here on we only calculate the yaw-signal, but the pitch-signal can be obtained in a similar way.

3.2 ASQ amplitude

When we laterally shift the beam on the OMC, each of its modal components in Eq.9 would generate "5o" modes. Using the formulas in LIGO-T990811-03[8], we can easily calculate the amplitude for these "5o" modes. However, there's one ambiguity: We don't know each of a_{mn} in the above equations. Rather, all we know is the square-sum of the degenerate components (see Tab.1), like

$$|a_{40}|^2 + |a_{22}|^2 + |a_{04}|^2 = 0.106. \tag{11}$$

However, since we're only interested in approximate number, here I simply assume that only one of "4e" modes is non-zero, and the same for "6e":

$$a_{22} = a_{04} = a_{42} = a_{24} = a_{06} = a_{31} = a_{51} = a_{33} = 0. \tag{12}$$

Under this assumption, when there's the lateral displacement of $\Delta a/w_0$, the amplitude of the induced "5o" field is written by:

$$E_{yaw}^{"5o"} \sim \left(-a_{40}\sqrt{5} + a_{60}\sqrt{6}\right)\frac{\Delta a}{w_0}U_{50}, \tag{13}$$

$$|a_{40}|^2 = 0.106, \tag{14}$$

$$|a_{60}|^2 = 0.067. \tag{15}$$

Though we don't know about the phase relation between the original U_{40} and U_{60} fields, we assume the worst case (i.e. a_{40} and a_{60} interferes constructively). In a similar way to

Eq.7, the resulting ASQ signal can be calculated as:

$$\begin{aligned}
|ASQ| &= \left|-a_{40}\sqrt{5}+a_{60}\sqrt{6}\right|^2 |ASQ_0(-5x_{\rm t}+0.5)| \left(\frac{\Delta a}{w_0}\right)^2 \\
&\sim 1.9\times 0.011\times\left(\frac{\Delta a}{w_0}\right)^2 \\
&= 0.021\times\left(\frac{\Delta a}{w_0}\right)^2. \qquad (16)
\end{aligned}$$

In the last equation, the value for "5o" at the center for U_{00} (i.e. $x=0$) was taken from Fig.5.

Combining Eqs8 and 16, we obtain the effective OMC length fluctuation signal induced by the misalignment:

$$\Delta L^{\rm eff}_{\rm OMC} \sim ASQ\times \left.\frac{{\rm d}ASQ(x)}{dx}\right|^{-1}_{(x\sim 0)} \times\frac{\lambda}{2} \sim 2.2\times 10^{-9}[{\rm m}]\times\left(\frac{\Delta a}{w_0}\right)^2. \qquad (17)$$

Fortunately, Peter F. has measured the coupling from the OMC length fluctuation to the equivalent differential length change in 4k arms[6]:

$$\frac{\Delta L_{\rm 4k}}{\Delta L_{\rm OMC}} \sim \frac{1}{1800}. \qquad (18)$$

Replacing the actual OMC length change ($\Delta L_{\rm OMC}$) with the effective signal induced by misalignment ($\Delta L^{\rm eff}_{\rm OMC}$), we can obtain the relation between the ASQ signal generated by the misalignment and the equivalent 4k arm length change:

$$\Delta L^{\rm eff}_{\rm 4k} \sim \frac{1}{1800}\times 2.2\times 10^{-9}[{\rm m}]\times\left(\frac{\Delta a}{w_0}\right)^2. \qquad (19)$$

Since the above equation is quadratic, we have to introduce some DC misalignment and the fluctuation to generate first-order coupling:

$$\Delta a = \Delta a_0 + \delta a(t). \qquad (20)$$

The resulting linear coupling constant from lateral-displacement-to-ASQ is:

$$\frac{\Delta L_{4\mathrm{k}}^{\mathrm{eff}}}{\delta a(t)} = \frac{2.4\times 10^{-12}[\mathrm{m}]}{w_0} \times \frac{\Delta a_0}{w_0} \sim 2.4\times 10^{-8} \times \frac{\Delta a_0}{w_0}. \tag{21}$$

For example, if the DC misalignment is $\Delta a_0/w_0 = 1/4$, which is not a huge number because the resulting decrease in TEM00 power is 6.3%, we'll have the coupling of 6×10^{-9}, which is equal to the measured data. If the DC misalignment is only 0.1, which corresponds to 1% power decrease in TEM00, will still produce strong coupling of 2.4×10^{-9}.

Since the coupling depend on the modal contents and the phase relations between various modes, we can easily understand that this mechanism potentially generates different coupling for yaw and pitch.

4 The IFO misalignment

It was observed by Nergis, Stefan, Luca, Joe and Keita on various occasions[9] that the coupling has a well-defined phase and amplitude when we intentionally misalign the OMC. As we refine the alignment, the coupling becomes somewhat smaller, but nobody successfully killed the coupling, and the phase of the coupling became unstable. This may be explained by the misalignment of the interferometer, not the OMC.

So far, we assumed that there's no misalignment in the interferometer. This means that there's no "5o" terms in the AS port apart from the ones generated by dithering. However, if any of the mirrors in the interferometer (ETMs, ITMs, BS and PRM) are misaligned, "5o" fields are induced. In this case, the induced fields have different amplitude for carrier and SBs (because of the different response of interferometer for different frequencies). However, as far as the ASQ is concerned, we should be able to handle this in a similar way as the dithering on the optical table, albeit with different coupling constants.

Basically, when we try to kill the coupling by aligning the beam on OMC, we're trying to compensate the "5o" modes in the incoming field by the alignment offset. Though that would succeed at one point, since the alignment of the interferometer itself changes with time, it's impossible to keep this compensation longer than the timescale of alignment fluctuation in the interferometer.

5 The OMC length offset

When the length of the OMC is not exactly on resonance with U_{00}, of course the ASQ signal contains corresponding offset. This offset is proportional to the power of the TEM00 component in the input field. Since the U_{00} power depends on the alignment of the OMC, this can potentially generate large coupling from the misalignment to the ASQ signal:

$$ASQ \sim ASQ_0(2\delta L/\lambda) \times |a_{00}|^2 \left[1 - \left(\frac{\Delta a}{w_0}\right)^2\right], \tag{22}$$

where δL is the offset in the OMC length. The coupling from misalignment to the ASQ in terms of arm length motion is represented by

$$\Delta L_{4\mathrm{k}}^{\mathrm{eff}} \sim \frac{\delta L}{1800} \times \left(\frac{\Delta a}{w_0}\right)^2. \tag{23}$$

If we take only the liner coupling term, this expression becomes

$$\frac{\Delta L_{4\mathrm{k}}^{\mathrm{eff}}}{\delta a(t)} \sim \frac{\delta L}{1800} \times 2\left(\frac{\Delta a_0}{w_0^2}\right) = \frac{1}{900} \times \frac{\delta L}{w_0} \times \frac{\Delta a}{w_0}. \tag{24}$$

Because of a large imbalance in the RF sidebands, this coupling can be large depending on the locking scheme of the OMC. Here three examples are given, i.e. maximization of the total throughput ("simple dither"), maximization of the sideband throughput ("two omega") and the equalization of USB/LSB ("ASI lock"). The first one would give coupling that is smaller than, but not far from, the worst-case estimation of the higher order mode coupling. In the second technique, the lock position is always $\delta L = 0$ and thus the coupling due to this effect becomes zero. The last one gives the largest offset of the three, which would cause the coupling comparable to the worst-case higher order mode coupling. Though these techniques were already analyzed by Daniel[10], here I repeat some simple analysis in relation to the alignment-ASQ coupling.

5.1 "Simple dither" locking

In this scheme, the OMC length is modulated while the transmitted power is monitored. A lock-in amplifier is used to demodulate the DC power after the OMC using the same frequency as the OMC length modulation. When the OMC length is controlled in such a way to null the output of the lock-in amplifier, the total throughput is maximized. The lock point therefore depends on the power of the carrier, upper- and lower-sidebands.

For example, it was measured by Stefan, Daniel and Peter F.[11] that the power ratio for the carrier and the SBs are 450(c):274(lsb):160(usb) for "detect".[5] This would give the offset of 0.6nm for "detect" (Fig.7), thus the coupling becomes

$$\frac{\Delta L_{4\mathrm{k}}^{\mathrm{eff}}}{\delta a(t)} \sim \frac{1}{900} \times \frac{6 \times 10^{-10}}{1 \times 10^{-4}} \times \frac{\Delta a}{w_0} = 6.7 \times 10^{-9} \times \frac{\Delta a}{w_0}, \tag{25}$$

which is roughly a factor of 4 smaller than the worst-case-coupling via higher-order mode. Nevertheless, this is not negligible and therefore the simple dither lock shouldn't be used whenever possible.

5.2 "2-omega" locking

The same dithering for the OMC as in "simple dithering" technique is also used here. The transmission from the OMC is picked-off and is led to an additional photo diode that monitors the second harmonics (2ω) of the modulation. This harmonic signal is doubly demodulated at 2ω and at the dithering frequency, and the resulting signal is used to maximize the "throughput" of the 2-omega signal. Since 2-omega harmonics is the cross-term of the upper- and the lower-sideband, the maximum-throughput point of this signal is not affected

[5] The notation for USB and LSB was arbitrary in their measurement.

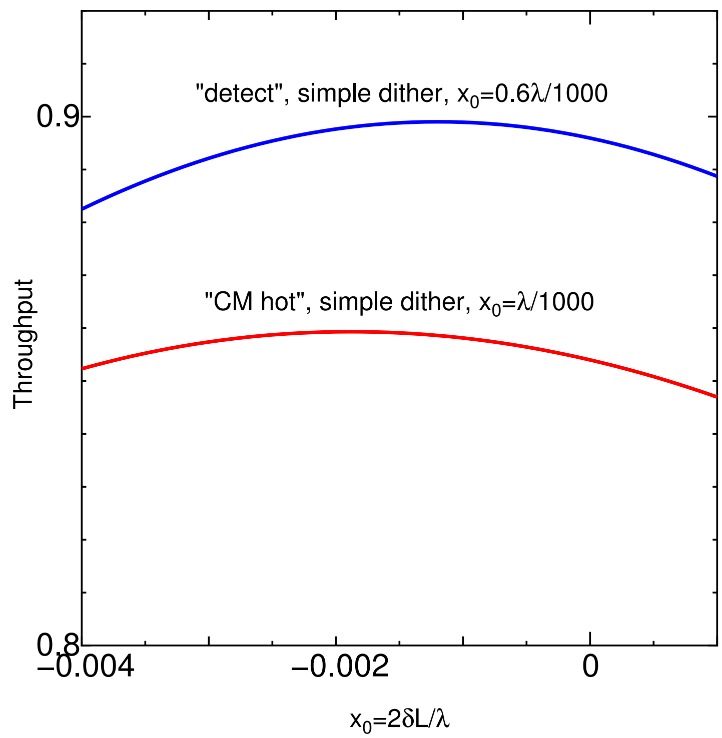

Figure 7: DC throughput calculation for "detect" (blue) and "common mode" hot (red) against the OMC length, using the measured power ratio for the carrier and the SBs. The curves are normalized by the total input power.

by the imbalances in the RF SBs, which is represented by the following equations:

$$
\begin{aligned}
E_{\mathrm{in}} &= c + m_+ e^{i(\omega_{\mathrm{m}} t + \phi_+)} + m_- e^{i(-\omega_{\mathrm{m}} t + \phi_-)} && (26) \\
E_{\mathrm{OMC}} &= c t_{\mathrm{OMC}}(x) + m_+ t_{\mathrm{OMC}}(x + \omega_{\mathrm{m}}/FSR) e^{i(\omega_{\mathrm{m}} t + \phi_+)} + m_- t_{\mathrm{OMC}}(x - \omega_{\mathrm{m}}/FSR) e^{i(-\omega_{\mathrm{m}} t + \phi_-)} && (27) \\
I_{\pm 2\omega_{\mathrm{m}}} &= 2 m_+ m_- \Re \left[t_{\mathrm{OMC}}(x + \omega_{\mathrm{m}}/FSR) t^*_{\mathrm{OMC}}(x - \omega_{\mathrm{m}}/FSR) e^{i(2\omega_{\mathrm{m}} t + \Delta\phi)} \right] \\
&= 2 m_+ m_- \times \left| t_{\mathrm{OMC}}(x + \omega_{\mathrm{m}}/FSR) t^*_{\mathrm{OMC}}(x - \omega_{\mathrm{m}}/FSR) \right| \times \cos\left[2\omega_{\mathrm{m}} t + \Delta\phi + \psi(x) \right] && (28) \\
\psi(x) &\equiv Arg\left[t_{\mathrm{OMC}}(x + \omega_{\mathrm{m}}/FSR) t^*_{\mathrm{OMC}}(x - \omega_{\mathrm{m}}/FSR) \right], && (29)
\end{aligned}
$$

where $m_\pm$and $\phi_\pm$are the amplitude and the static phase of the SBs, $t_{\mathrm{OMC}}$is the transmittance of the OMC and$\Delta\phi$ is the difference in the static phase of the SBs.

From the above expression, one can immediately see that the "throughput" of 2ω signal is not dependent on the amplitude and the phase of the SBs. From the symmetry of $t_{\mathrm{OMC}}(x)$, the "throughput" of 2ω is always maximized at $x = 0$. Strictly speaking, of course, the signal depends on the demodulation phase. However, note that both $|t_{\mathrm{OMC}}(x + \omega_{\mathrm{m}}/FSR) t^*_{\mathrm{OMC}}(x - \omega_{\mathrm{m}}/FSR)|$ and $\psi(x)$ are even functions of x. Because of this, any kind of demodulation phase would result in either maxima or minima at the center ($x = 0$). Therefore a slight change in the demodulation phase wouldn't change the lock point (Fig.8).

5.3 "ASI" locking

There's another technique in which the length of the OMC is adjusted to minimize the ASI signal ("ASI" locking). The intuitive reasoning behind this is to equalize the imbalanced RF SBs by intentionally introducing a large offset in the OMC length. A change in the lock position means a change in the optimal demodulation phase, therefore the adjustment of the demodulation phase should be done iteratively in the initial setup process to maximize the GW signal in ASQ. As far as this is done properly, this technique should work to some extent. However, a large offset in the lock position is apparently not desirable for the alignment-ASQ

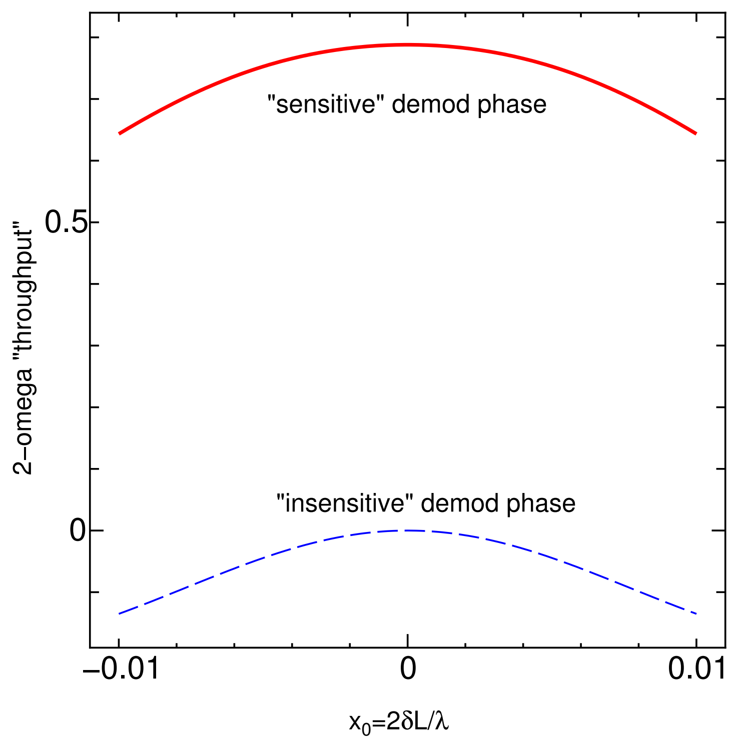

Figure 8: The 2-omega "throughput" for the most sensitive and the most insensitive demodulation phase (i.e. $\Delta\phi + \psi(0)$ and its quadrature-phase). Because of the symmetry of these curves, any demodulation phase has either maxima or minima at $x = 0$, therefore a slight shift in the demodulation phase wouldn't change the lock point.

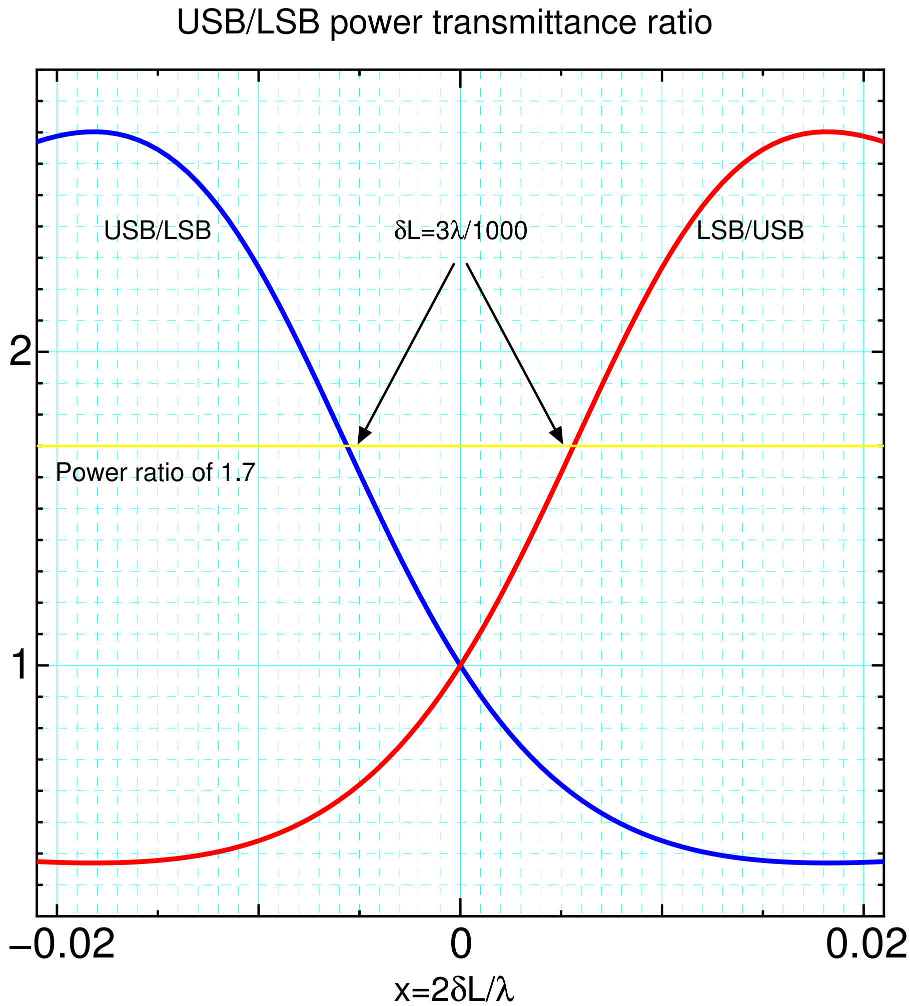

Figure 9: The ratio of the intensity transmittance for USB and LSB. In "detect", the OMC is locked to $\delta L \sim 3\lambda/1000$ because of the imbalance in the SBs.

coupling as we saw. For example, in "detect", the power ratio for USB/LSB was measured to be 274/160=1.71. To equalize them, one has to offset the lock position by roughly $3\lambda/1000$ (Fig.9), which is factor of 3 to 4 larger than in "simple dither" locking, and is slightly worse than the worst case higher order mode coupling:

$$\frac{\Delta L_{4k}^{\rm eff}}{\delta a(t)} \sim \frac{1}{900} \times \frac{3 \times 10^{-9}}{1 \times 10^{-4}} \times \frac{\Delta a}{w_0} = 3.3 \times 10^{-8} \times \frac{\Delta a}{w_0}. \tag{30}$$

Therefore "ASI lock" technique should be avoided unless the imbalance in RF SBs is considerably improved by other means.

6 Mitigation of the alignment-ASQ coupling by a better mode matching

One of the reasons why we have a large coupling from the misalignment is that the AS field itself contains a large power for "4" and "6" modes. For this reason, we're studying the origin of these higher order modes. At the moment, at least a part of this seems to be due to the large astigmatism in the AS field.[6] Indeed, it was measured that the AS beam is astigmatic, and the waist position for one direction and the other is different by more than Rayleigh range. It is still not clear why the astigmatism is so large, though. At the time of this writing, to help understanding the origin of these higher order modes, Luca is working on a model which seems to explain the modal content of the bright MICH field. Mike Smith is expected to help to see if the IFO optics can cause such a large astigmatism, even in bright MICH.

For "detect" and "common mode", things get more complicated because the amplitude of TEM00 gets considerably smaller than in bright MICH. An important thing to do is to study the modal structures of the carrier and the SBs separately by phasecamera. A complementary approach is to study "why things are such", and it seems to be a natural route to start from a "bull's eye" DC pattern, and incorporate the astigmatism. Probably it is useful to ask for help from simulation team.

Once it is established that the origin of the higher order modes are the astigmatism of the AS field, a better mode matching setup should be implemented. At the moment it is difficult to give any number as for how much the coupling is mitigated by mode matching, though.

[6] One has to remember that the problem of the astigmatism in the AS field is different from the problem of the astigmatism of the OMC itself, which is discussed later separately in Sec.7.3.2. They are not completely orthogonal, but they are still separate issues.

7 Mitigation of the coupling by a new OMC design

The problems of the current OMC design are:

1. "5o" is close to the resonance of the fundamental mode.
2. The width of the OMC is neither narrow nor wide enough.
3. The OMC has the different response for the carrier, USB and LSB.

In the (distant) future, an OMC with half the modulation wavelength (6m) is preferable because the ASQ signal of such a cavity doesn't have any sensitivity to the OMC length change. However, at the moment we have to think of something that can be implemented in current LIGO system. Therefore I propose the followings:

- Use of 4-mirror cavity instead of 3
- A slight change in g-factor.
- A higher finesse (useful only when implemented together with the change mentioned above).
- Shorter length.

As starting points to begin discussion with, three alternative designs are presented:

- Non-planar configuration with four identical spherical mirrors.
- Planar configuration with two different cylindrical mirrors.
- Planar configuration with four plane mirrors and a lens inside the cavity[]

7.1 4-mirror cavity and different g-factor

In linear cavities with 2 mirrors, modes with the same order (i.e. a set of $\{U_{nm}\}$ with the same $n+m$) are degenerate. However, in triangular cavity, this degeneracy is broken: Assuming the same g-factor, the resonance of U_{nm} for $n = 2k+1$ (k: non-negative integer) for a 3-mirror cavity is shifted by FSR/2 compared with the linear-cavity counterpart, while $n = 2k$ are the same.[7] This is the reason why the "5o" modes are close to TEM00 in our triangular OMC. On the other hand, in cavities with even-number of mirrors, this is not the case. "5o" and "5e" are degenerate.

Figure 10 shows ASQ_0 curves against transverse mode spacing x_{t} for up to 8th order modes for triangular cavity. Since the curves are normalized by the power of the modes, each curve roughly shows the quantity that is proportional to "coupling strength" from a particular mode. As can be seen from the plot, no matter how the transverse mode spacing is chosen, it is almost impossible to improve the coupling by an order of magnitude. The only exception is where the higher order modes become degenerate with TEM00 (like $x_{\mathrm{t}} = 0.3$ for "5o"): Modes that are completely degenerate with TEM00 don't create the ASQ signal when the cavity is locked to the resonance of TEM00. However, the tolerance for the error in g-factor becomes very strict for such positions: For example, if we're to choose $x_{\mathrm{t}} = 0.3$, the error of 2×10^{-4} in x_{t} easily degrades the coupling to the current level. Therefore this route should be avoided.

On the other hand, Fig.11 shows ASQ_0 curves for a cavity with even number of mirrors. Since the "e" and "o" modes are degenerate, by simply changing to 4-mirror design we would instantly get a factor of 7.5 improvement, assuming other parameters (*i.e.*the finesse, the transverse mode spacing and the round-trip length of the cavity) being equal. If we're to change the g-factor, both $x_{\mathrm{t}} = 0.2225$ and 0.3015 looks interesting because these points give

[7] When the field is reflected on the mirror, the spacial symmetry of the field changes from right- to left-handed (or from left to right). When there're only two mirrors, the field is bounced by mirrors two times per round trip, thus the spacial symmetry of the beam always stays the same no matter how many roundtrips it makes in the cavity. However, when there are 3 mirrors, if the beam path is in xy plane, one round trip would transform $U_{nm}(x, y, z)$ to $U_{nm}(-x, y, z)$. Since $U_{nm}(x, y, z)$ is an odd or even function of x depending on n, that causes FSR/2 shift for the modes with odd n.

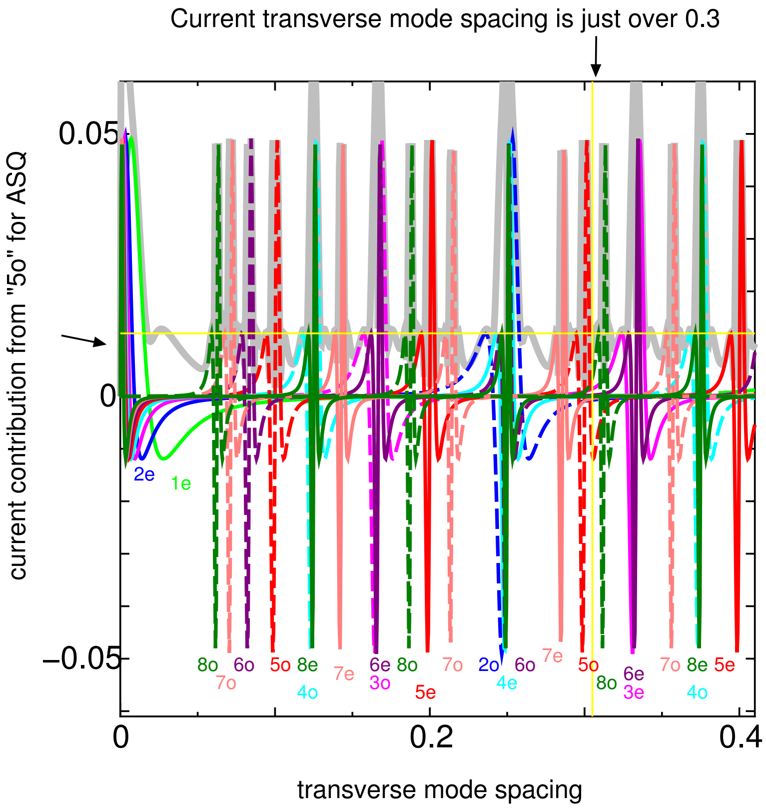

Figure 10: ASQ_0 curves for all of the modes from "1" up to "8". Since the curves are normalized by the power of the modes, each curve roughly shows the quantity that is proportional to "coupling strength" from a particular mode. The thick grey line is the RMS of all of the modes up to 8. It is almost impossible to obtain anything better than a factor of 2 improvement.

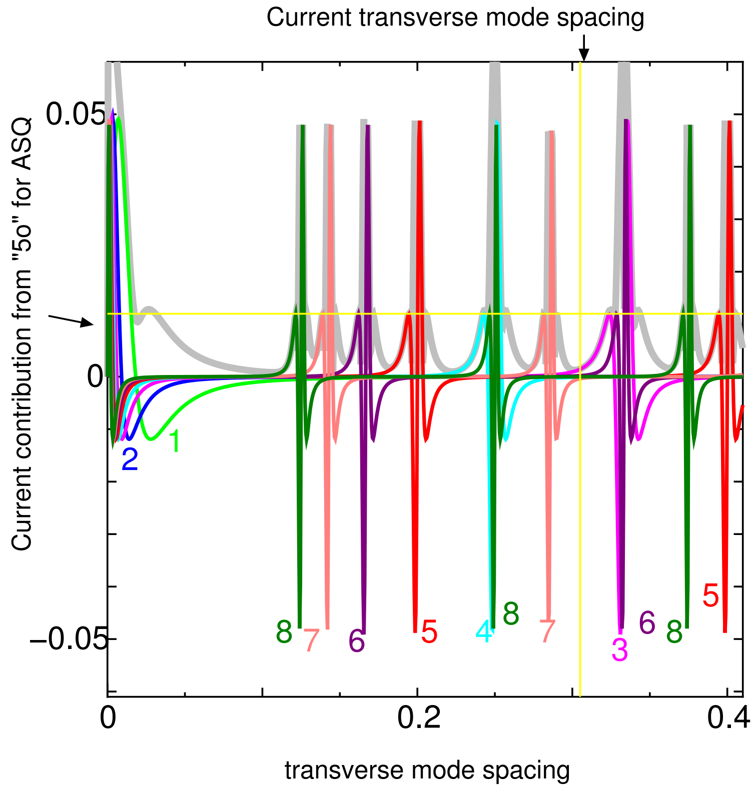

Figure 11: ASQ_0 curves for cavities with even number of mirrors (linear, 4-mirrored etc.) up to 8th order. Other conditions being equal, by simply changing to 4-mirror cavity we'd be able to obtain factor of 7.5 improvement. Also note that the tolerance for the errors in transverse mode spacing seems to be reasonable.

the improvement roughly by a factor of 10, and the tolerance for the transverse mode spacing doesn't seem to be stringent. For this reason I propose to change the configuration from triangular to four-mirrored.

7.2 Higher finesse

It should also help to increase the finesse of the cavity to have larger discrimination of the higher order modes, but one has to be very careful: Simply going for a higher finesse without changing to 4-mirror design makes the problem worse. Here the case study for triangular and the 4-mirror cavity are made.

7.2.1 Higher finesse for triangular configuration makes things worse

One might easily think that a higher finesse (i.e. more discrimination) is the way to go, because the ASQ would become insensitive to "5o" if that mode is far from the resonance. However, this is only true for a cavity in which "5o" (and any other mode) is well far away from the resonance. For this to be true, we have to have really high finesse (on the order of 1000) for the current configuration. Let's see what happens if we go for a higher finesse with the triangular OMC.

The sensitivity of ASQ against the GW signal is proportional to the transmittance of the OMC for SBs: When we make the finesse higher, we make the GW signal smaller. On the other hand, the sensitivity of ASQ against the OMC length is proportional to the difference in the OMC transmittance for the carrier and the SBs: To some extent, the higher finesse makes the amplitude of the error signal generated by "5o" larger, despite the fact that the mean transmittance for that mode decreases. Since the coupling, $\Delta L_{4\mathrm{k}}^{\mathrm{eff}}/\delta a(t)$, is inversely proportional to the ASQ sensitivity against the GW signal and is proportional to the sensitivity against the OMC length, the "5o" has to almost completely goes away from the resonance if we're to avoid the coupling by making the finesse higher.

Figure 12 shows the "5o" error signal against the OMC length change, normalized by the transmittance for the TEM00 SBs for various finesse. Basically, by going for higher-finesse, one has to live with the decrease in the GW signal amplitude. For example, compared with the current value of $\mathcal{F} = 30$, a factor of 5 improvement in the coupling is obtained if we increase the finesse to 1000 at the expense of making GW signal level smaller by a factor of 15. A factor of 10 improvement is only obtained by $\mathcal{F} = 2000$.

One could still try to kill the coupling by fine-tuning the finesse to somewhere around 21 ($R = 86.11\%$). Of course we cannot rely on this theoretical limit because we surely have some measurement error in the transverse mode spacing, and we don't have fine control over the reflectivity of the OMC mirrors. A reasonable tolerance of $R = 86 \pm 1\%$ still yields a factor of 5 improvement, without taking the astigmatism of the OMC itself into account.

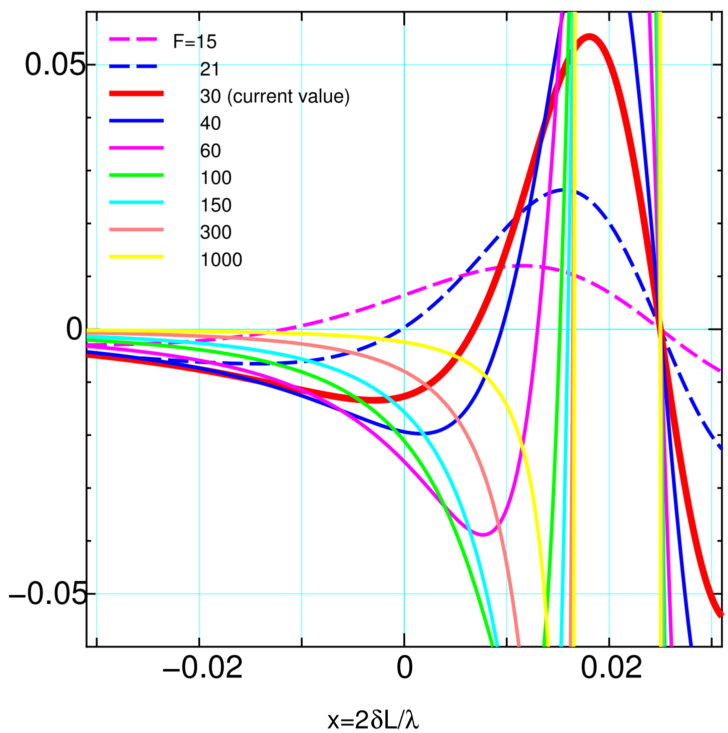

Figure 12: ASQ curve of the current OMC geometry for "5o" for various finesse, normalized by the transmittance of the RF SB for TEM00, i.e. $ASQ_0(x{+}5x_{\mathrm{t}}{+}0.5)/t_{\mathrm{OMC}}(25\mathrm{MHz}/3\mathrm{GHz})$. Higher finesse design necessitates the REALLY higher finesse: For example a factor of 5 improvement is obtained by the finesse of 1000. On the other hand, the coupling would be zero at around $\mathcal{F} \sim 21$, and a factor of 6 improvement is still obtained by the finesse of 20.

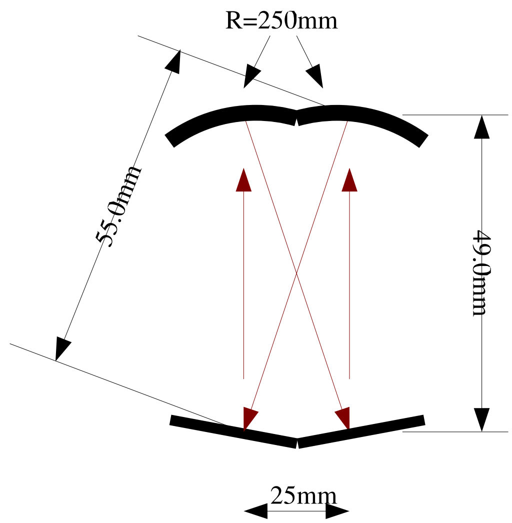

Figure 13: A four-mirrored cavity with $x_{\mathrm{t}} \sim 0.302$.

However, my recommendation is to stay away from such a tricky tuning on a quantity (i.e. the reflectivity of the mirror) of which we don't have any dynamic control.

As we have seen before, in triangular cavity it is practically impossible to find a transverse mode spacing for which all of the higher order modes up to 8 don't come into resonance. Therefore, to obtain an order of improvement we'll have to increase the finesse to the order of 1000 no matter how we choose the $g-$factor. This is another basis for my recommendation on 4-mirror configuration.

7.2.2 Higher finesse for 4-mirror configuration is useful

Let's take a look at the effect of the finesse on the coupling in 4-mirrored cavity. As a simple example, Fig.13 shows a 4-mirror cavity with the transverse mode spacing of $x_{\mathrm{t}} \sim 0.302$.

Figure 14 shows a plot similar to Fig.12 for this example configuration. Since there are two important terms in this case, i.e. "3" and "7" modes, both are plotted on the same graph. Unlike the triangular design, the coupling decreases with the finesse. This is due to the fact that all of the higher order modes can be kept away from the resonance in 4-mirror configuration.

Since the overall size of the OMC was kept roughly the same with the current one, the

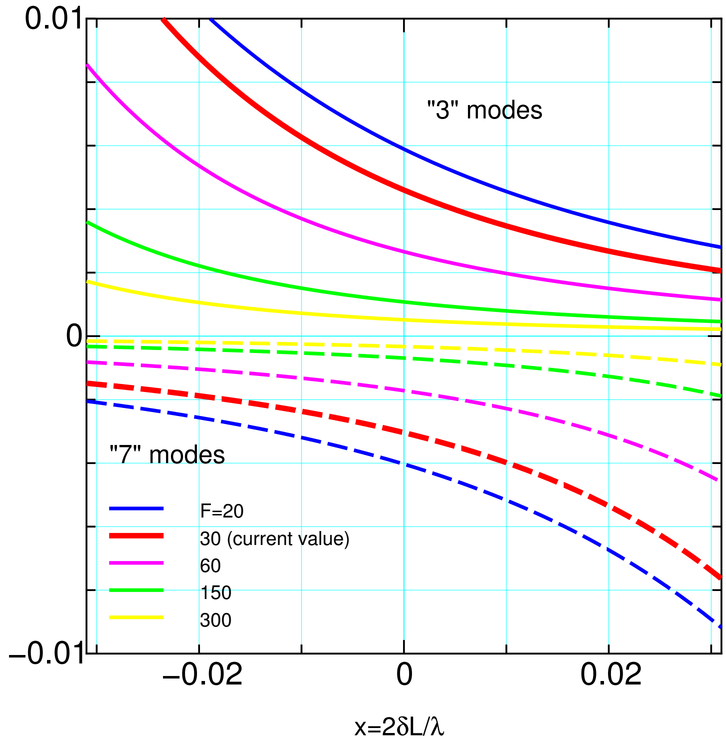

Figure 14: ASQ curve of the four-mirror OMC geometry for "3" and "7" against various finesse, normalized by the transmittance of the RF SB for TEM00.

actual roundtrip length is doubled compared with similar triangular design. This makes the FSR of the cavity smaller, which in turn makes the sensitivity of the ASQ against the length larger. For this reason, the decrease in the coupling on the plot is not that drastic. Still, by changing finesse to 150, we'll be able to obtain a factor of 10 improvement. Contrast this to the triangular configuration where the same improvement is impossible unless we make the finesse much higher, i.e. $\mathcal{F} = 2000$. Also we have to note that the waist size is larger ($\sim 185\mu$m) in this example to keep the transverse mode spacing at around 0.3.

7.2.3 Coupling via the OMC length offset becomes worse by higher finesse

Though the higher finesse with 4-mirror configuration helps reducing the higher order mode coupling, the coupling via the OMC length offset becomes worse. This is because of the simple fact that the alignment-ASQ coupling via the offset is proportional to the ASQ sensitivity against the to the OMC length. Though this can be mitigated to some extent by a proper choice of locking scheme (Sec.5) and a large bandwidth in OMC length control, we have to remember that this poses some restriction on finesse.

Lower plot in Fig. 15 shows the amplitude transmittance of the OMC for RF SBs, the quantity that is proportional to the signal amplitude corresponding to the GW. Upper plot in the same figure shows the length sensitivity of the OMC (i.e. $\mathrm{d}ASQ_0/\mathrm{d}L_{\mathrm{OMC}}$) normalized by the transmittance of the RF SBs. This is proportional to the coupling from the OMC length to the GW signal. Both of the figures are plotted against the finesse for several round trip lengths (i.e. FSRs). As the upper plot shows, the coupling has a very strong dependence on finesse even though the GW signal is not so strongly affected. This is because the $ASQ_0(x)$ is proportional to $\mathcal{F}^3$ in the lower-finesse limit (see Eqs.46 and 47 for details). Increasing the finesse to 100 without making the cavity shorter, the coupling would increase by more than a factor of 30. Since we cannot make the cavity much shorter than, say, 4cm for practical reasons, it seems to be better to stay within $\mathcal{F} < 100$.

7.3 Making the OMC length shorter while keeping good degeneracy of the modes

7.3.1 Making the cavity length shorter helps

As we saw in Fig.15, it seems to be a good idea to make the OMC length shorter and thus making the FSR larger. The transmittance of the higher order modes are not affected by this, but the difference in the transmittance of the OMC for the carrier and the SBs become smaller, which in turn makes the sensitivity of ASQ against the OMC length smaller. At a first glance, one might think that a care has to be taken because the smaller cavity in general means the smaller waist size, which might increase the sensitivity to the lateral beam displacement. This turns out to be not so serious, because (as we already saw) a four-mirror cavity has the larger waist size compared with that of the triangular cavity with an identical transverse mode spacing and the same physical dimensions.

However, due to physical constraints (i.e. fixed size mirror diameter etc.), a shorter

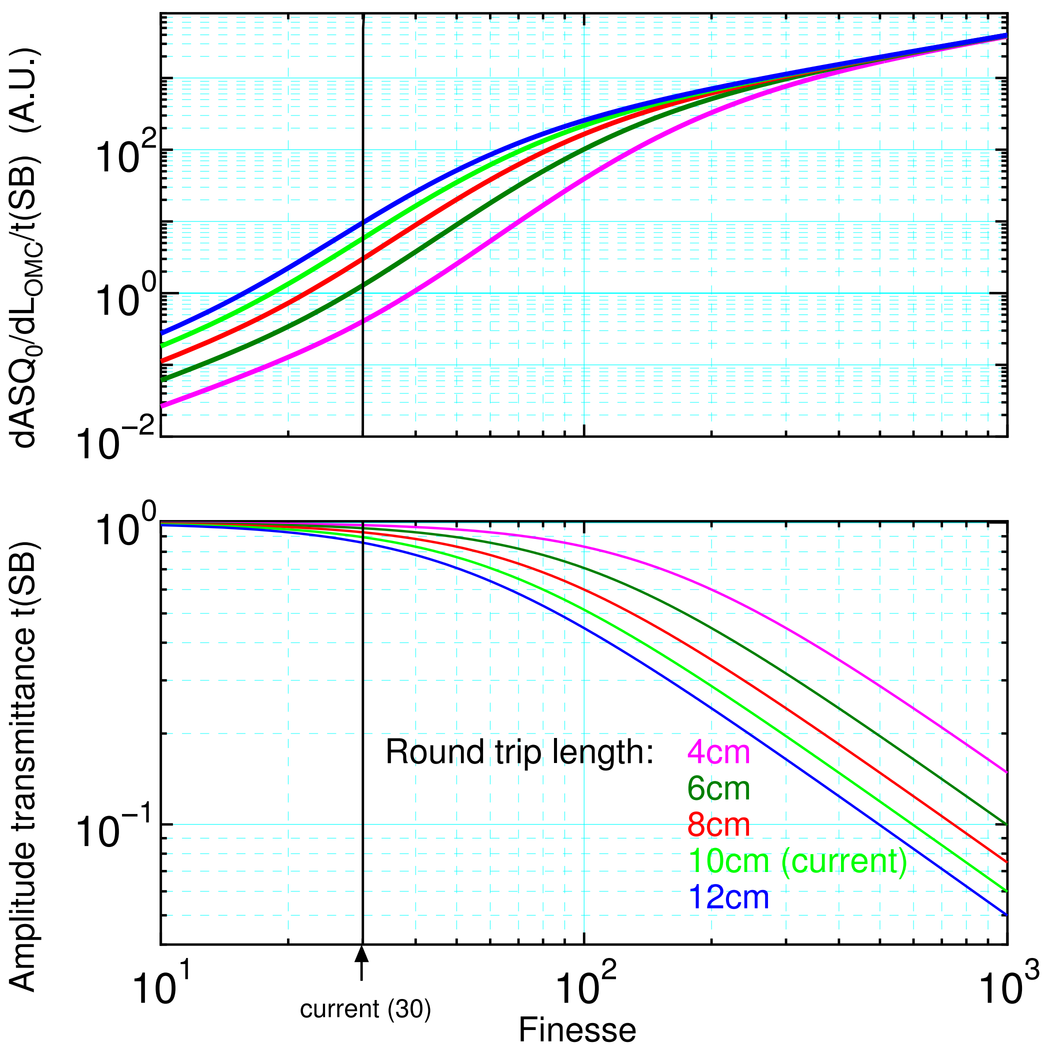

Figure 15: The sensitivity of the ASQ to the OMC length against finesse, normalized by the transmittance for the RF SBs (upper plot), and the amplitude transmittance of the OMC for RF SBs (lower).

ring quite often means a larger astigmatism and/or a larger difference in Gouy phase for horizontal and vertical direction. This breaks the degeneracy of the otherwise completely degenerate modes, which is discussed in the following section.

7.3.2 Astigmatism and Gouy phase difference of the OMC itself compromise the merit of shorter length

So far we have ignored the asymmetry of the OMC for horizontal and vertical direction, i.e. astigmatism and the difference of the Gouy phase shift per round trip. However, in reality the OMC is asymmetric due to finite incident angle on curved mirror(s).[8] This makes it more difficult to design a small-coupling cavity, as the difference in Gouy phase shift for horizontal and vertical direction means the split in the same order, same parity modes. If the mode spacing for horizontal and vertical direction are represented by $x_{\mathrm{t}}^{\mathrm{H}}$ and $x_{\mathrm{t}}^{\mathrm{V}}$, each of the components in, for example "5o" group, has slightly different resonant positions, i.e. U_{50} has the resonant point of $5x_{\mathrm{t}}^{\mathrm{H}}+0.5$ while that of U_{14} is $x_{\mathrm{t}}^{\mathrm{H}}+4x_{\mathrm{t}}^{V}+0.5$. In the current OMC, the spacings are expected to be $x_{\mathrm{t}}^{\mathrm{H}} \sim 0.309$ and $x_{\mathrm{t}}^{\mathrm{V}} \sim 0.,302$ for horizontal and vertical direction, even though the "mean" spacing was measured to be 0.305. This is not negligible, especially when "5o" modes are so close to the resonance (see Fig.16).

Even though this is more visible in triangular configuration, we still cannot ignore this effect in four-mirror cavity, either. For example, when we take the asymmetry into account in the model cavity in Fig.13, the spacings for x and y direction becomes $x_{\mathrm{t}}^{\mathrm{H}} \sim 0.306$ and $x_{\mathrm{t}}^{\mathrm{V}} \sim 0.297$, respectively. The "split" in this case for "7" modes are as large as $7(x_{\mathrm{t}}^{\mathrm{H}}-x_{\mathrm{t}}^{\mathrm{V}}) \sim 0.063$. This is not that bad compared with the triangular cavity, but still not negligible. As Fig.17 shows, the coupling could potentially be a factor of two larger than the one obtained using the "mean" transverse mode spacing, depending on the modal contents of "2", "4", and "6" in the AS field (the information which we don't know of yet). Therefore, to assure the improvement by a factor of 10, for example, its safer to have the finesse of 300 instead

[8] The problem of the astigmatism of the OMC itself should never be confused with the problem of the astigmatism of the AS field.

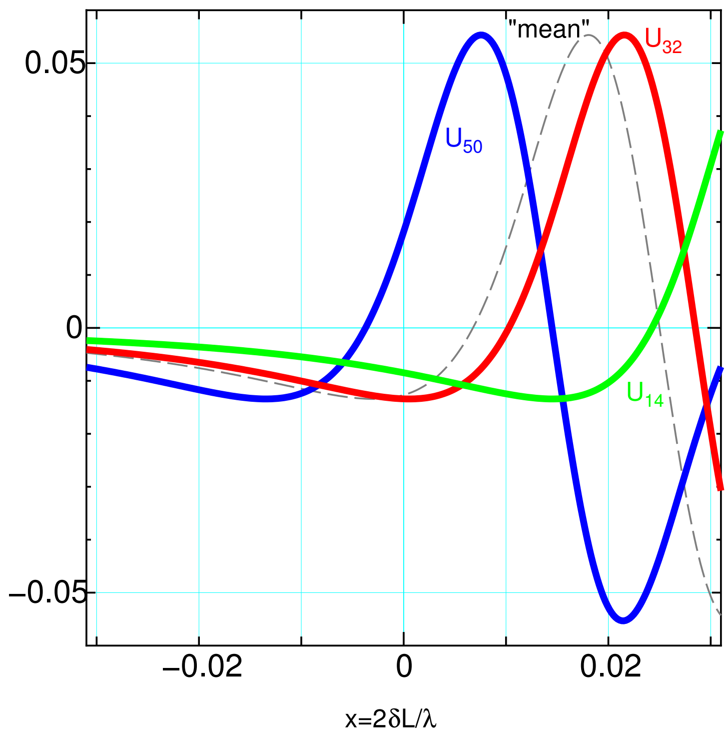

Figure 16: ASQ curves for "5o" modes normalized by the transmittance of the RF SB for TEM00 for the current OMC, taking the Gouy shift difference into account. The "5o" modes, which are otherwise degenerate in the "mean" position, are split into three in the current OMC.

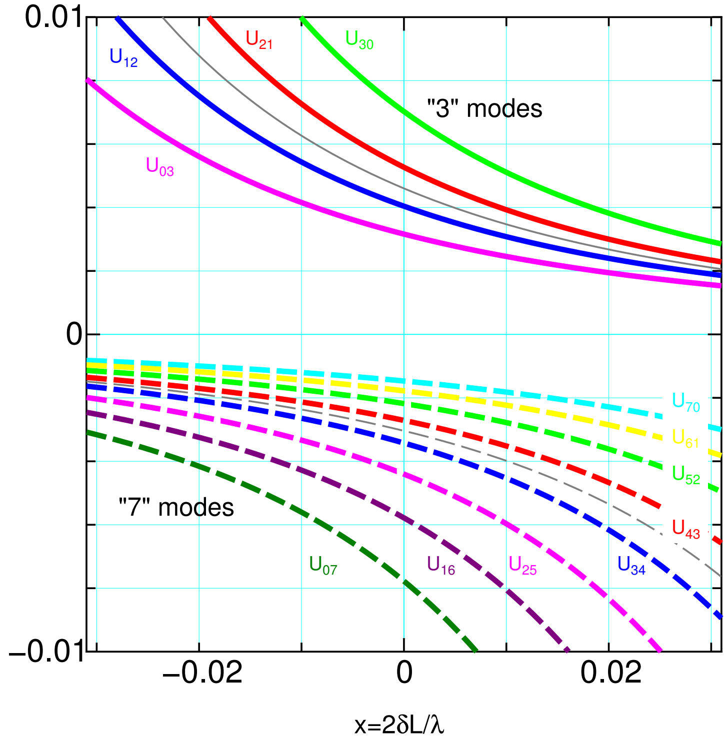

Figure 17: ASQ curve of the astigmatic four-mirror OMC with the finesse of 30 for "3" and "7", normalized by the transmittance of the RF SB for TEM00. The gray lines are the "mean" curves. The "3" is split into 4 curves while "7" is into 8.

of 150. Making the length shorter also means making the astigmatism and/or Gouy phase difference larger at the same time, which compromise the whole point of making it shorter to provide a good alignment-ASQ separation.

7.4 Proposed designs

Three alternative designs based on the same design principle (i.e. to keep the good degeneracy of the higher order modes) are presented here.All of them can eliminate the split of the degenerate modes completely in theory.

The first one uses cylindrical mirrors, and allows some waist size difference for horizontal and vertical modes while the waist positions are the same. The second one, all-flat mirrors with an internal lens, completely eliminates the waist size difference as well as the astigmatism. The last one, a non-planar design with six mirrors, allows a small astigmatism while the waist radii are the same. former makes the astigmatism smaller compared with the latter, at the expense of considerably more complex beam trajectory.

The mitigation effect of these three are not drastically different. However, since the first one makes the smallest cavity with standard optics, my recommendation is to try the first design.

7.4.1 Planar design with cylindrical mirrors

The design presented here uses cylindrical optics instead of the spherical ones. The different radius of curvature for horizontal and vertical direction allow one to keep good degeneracy of the higher order modes. The incident angles and the radii of curvature for horizontally and vertically curved mirrors should satisfy the following relation:

$$\frac{R_y}{R_x} = \cos\theta_x \cos\theta_y \tag{31}$$

where R_x, θ_x, R_y and θ_y are the radius curvature and the incident angle for horizontally and vertically curved mirror, respectively.

Figure 18 shows such a configuration with the equal incident angle of $\pi/6$. By carefully choosing the round trip length (71.1mm, i.e. FSR of 4.2GHz) and the radii of curvature ($R_x = 101.7$mm and $R_y = 76.3$mm[9]), one can obtain the desired transverse mode spacing of 0.303. As one can see in Fig.19, this cavity has different waist size (123μm and 98μm) for horizontal and vertical direction. However, Eq.31, which in this case means that $R_x = 4R_y/3$, assures that the Gouy shift per round trip is the same for both directions, therefore the higher order modes are still degenerate.

The difference in the waist size, in general, makes it more difficult to properly modematch the incoming light to the cavity. In this case, however, the difference is small enough. For example, if one match the incoming field to the "mean" round mode with the waist size of (123+98)/2=110.5μm, the mismatching parameter is 0.1 for both of the directions. This

[9]These are the standard curvatures for CVI cylindrical lens.

means that only 2% of the power of the TEM00 component in the incoming field couples to the second order OMC modes. Compared with the current situation (the power of the TEM00 and the second order modes are 43% and 23% of the total power, respectively), this is going to become an issue only after a considerable amount of improvement in the mode structure of the AS port is achieved. Even at that point, we should still be able to use cylindrical matching optics to have a better results.

Figure 20 shows the ASQ curves for dominant higher order modes (third and seventh) in this configuration. Due to 4-mirror configuration with shorter length, an improvement by a factor of 20 is obtained by the same finesse as the current one. With the finesse of 60, we would be able to achieve an improvement by a factor of 30, at the expense of making the coupling via OMC length offset worse by a factor of 5.

The characteristics of this configuration is as follows:

- The higher order modes with the same order are completely degenerate.
- Smaller round trip length (7.11cm, i.e. FSR of 2.1GHz) makes the mitigation effective.
- Waist sizes are slightly different for horizontal and vertical directions, but this is still reasonable.
- A smaller incident angle on the input/output mirror ($\pi/6$) might mean larger backscatter than $\pi/4$ incident.
- Less choice of optics due to cylindrical mirrors.

7.4.2 Flat mirrors with an internal lens

Bill Kells suggested me that it might be worth studying a cavity with flat mirrors and an internal lens to eliminate the astigmatism. By doing this, one can completely eliminate the astigmatism and the waist size difference. Figure 21 shows such a configuration that gives similar transverse mode spacing as the previous configuration. Figure 22 shows the beam

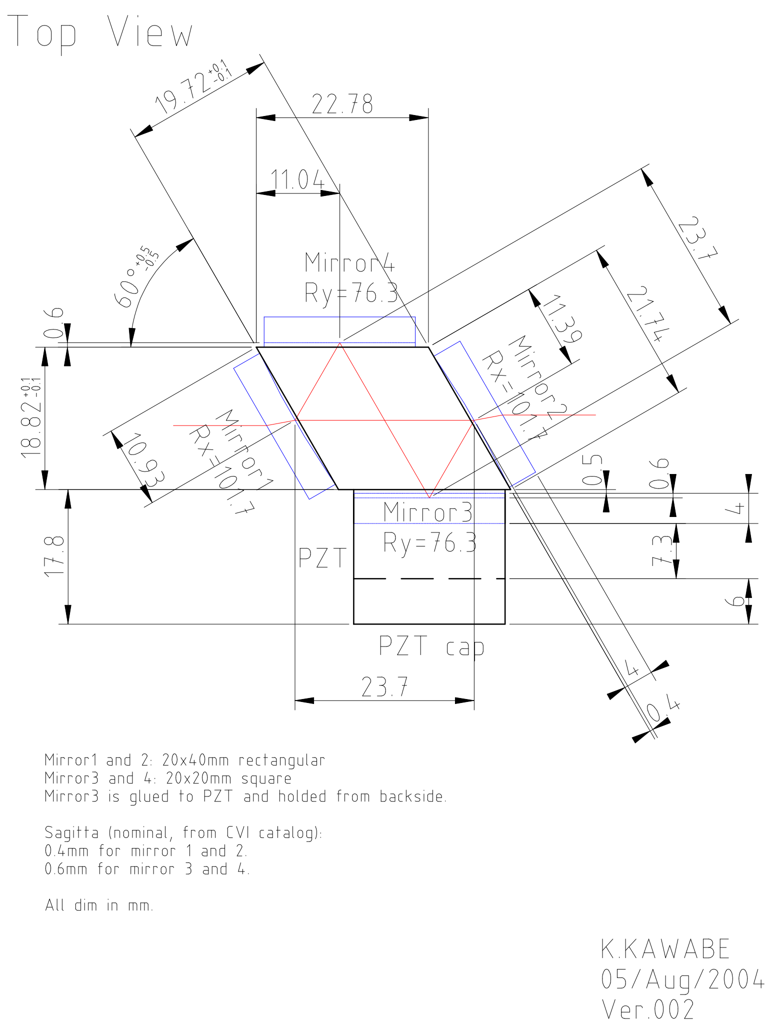

Figure 18: A four-mirror cavity with two different kinds of cylindrical mirrors. Mirror 1 and 2 are curved in vertical direction, while mirror 3 and 4 are in horizontal direction. The round trip length is 71.1mm.

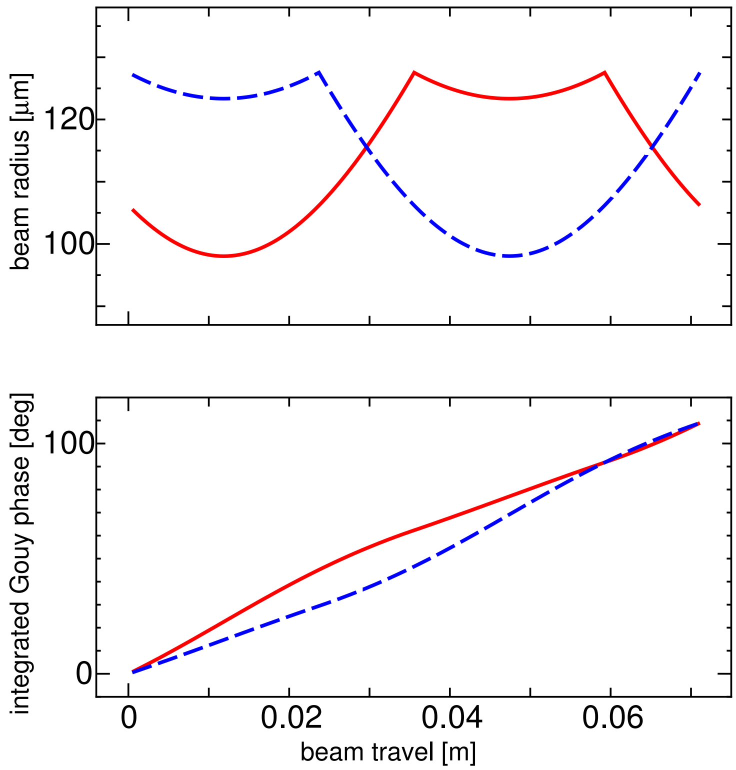

Figure 19: Beam propagation inside the cavity using cylindrical optics.

size and the Gouy phase along the beam path for one round trip. [10] According to Bill, the back reflection/scattering of the lens are less problematic than that of the mode matching lens between IFO and OMC, because there is a mirror between IFO and the internal lens. On the other hand, there still seems to be some unknown effects. For example, light inside the cavity can be reflected on the lens surface, reflected again after N roundtrip (in reverse direction) and then interferes with the main beam. Such effects should be studied carefully.

Figure 23 shows the ASQ curves for dominant higher order modes, normalized by the transmittance of the OMC for RF SBs, in this configuration. Since the round trip length is larger, the coupling is worse than that of the "cylindrical mirrors" design. With the finesse of 60, we would be able to achieve an improvement by a factor of 20 while the coupling via OMC length offset would become by a factor of 8.6 worse.

[10] Due to practical constraint (like the size of the input/output mirrors and the size of the lens holder), the round trip length is 94mm, which is 30% larger than that of the cylindrical configuration. The size of the lens holder can be reduced if we design it by ourselves, though.

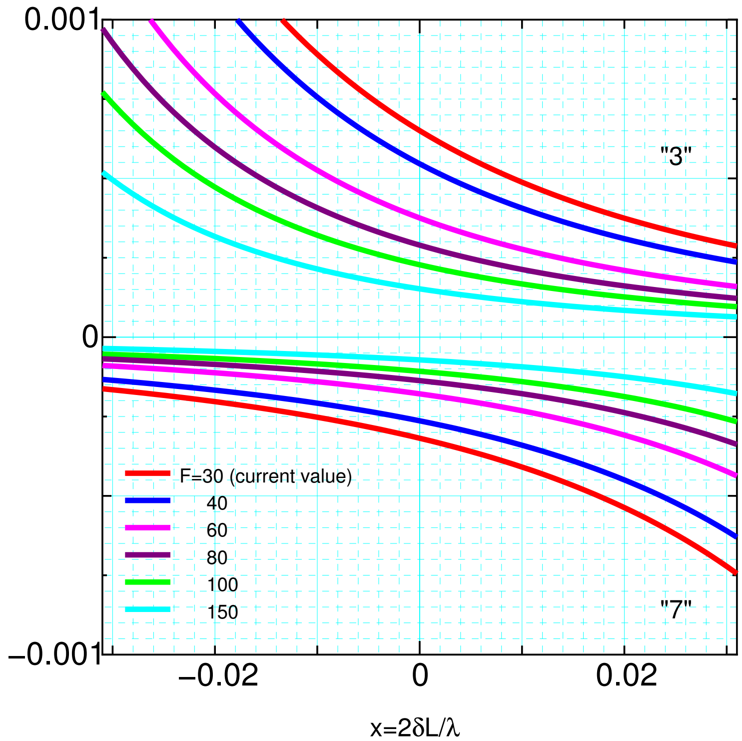

Figure 20: ASQ curves for "3" and "7" modes, normalized by the transmittance of the RF SB for TEM00, for various finesse using the planar 4-mirror design with cylindrical mirrors.

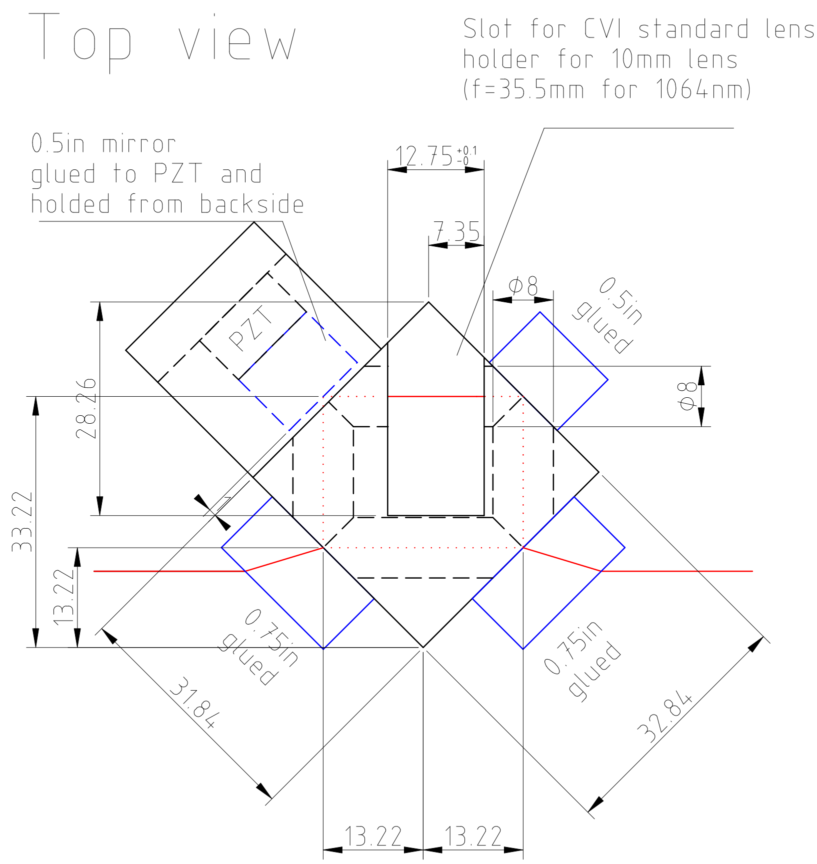

Figure 21: A four-mirror cavity with all flat mirrors and an internal lens. This completely eliminates the astigmatism and waist size difference of the OMC. The optical thickness of the lens was taken into account to obtain a desired transverse mode spacing.

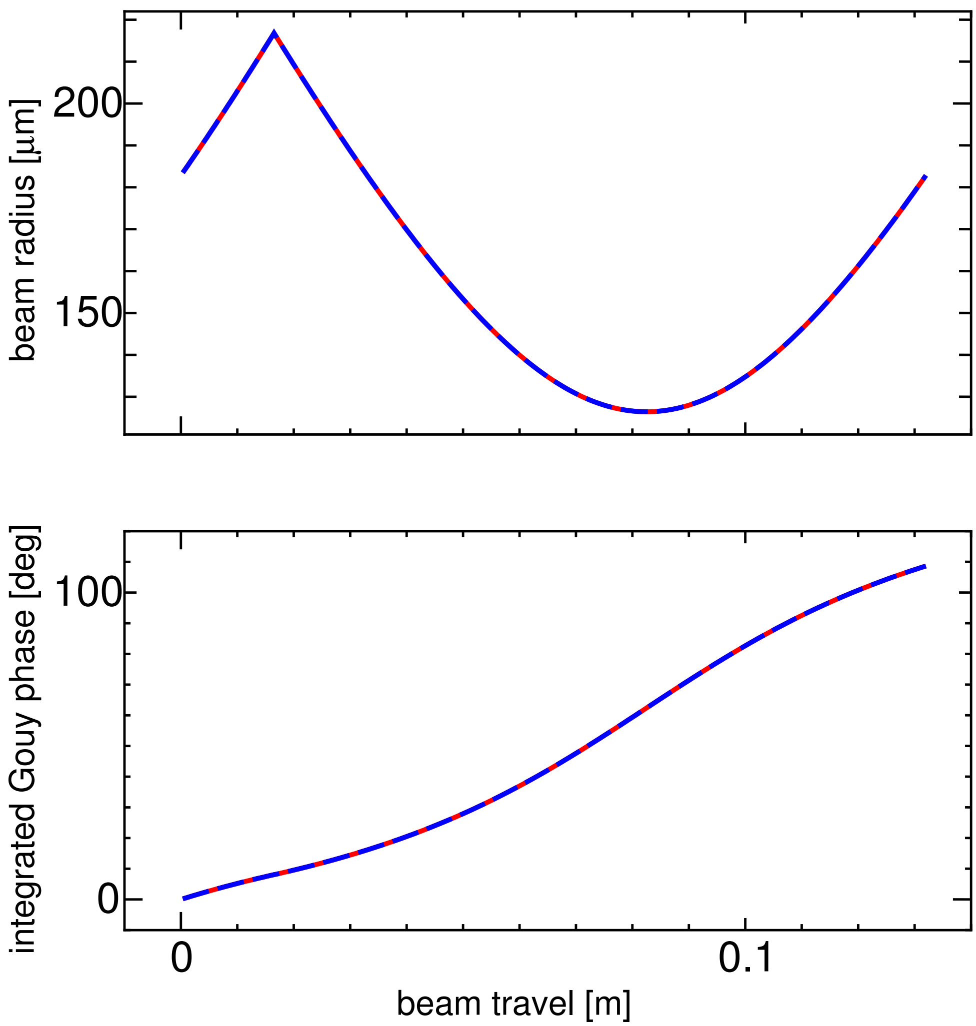

Figure 22: Beam propagation of the internal lens design.

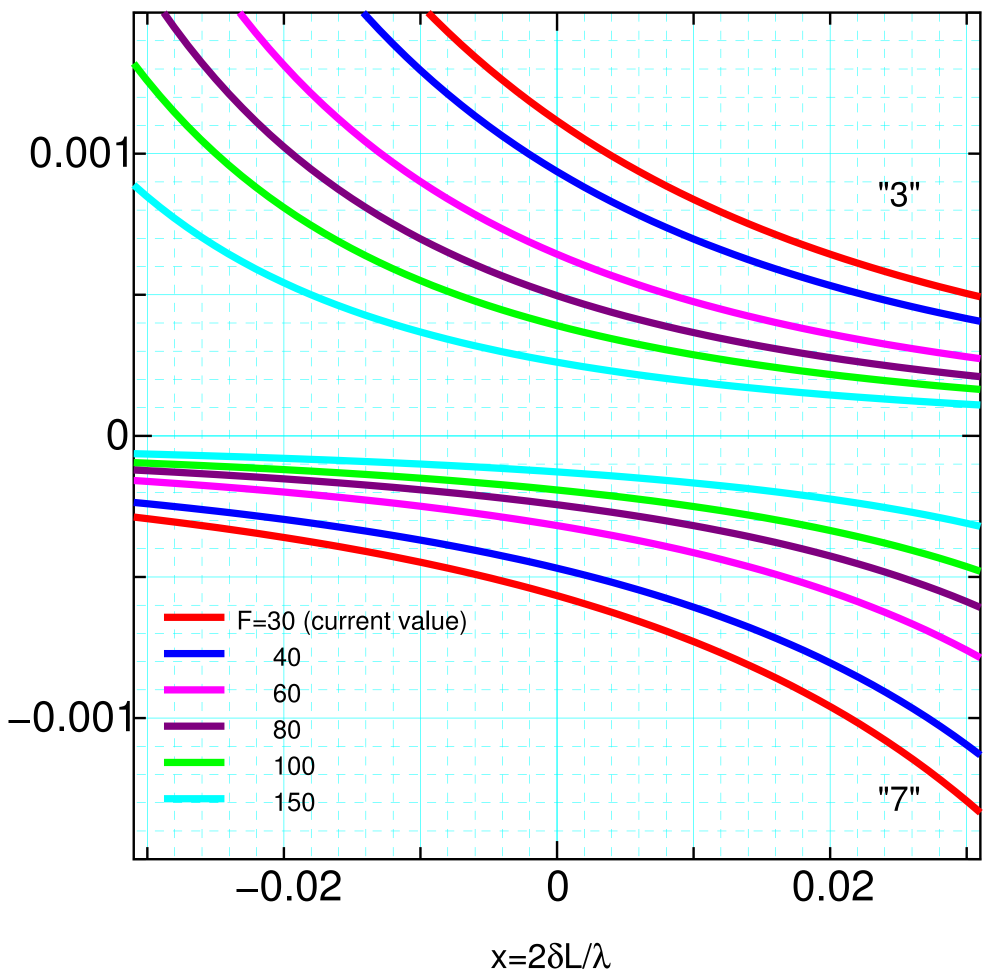

Figure 23: ASQ curves for "3" and "7" modes, normalized by the transmittance of the RF SB for TEM00, for various finesse using the flat 4-mirror design with an internal lens.

Characteristics of this configuration is:

- No astigmatism, no waist size difference.
- Simpler beam path makes it easier to fabricate the cavity.
- Incident angles for the mirrors are $\pi/4$, thus smaller back scattering is expected.
- Slightly longer roundtrip length, making the mitigation slightly worse (though this can be improved by a better mechanical design).
- Potentially dangerous unknown effects (like double reflection on AR) that should be studied further.

7.4.3 Non-planar design with six mirrors[11]

The cavity comprises two flat and four spherical mirrors (Fig.24). All of the four curved mirrors have the same radius of curvature of 200mm. All of the mirrors has the same incident angle of $\pi/4$. The center of four mirrors (labelled Mirror1, 2, 3, and 6, respectively) are on the $xy-$plane, while the other two (Mirror 4 and 5) are located above the mirrors 3 and 6. As shown in Fig.25, the "horizontal" and "vertical" eigenmode structures are different, yet they experience almost the same (theoretically, exactly the same) amount of Gouy phase shift per one round trip. Though the cavity is astigmatic, the waist size are the same for both directions. Also, astigmatism itself is not large. For Rayleigh range of 4cm, the difference in the waist position is 4.6mm. The "mean" mode that has the same waist size and the mean waist position would still couple to the OMC mode with more than 99.3% power throughput.

Since the round trip length of this configuration is somewhere between the "cylindrical" and "internal lens" designs, the mitigation effect of this configuration is also between these two. A factor of 24 improvement should be obtained with the finesse of 60 (Fig.26) while the

[11] This design is similar to the original non-planar design with four mirrors that turned out to have critical design error. The six mirror configuration itself was originally rejected by the author, but was added by request of Rana.

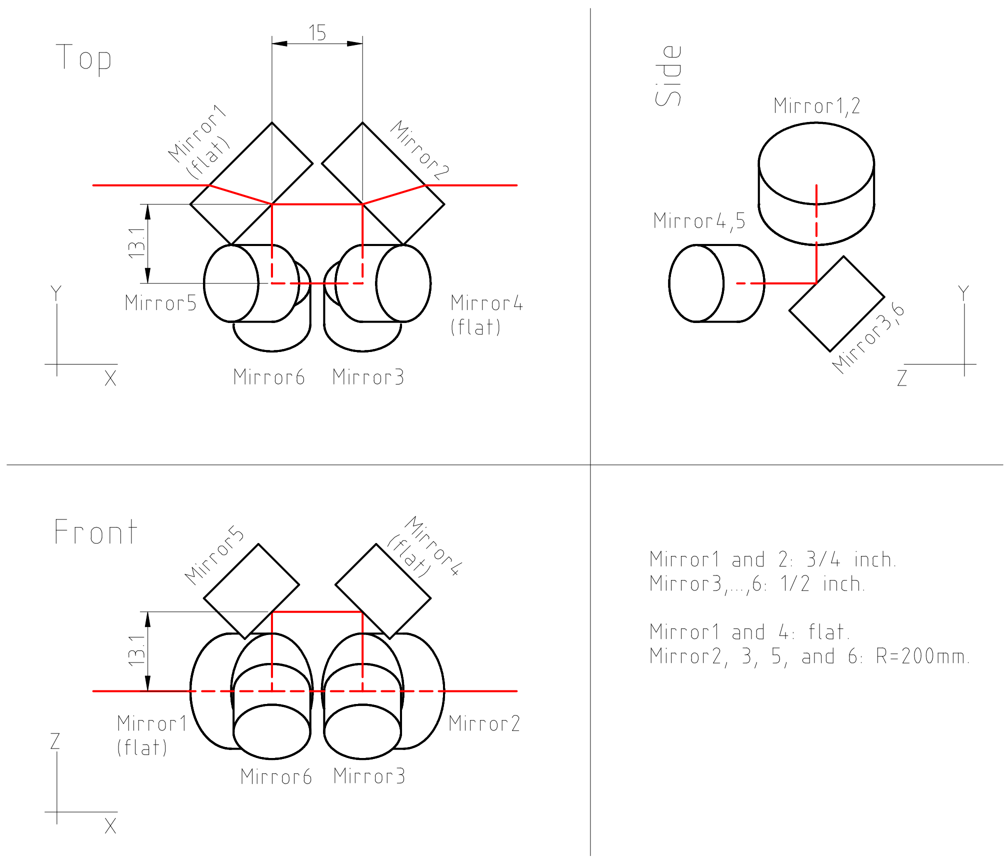

Figure 24: Non-planar OMC with six mirrors. Two of the mirrors (Mirror1 and 4) are flat and others have the radius of curvature of 200mm. The flat and curved mirrors are located in such a way to minimize the $x - y$ asymmetry.

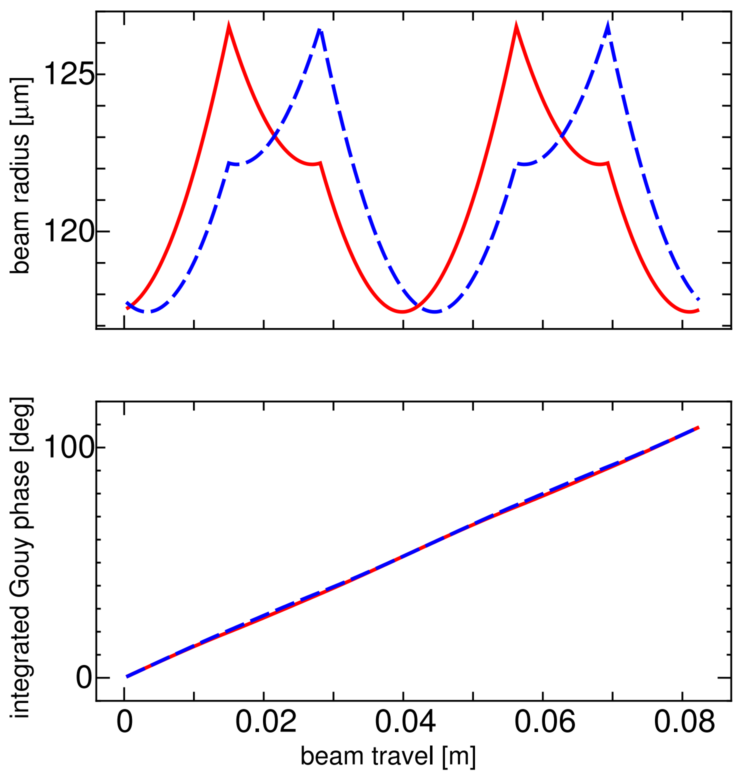

Figure 25: Beam propagation in the non-planar configuration of Fig.24. Red and blue lines represents the "horizontal" and "vertical" beam diameter and the Gouy phase shift. Though the cavity is slightly astigmatic, the Gouy phase shift per roundtrip is the same for both horizontal and vertical direction.

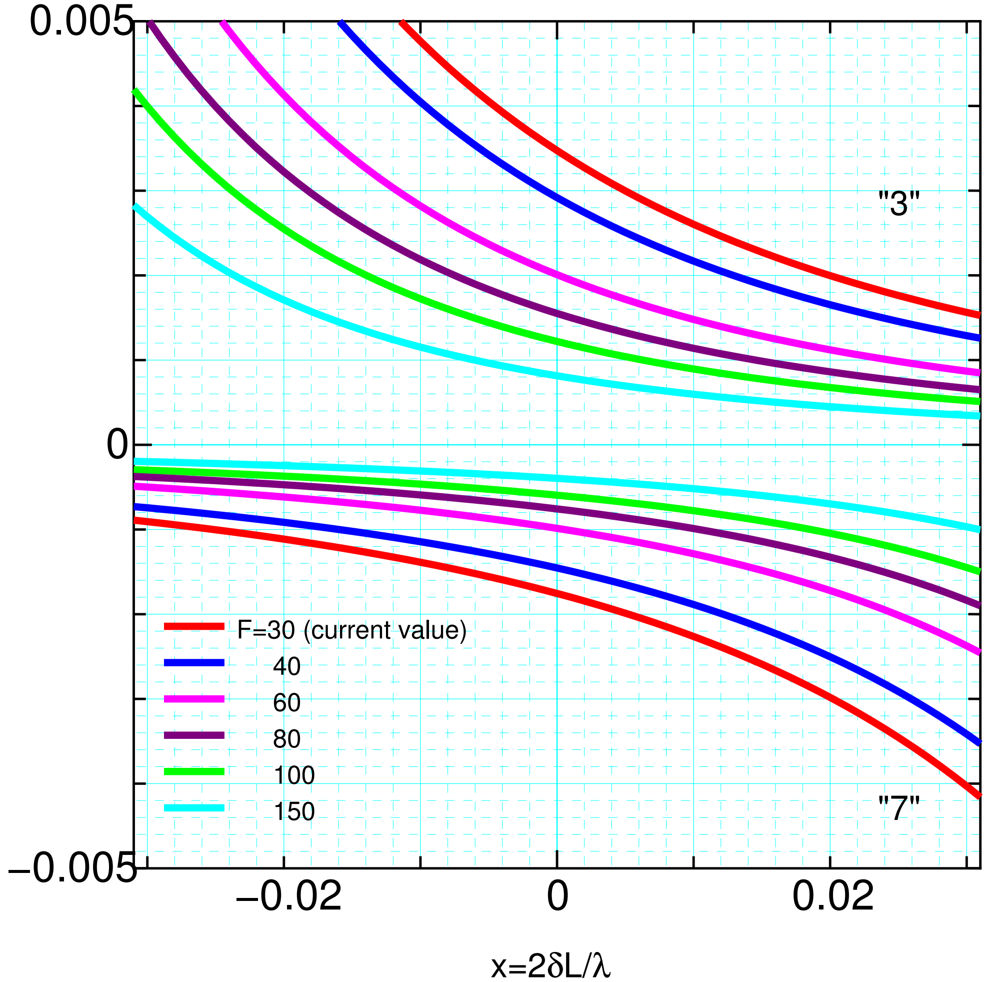

Figure 26: ASQ curve for non-planar 4-mirror geometry.

coupling via the OMC offset would become by a factor of 6.6 worse. Due to the degeneracy of the same order modes, we don't suffer from the split of the resonant position for the mode of the same order.

Characteristics of this configuration is summarized as:

- Astigmatism is reasonably small. The waist positions for horizontal and vertical direction are reasonably close (4.6mm for $z_0 \sim$ 4cm), while the waist radiuses are the same.
- Incident angles for the mirrors are $\pi/4$, thus smaller back scattering is expected.
- Due to the complex beam path, fabrication of the spacer can be difficult.

7.5 Other possibilities

Without any physical/time/budget constraint, of course, the ideal implementation is to make the OMC length equal to half of the RF sideband wavelength, i.e. 6m in our case. In this way, it is assured that the response of the cavity for the carrier and the sidebands are the same for ALL of the modal components, thus ASQ becomes insensitive to the OMC length in the first place. In this configuration, since the TEM00 RF SBs as well as the carrier are resonant at the same time, we can increase the finesse without decreasing the GW signal. Dick G is working on such a design.

Also, Nergis mentioned the use of optical fiber as the mode selector, which might be worth studying.[12]

8 Summary

A model that gives the alignment-ASQ coupling after the OMC that is on the same order as the measured value was presented.

In order to mitigate the alignment-ASQ coupling, a new design as well as the better mode matching is needed. With three-mirrored design, it seems to be very difficult to obtain anything more than a factor of 2 improvement, because the degeneracy of the same order modes ($\{U_{nm}\}$ for the same $n+m$) is broken. A higher finesse doesn't help unless we make the finesse REALLY high (like $\mathcal{F} \sim 2000$).

A 4-mirror (or 6-mirror) configuration was proposed to keep the degeneracy of the same order modes. With such a configuration, it is more straightforward to make the finesse larger to have better discrimination of the higher order modes. However, due to the astigmatism and/or the waist size difference in the OMC itself, the modes with the same order split again. To prevent this from happening, three designs were proposed: Planar with cylindrical optics, planar with flat mirrors and an internal lens, and a non-planar 6-mirror configuration.

[12] According to Bill K, there is no strong mode selectivity, as "the fiber waveguide modes are not anything like the free space paraxial modes we are interested in".

It seems to be possible to mitigate the coupling by a factor of 20 or 30 with the proposed designs with slightly higher finesse ($\mathcal{F} = 60$), or one could go for higher finesse (e.g. $\mathcal{F} = 150$ for a factor of 80 improvement) at the expense of smaller RFSB throughput. Among these three designs, my recommendation is to try the first one ("cylindrical optics"), because it was most straightforward to make a small cavity using standard optics with this design.

The offset in the OMC length from the resonant point of U_{00} carrier is also an important factor for this coupling. Different locking techniques give different offsets, and it seems that only "2-omega" scheme can theoretically lock the OMC at the "center". Nevertheless, even with the use of "2-omega" scheme, probably this will become the dominant factor of the coupling once the higher order mode coupling is sufficiently mitigated. This is more true with higher finesse (Sec.7.2.3). For example, in the first design with cylindrical mirrors, the finesse of 60 would make this coupling worse by a factor of 5.

Any inputs are welcome, by the way.

9 Basic Calculations

9.1 Single mode ASQ against the OMC length

Suppose that the AS field is written by

$$E_{\mathrm{in}} = e^{i\Omega t}\left(C_0 e^{i\phi_{\mathrm{GW}}} + im\sin\omega_{\mathrm{m}}t\right), \tag{32}$$

where c_0 is the real amplitude of the carrier, ϕ_{GW} is the phase change in the carrier corresponding to the gravitational wave signal, $m/2$ is the amplitude of the upper- and lower-sideband, and ω_{m} is the angular frequency of the modulation. The cross term of the carrier and the SBs in the intensity of this field before the OMC is

$$I^0_{\pm\omega_{\mathrm{m}}} = -2mC_0\sin\phi_{\mathrm{GW}}\sin\omega_{\mathrm{m}}t. \tag{33}$$

After passing the OMC, the field becomes

$$E_{\mathrm{OMC}} = e^{i\Omega t}\left[C_0 t_0^{\mathrm{c}}(x) e^{i\phi_{\mathrm{GW}}} + \frac{m}{2} t_0^{+}(x) e^{i\omega_{\mathrm{m}} t} - \frac{m}{2} t_0^{-}(x) e^{-i\omega_{\mathrm{m}} t}\right], \tag{34}$$

where t_0^{c}, t_0^{+}, and t_0^{-} are the transmittance of the OMC for the carrier, the upper- and the lower-sideband, respectively, and x denotes the deviation of the OMC length from the resonance of the carrier. The linear terms in the intensity proportional to $e^{\pm i\omega_{\mathrm{m}} t}$ in this case are written by

$$\begin{aligned} I_{\pm\omega_{\mathrm{m}}} &= -2mC_0\left\{\Im\left[t_0^{\mathrm{c}*}(x)\overline{t_0}(x)e^{-i\phi_{\mathrm{GW}}}\right]\sin\omega_{\mathrm{m}} t - \Re\left[t_0^{\mathrm{c}*}(x)\Delta t_0(x)e^{-i\phi_{\mathrm{GW}}}\right]\cos\omega_{\mathrm{m}} t\right\}, && (35)\\ \overline{t_0}(x) &\equiv \frac{t_0^{+}(x)+t_0^{-}(x)}{2}, && (36)\\ \Delta t_0(x) &\equiv \frac{t_0^{+}(x)-t_0^{-}(x)}{2}. && (37) \end{aligned}$$

When the OMC is locked to the resonance of the carrier, t_0^{c} and $\overline{t_0}$ become real while Δt_0 becomes imaginary, respectively, and the error signal is simplified as

$$\begin{aligned} I_{\pm\omega_{\mathrm{m}}}(x=0) &= -2mC_0\left|t_0^{\mathrm{c}}(0)t_0^{+}(0)\right|\sin\phi_{\mathrm{GW}}\sin(\omega_{\mathrm{m}} t+\phi_{\mathrm{m}}), && (38)\\ \phi_{\mathrm{m}} &\equiv \angle t_0^{+}(0), && (39) \end{aligned}$$

where ϕ_{m} is the optimal demodulation phase. When the finesse, the modulation frequency and the FSR are 30, 25MHz and 3GHz respectively, ϕ_{m} is roughly -26 degrees or -0.144π.

Usually in the AS port, the "quadrature" demodulation phase is defined as the phase to maximize the GW signal. Since the above equation gives the static optimal demodulation phase for GW signal, the Q-phase part of the AS signal against x should be rewritten as

$$\begin{aligned} ASQ(x) &= -2mC_0\left\{\Im\left[t_0^{\mathrm{c}*}(x)\overline{t_0}(x)\right]\cos\phi_{\mathrm{m}} - \Re\left[t_0^{\mathrm{c}*}(x)\Delta t_0(x)\right]\sin\phi_{\mathrm{m}}\right\}\sin(\omega_{\mathrm{m}}\, t+\phi_{\mathrm{m}})\\ &\equiv 2mC_0\sin(\omega_{\mathrm{m}}\, t+\phi_{\mathrm{m}})ASQ_0(x). && (40) \end{aligned}$$

Similarly, the ASI signal is written by

$$\begin{aligned} ASI(x) &= -2mC_0 \left\{ -\Im\left[t_0^{c*}(x)\overline{t_0}(x) \right] \sin\phi_{\rm m} - \Re\left[t_0^{c*}(x)\Delta t_0(x) \right] \cos\phi_{\rm m} \right\} \cos(\omega_{\rm m} t + \phi_{\rm m}) \\ &\equiv 2mC_0 \cos(\omega_{\rm m} t + \phi_{\rm m}) ASI_0(x). \end{aligned} \tag{41}$$

9.1.1 Small deviation approximation

When the deviation of the OMC length is much smaller than the width of the cavity (i.e. $\mathcal{F}x \ll 1$) and the modulation frequency is much smaller than the FSR (i.e. $\nu_{\rm m} \ll c/2l$ where c is the speed of light and l is the length of the OMC), these expressions are simplified considerably. The transmittances are approximated by

$$\begin{aligned} t_0^{\rm c}(x) &\sim \frac{1}{1+2i\mathcal{F}x} \\ &\sim 1 - 2i\mathcal{F}x, & (42) \\ t_0^{\pm}(x) &\sim \frac{1}{1+2i\mathcal{F}(x \pm x_{\rm m})} \\ &\sim \frac{1}{1 \pm i\nu_{\rm m}/\nu_{\rm c}} \left(1 - \frac{2i\mathcal{F}x}{1 \pm i\nu_{\rm m}/\nu_{\rm c}} \right), & (43) \\ \overline{t_0}(x) &\sim \frac{1}{1+(\nu_{\rm m}/\nu_{\rm c})^2} - \frac{1-(\nu_{\rm m}/\nu_{\rm c})^2}{\left[1+(\nu_{\rm m}/\nu_{\rm c})^2\right]^2} \times 2i\mathcal{F}x, & (44) \\ \Delta t_0(x) &\sim \frac{-i\nu_{\rm m}/\nu_{\rm c}}{1+(\nu_{\rm m}/\nu_{\rm c})^2} - \frac{4\nu_{\rm m}/\nu_{\rm c}}{\left[1+(\nu_{\rm m}/\nu_{\rm c})^2\right]^2} \times \mathcal{F}x & (45) \\ \phi_{\rm m} &\sim = -\arctan(\nu_{\rm m}/\nu_{\rm c}). & (46) \end{aligned}$$

where $\nu_{\rm m}$ and $\nu_{\rm c}$ are the frequency of modulation and the cut-off frequency of the cavity, and $x_{\rm m} \equiv 2l\omega_{\rm m}/c$ is the modulation frequency normalized by the free spectral range. Since the only assumptions we made to derive these approximations are $\mathcal{F}x \ll 1$ and $x_m \ll 1$, the above expressions are valid even when $2\mathcal{F}x_{\rm m} = \nu_{\rm m}/\nu_{\rm c}$ is on the order of or larger than 1.

Using these simplified expressions, the ASQ and ASI signals are now represented by

$$ASQ_0(x) \sim -4\mathcal{F}x\left\{\frac{(\nu_{\rm m}/\nu_{\rm c})^2}{[1+(\nu_{\rm m}/\nu_{\rm c})^2]^2}\cos\phi_{\rm m} - \frac{(\nu_{\rm m}/\nu_{\rm c})\,[-1+(\nu_{\rm m}/\nu_{\rm c})^2]}{[1+(\nu_{\rm m}/\nu_{\rm c})^2]^2}\sin\phi_{\rm m}\right\}, \quad (47)$$

$$ASI_0(x) \sim 4\mathcal{F}x\left\{\frac{(\nu_{\rm m}/\nu_{\rm c})^2}{[1+(\nu_{\rm m}/\nu_{\rm c})^2]^2}\sin\phi_{\rm m} + \frac{(\nu_{\rm m}/\nu_{\rm c})\,[-1+(\nu_{\rm m}/\nu_{\rm c})^2]}{[1+(\nu_{\rm m}/\nu_{\rm c})^2]^2}\cos\phi_{\rm m}\right\}. \quad (48)$$

9.2 Multimode ASQ

Suppose that, even though there are several modes, the RF SBs have the same modal structure as the carrier. Ignoring the GW phase, Eq.32 is now rewritten by

$$E_{\rm in} = e^{i\Omega t}\left(C_0 + im\sin\omega_{\rm m}t\right)\sum a_{nm}U_{nm} \quad (49)$$

where a_{nm} is the amplitude of nm-mode. Using Eqs.49 and 40, the ASQ error signal under the presence of multiple mode is written by

$$ASQ(x) = 2mC_0\sin(\omega_{\rm m}t+\phi_{\rm m})\sum|a_{nm}|^2\,ASQ_0(x+x_{nm}), \quad (50)$$

where x_{nm} is the resonant position of U_{nm}. In linear and four-mirrored cavities, the modes with the same order (i.e. the same $n+m$ value) share the same resonant position, i.e. $x_{nm}=(n+m)x_{\rm t}$ where $x_{\rm t}$ is the so called transverse mode spacing. In triangular cavities, the same order modes with different horizontal parity have different resonant position, i.e. $x_{nm}=(n+m)x_{\rm t}$ for $n=2l$ and $x_{nm}=(n+m)x_{\rm t}+0.5$ for $n=2l+1$.

10 References

1. Keita, Joe and Luca, Keita's elog on Jun 15 2004.

2. Peter Fritschel, "Output Mode Cleaner Design", LIGO-T040018-00-D.

3. Keita, Joe and Stefan, Keita's elog on May 26 2004.

4. Stefan and Nergis, Stefan's elog on May 19 2004.

5. Luca's elog on May 21 2004.

6. Peter F. and Stefan, Peter F's elog on May 10 2004.

7. Peter F., Daniel and Stefan, Stefan's elog on May 8 2004.

8. Biplab, Matt, Malik and Hiro, "Time Domain Modal Model in End-to-End simulation package", LIGO-T990811-03.

9. Among others, see Stefan's elog on May 18 and Luca's on Jun 12 2004.

10. Daniel's elog entry on May 8 2004 [2004:05:08:22:14:29-daniel].

11. Peter F., Daniel and Stefan, Stefan's elog on May 8 2004 [2004:05:08:03:42:54-stefan]

11 History

LIGO-T040158-00-D (20040811/20040816):

- Obtained a DCC number.
- Corrected typos and such.

20040802:

- Reference to Peter F's original design documentation [2].
- Included Luca and JoeB in the author list.

20040729:

- Recommendation was made to go for "cylindrical" design. This is solely based on some practical constraints (like the availability of the right optics, ease of design etc.).

- Serious efforts were made for “Cylindrical” and “internal lens” designs to give shorter length and larger FSR.
- Instead of the previous “non-planar” design, a new non-planar design with 6 mirrors was added (request from Rana).
- Detailed parameters for each of the designs.
- In the previous version, the term “astigmatism” was used for any kind of asymmetry for horizontal and vertical modal structure, even if the waist position is the same. This seemed to cause some confusion among some readers. In this version, “astigmatism” means something with the waist position difference.

20040715 (not distributed):

- It turned out that the non-planar design has a critical flaw, so it is temporarily deleted.
- Added a short note about the impact of higher finesse on the coupling via the OMC length offset (Sec.7.2.3).
- Added a reference to Daniel’s elog on different locking techniques.
- Corrected all of “planer” to “planar” (http://www.sawdustmaking.com/Planer/planer.htm).

20040714:

- Added two alternative designs, i.e. planar 4-mirror cavity with cylindrical mirrors, and planar cavity with four plane mirrors and a lens inside.
- Added the comment of Bill Kells about the fiber as the mode selector.

20040712:

- Added two figures for the discussion for astigmatism of the OMC.
- Added Bill K’s suggestion.

20040708:

- Finally announced the availability and the URI.