LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

LIGO- T040110-01-K *Advanced LIGO UK* May 2004

Input to the OSEM selection review decision

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This is an internal working note
of the Advanced LIGO Project, prepared by members of the UK team.

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1 Introduction and scope

1.1 Purpose

This document is intended to supplement LIGO E040108-01 by providing answers to 5 specific issues raised when that document was reviewed. This version (01) includes a statement on OSEM range and minor rewording.

1.2 Scope

The document discusses the displacement sensors of the OSEMs to be fitted to the major suspensions within Advanced LIGO (i.e. mode cleaner, recycling mirror, beam splitter, folding mirror, and test mass suspensions). It also addresses some aspects of the use of eddy-current damping. Numerical analysis is restricted to the test mass (TM) quad suspensions, although some conclusions are extrapolated from this for the other core-optics suspensions.

In summary the document assesses whether basic “Initial LIGO” or “Hybrid” OSEM sensors (which are equivalent) can be used together with enhanced controllers and eddy-current damping to provide all required damping of the Advanced LIGO suspensions.

1.3 Terminology

1.4 Acronyms

ISC	interferometer sensing and control
LSC	length sensing and control
OSEM	optical sensor electromagnetic actuator
ECD	eddy current damping

2 References

The document depends on

LIGO-T000051-01-D

G010086

E960050-A-E

G030339 (ALUKGLA0020).

E960020-B-E

T040001

T040045-00-K

T040106-01-K

3 Formulating figures of merit

3.1 Summary

The requirements in T000051-01 need to be clarified/interpreted to allow their safe application in the present decision process. This was done in consultation with Peter Fritschel and others, and the final outcome is described here. It would be reasonable to update the DRD at some stage to reflect this way of looking at the requirements.

The requirements that determine the required degree of damping are restated in the form of 3 states (or aspects, as one operational state could meet two or even all three requirements) of operation of the detector system. The terminology used is meant to be descriptive not definitive.

- **Emergency/Installation/Pre-Alignment**. If there is an earthquake, major adjustment, or inappropriate human action we want enough damping to bring the suspension quickly to rest. There are no considerations of noise in this case. Local control should switch to this mode whenever another mode becomes inappropriate due to some disturbance. It should be the startup mode. This may be the same state as for acquisition, but that should not be taken as a requirement.
- **Acquisition.** The key requirement is that the fringes are sufficiently slow to allow the ISC controllers to act before the fringe has passed. Unfortunately the present decision will be made long before the controllers are designed and so the only reasonable approach is to state that the suspension local damping must minimize the "average" speed of the mirrors and that the result must be very close to the minimum possible given the input vibration from the SEI subsystem. (See below for a more quantitative approach to this issue.)
- **Detection/Science mode.** The in-band noise requirements as set out in the DRD must be met in this mode. Additionally it is necessary to restrict the required control-band feedback forces to a reasonable minimum. The latter consideration is related to the velocity requirement above but with two differences: 1) very low frequency motion is unimportant because feedback can be applied in the SEI stage rather than the suspension and 2) force (hence acceleration) is a truer measure of the problem than velocity. In principle the analysis could become very complex, taking into account every detail of the multi-output control system, but in practice the important forces in the significant part of the control band frequency range will be applied via the actuators on the penultimate mass.

Separating out these three functions makes it easier to justify the use of eddy current damping (ECD) techniques to provide the damping required for acquisition mode (and hence also detection mode as it is assumed that the level of ECD is not variable). If the amount of damping employed results in a settling time too long for "emergency" mode, active damping would be added to supplement the ECD. This gives maximum flexibility as a fall-back in case the more complicated solution presented below proves problematic.

3.2 Range

The hybrid OSEMs were measured to have a range of 0.7 mm peak to peak (see T040106-01-K). The interferometric sensor has a range exceeding 3 mm peak to peak. The question arises as to whether this difference is significant for the selection.

3.2.1 Required range

The OSEM range must exceed the sum of three components. Firstly there will be some inaccuracy in the location of the OSEM due to resolution of the adjustment process and, perhaps, limited possibilities to zero each OSEM. It is expected that these errors will be kept to less than 0.2 mm worst case. The second component arises due to errors in setting up the suspension to have the correct pitch (and possibly yaw) when installed in the interferometer. The bias-coils will be used to correct for this and the OSEM range must allow for such adjustments. The final component of the range is that required to cope with any drifts in the suspension (e.g. temperature dependence of the relative angles of mirror and top masses). This last component is expected to be quite small in comparison to the others, especially given the good temperature regulation of the suspension's environment.

Informal discussions have led to the adoption of 1 mrad pk-pk as the requirement for angular correction available using the bias coils, with the understanding that the surveying errors and suspension set-up errors must be controlled to be, individually, small compared to this.

The angle range (1 mrad) needs to be translated into a worst case OSEM displacement range according to the greatest distance from axis of rotation to an OSEM. The final layout of OSEMs has not yet been established for all suspensions, but it seems likely that we can avoid mounting an OSEM more than 150 mm from the axis (the worst case in present conceptual designs). This component of the range is, therefore, no more than 0.15 mm.

In summary the range requirement is estimated to be 0.35 mm.

4 Modeling

4.1 Important suspensions and degrees of freedom

Only those particular suspensions and degrees of freedom that could possibly require better sensors than the ones under consideration are analyzed here. See E040108-01 for the background.

The test mass suspensions have critical requirements in both acquisition mode (longitudinal motion) and detection mode (vertical motion which cross-couples to longitudinal) and these two situations are analyzed. The BS/FM suspensions are only a little different in important mode frequencies (in spite of being triple pendulums, the overall length and vertical spring frequencies are not very different), but the requirements for these suspensions are much relaxed compared to the TMs, so no special consideration is given them. The MC suspensions must exhibit good detection band noise performance.

4.2 A strategy to minimize risk

The recommendation presented in E040108-01 was intended to minimize risk (at least from a particular point of view), and care is required to keep the risk increase to a minimum in any replacement strategy. The two key features that are lost if the interferometric sensor is not used are a) considerable margin in noise around 10 Hz and b) control band noise less than the SEI platform noise at each Fourier frequency. However, it should be possible to meet the requirements using other approaches based on a more sophisticated solution to the overall problem (but simpler sensors). The rewards would be removal of almost all risk directly associated with production of the sensors, and probably also reduced cost of manufacture.

Several strategies have been considered and 3 are noted here. One of these, expected to carry least risk, is carried forward for full analysis, but the others are by no means excluded for future consideration.

4.2.1 Low noise sensor

This is the approach originally outlined, and puts nearly all of the risk into sensor development. Unfortunately the only acceptable sensor solution is the complex interferometer, which requires considerable development before it is ready for use. It is, however, thought that this approach is valid.

4.2.2 Eddy current damping of key degrees of freedom

An alternative approach focuses on achieving the detection mode, detection band noise performance. This places eddy current damping on all degrees of freedom that require lower noise damping than the standard sensors. Other degrees of freedom are damped actively. Eddy current dampers could be supplemented by active damping whenever necessary (e.g. in an emergency), as the required OSEMs would be fitted. The main difficulties with this approach are a) obtaining sufficient ECD to damp the low frequency modes of the suspension and b) possibly some excess control band noise in the actively damped degrees of freedom. (Given the success of the option presented in 4.2.3 there was no detailed analysis of this problem.)

4.2.3 Hybrid damping of important degrees of freedom

This third option employs a mixture of active and eddy-current damping together on many degrees of freedom on the suspensions.

ECD is extremely successful at providing sufficient damping of the higher-frequency suspension modes (those above ~1 Hz), neither shorting out isolation nor adding too much thermal noise. The amount of ECD needed to optimally damp the lower frequency modes (1 Hz and below) is problematic. The dampers become large and heavy, and the risk of reducing isolation is increased along with thermal noise (although not necessarily to a prohibitive extent, hence the option presented in 4.2.2 could prove perfectly adequate given the right balance of parameters). Active damping is excellent at damping the low frequency modes, but noisy sensors carry the risk of adding noise at 10 Hz unless sufficiently sophisticated low pass filtering can be installed. It is obvious that the design of such low pass filters can be substantially eased if the highest unity gain point of the damping controller can be lowered by a significant factor. Damping the modes around 2~4 Hz with ECD allows the highest UGP to be reduced from around 5 Hz to around 1~2 Hz. This removes concern a) in 4.2.2, at the potential expense of adding ECD in more locations (but note each damper would be smaller). The concern over control noise remains to be addressed.

4.3 Analysis plan

A key point of either of the strategies based on simple sensors is the need to handle the noise they produce in the control band. This was a main focus of measurement (to establish the noise) and analysis (to understand its effect). The results of a new measurement by Nick Lockerbie are given in T040106-00-K. These confirm earlier results from MIT.

The hybrid approach is more flexible than the alternative pure-ECD based approach, and can be regarded as a (slight) generalization of the other approach. It is taken as the focus of the present analysis (the active part can be turned down and the ECD part turned up until the approach merges into the pure ECD one, although optimization might require additional ECD).

Pending the measurement of typical sensor noise, an example model was used to allow initial simulation. The example combined white noise at $10^{-10}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$ with an additional $10^{-9}/f\ \mathrm{m}/\sqrt{\mathrm{Hz}}$ (where *f* is the frequency – steeper than 1/*f* power noise). This model turned out to be just a little worse than the measurements at all frequencies. It did not seem worthwhile to do the calculations again with the measured data (there is a little margin in the results).

4.4 Reanalysis with measured noise data

Not required, as the initial results were confirmed.

5 Outcomes

Emergency mode requires no special consideration as actuator strength and the design of the suspension determine the rate at which energy can be extracted. The other modes of operation have been considered in some detail. Summary results are presented immediately below, while for the interested, a collection of thoughts comments and results can be found in the Appendix (6.1.3 is the most important section).

5.1 Acquisition and detection modes

5.1.1 TM suspensions – vertical motion

A hybrid ECD/active solution has been found which works with the example sensor to provide 10s settling time at nominal gain (there is gain margin) and a lower-noise operating mode suitable for acquisition and detection. In the low noise mode the *rms* velocity and acceleration (in the important part of the control band) are dominantly produced by SEI motion not sensor noise. There should be little difficulty implementing the controller in digital form. Note that the controller must be tuned, to march the individual suspension, to about 5% accuracy in several parameters (to be checked). The solution found for vertical damping is not optimum but is probably good enough to require only minor optimization to allow its use in trials.

5.1.2 TM suspensions – longitudinal motion

A hybrid ECD/active solution has been found which works with the example sensor to provide 20s settling time at nominal gain (with gain margin) and a lower-noise operating mode suitable for acquisition and detection (sensor noise can be eliminated by turning the active part off in detection mode). In the low noise mode the *rms* velocity and acceleration (in the important part of the control band) are dominantly produced by SEI motion not sensor noise. There should be little difficulty implementing the controller in digital form. Note that the controller must be tuned, to march each individual suspension. Slightly better tuning is required in longitudinal than in vertical, as there is a more complex mode structure (4 modes, closely spaced). The solution found is not all that good. The transfer function around both 0.5 and 1 Hz modes is not nearly optimum. The damping does not, therefore, meet the requirements for settling time. Solving this is thought to be just a matter of finding a robust method of optimizing the servo response, and not a fundamental issue.

5.1.3 Other suspensions

No new modeling of other suspensions was carried out in the preparation of this document. It is expected that the hybrid approach would work very well in all suspensions (given the performance seen when modeling the TM suspensions). Some reasoning is given here.

5.1.3.1 BS and FM suspensions

The requirements are, mostly, relaxed (compared to the TM suspensions) and the isolation is reduced, but by a smaller factor. There should be no difficulty meeting damping time or detection band noise performance targets (look at the results for the TMs to judge this). Acquisition velocity requirements are much less stringent for these suspensions, but the performance obtained from hybrid damping should be similar to that shown for the TM suspensions (actually the performance is likely to be slightly better for the BS and FM suspensions in their latest form, since the relative lengths of the 3 stages are better suited to damping at the top stage).

5.1.3.2 MC suspensions

Hybrid damping should work, but another option (avoiding the need to fit ECD into the already relatively mature design) would be relaxation of the requirements on detection band noise, and/or more aggressive filtering of the sensor noise. A final recommendation is beyond the scope of the present document as in neither case are low noise sensors required.

6 Appendix: some detail of the modeling work carried out

This section presents rough notes on the choices and changes made during modeling of the TM suspensions and other calculations. A log of the work carried out is given in 6.1. The details of the model, analysis code and pendulum versions are given in 6.2 to 6.4. It is all very much “work in progress” and it might take some time to fish out the useful content, but I have tried to document as much as possible of the thought processes in case it is not revisited for some time. The key results are found in 6.1.3.

6.1 Suspension modeling

Norna Robertson provided the TM Quad suspension model packaged as a zip archive. This is available from Norna.

I made the following changes to simplify operation: remove step generator, scopes, transverse-roll model and yaw model from the SIMULINK diagram (done to speed the start of analysis – MATLAB was crashing very frequently for some reason). I replaced all of the existing inputs with 4, for longitudinal and vertical suspension point, and sensor noise inputs, and created only two outputs – those for the lowest stage of each part of the model. (See section 6.2.)

The model can be used to represent either active or passive (eddy-current) control, and both modes were investigated. The passive damping was changed in amount, as noted in the results sections. The active damping algorithm was changed significantly, as noted in various places.

A MATLAB script (see section 6.3) was written to allow the following actions on a model exported from LTIviewer (i.e. a specific instance of ECD and active gain):

- Select appropriate input
- Select appropriate output

- Select frequency weighting function applied (e.g. velocity for acquisition)
- Select SEI or sensor noise model (automatically)
- Calculate correct suspension TF and weight with appropriate function (velocity for acquisition, acceleration for detection) and low frequency cutoff (as input)
- Plot resultant longitudinal mirror motion (vertical has cross coupling factor), as both spectrum and integrated mirror motion (from high frequency down to each point plotted, neglecting any correlations).
- Example outputs are given below, showing near final results.

6.1.1 Observations TM quad modeling – active damping (original method)

- Vertical contribution to longitudinal velocity and acceleration is negligible, so for vertical acting sensors we care only about the noise at 10 Hz.
- Longitudinal acting sensors can be turned down in detection mode, so we do not care much about the noise at 10 Hz.
- TM ***velocity*** has 2 dominant contributions namely
- around 0.45 Hz $\geq 10^{-8}\,\mathrm{m/s}$ from SEI platform motion (~ equal with default gains, or larger depending on Q), sensor noise would have to be really terrible to make this worse (in an integrated sense)
- around 1.00 Hz >~$10^{-10}\,\mathrm{m/s}$ from SEI platform motion, to which sensor noise can easily add a significant amount, for example the standard displacement sensor adds $3\times10^{-9}\,\mathrm{m/s}$ in a band near 1 Hz.
- The factor of 2.2 between these frequencies small enough that neither any proposed controller, nor any reasonable weighting function, will distinguish between them in any significant way (remember that there is already velocity weighting applied).
- The 3rd mode, at ~2 Hz is excited much less by platform or sensor (e.g. $3\times10^{-11}\,\mathrm{m/s}$ velocity from the test sensor model).
- Again the small frequency interval and large amplitude change render the higher frequency contribution negligible.
- In detection mode, with an acceleration weighting applied to the perturbations, the standard sensor adds a force contribution (mainly at 1 Hz), which is slightly smaller than the contribution (mainly at 0.45 Hz) from the SEI, but this is with full gain on the control. Reducing the gain from nominal by a factor of 100 would render the control band contribution completely unimportant, and a factor of ~1000 reduction meets the noise requirement at 10 Hz. (Note that thus far these results are with the standard controller.)

6.1.2 Observations TM quad modeling – passive damping (ECD)

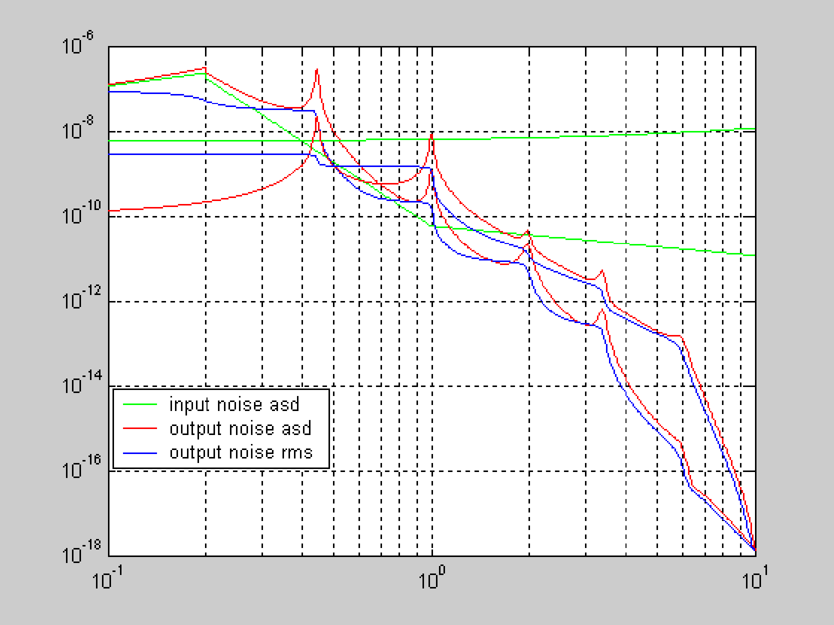
The same basic observations apply and only areas where there could be differences are noted here

- Eddy current damping of vertical degrees of freedom (at the nominal level in the distributed model) gives similar results to active damping, and the small cross-coupling to longitudinal renders the detail unimportant for the present decision.
- Eddy current damping could be used to provide some level of longitudinal damping, but offers little advantage, except as a type of fail-safe (power failure, accidental disconnection etc.)
- Since the baseline plan is to fit some kind of sensor and actuator (OSEM) for each degree of freedom in any case, it could very possibly cost less combine reduced (from nominal) eddy current damping with add active damping of the lowest mode (around 0.5 Hz), provided sensor noise allows this to work. This would allow the sensor noise to be more aggressively rolled off (completely removing its high frequency components from consideration as a problem in detection mode).
- See next section.

6.1.3 New model for hybrid damping

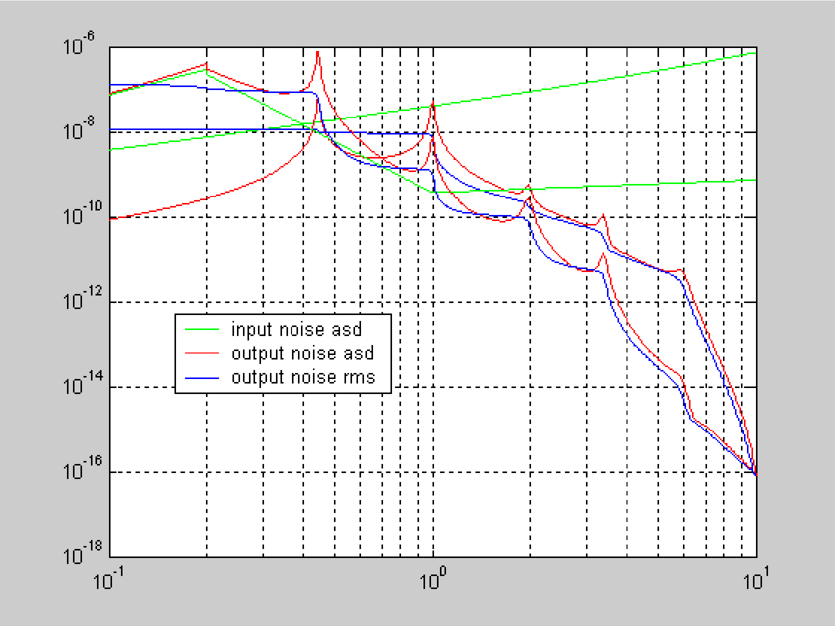
- Simulink model (pende.mdl -> pende2.mdl) was rewritten to directly use “b” values to represent ECD, *absolutely no* changes to “ssmake????” or “pend”.
- Controllers rewritten to work in hybrid form along with ECD (iterative development, detail not given here as method intuitive rather than algorithmic)
- New longitudinal and vertical controller for TM quad, each tuned to specific suspension parameters.
- N.B. changes in suspension design will require re-optimization of the controllers; they are no longer generic.
- Script (generate_simulink.m) rewritten to ease the current project (absorb localdamp.m into generate_simulink and write more versions of local4*.m to deal with different cases.
- First useful controllers are stable with reasonable gain margin (in the sense that the lowest mode approaches the lowest achievable Q at less than half the maximum stable gain). (Later improvements to reduce noise at 10 Hz reduce the stability somewhat – there is a trade-off here.)
- **First analyze vertical.**
- With b = 27 kg/s on ECD (one “standard block” of 16 magnets) and active gain 40 (arbitrary factor) the settling time in vertical is 10s and the feed-through of sensor noise does not exceed 5×10^{-10} above 10.0 Hz (includes 0.001 cross-coupling).
- In this state the longitudinal control band *rms* velocity contributions are 1e-11 m/s from the sensor and 2e-12 m/s from the SEI platform.
- If the active gain is reduced by a factor of 5, to tolerate a $10^{-10}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$ sensor, the settling time becomes 25s, the sensor and SEI *rms* velocity contributions are now both $3\times10^{-12}\,\mathrm{m/s}$. (Note the integral over long times is $1\times10^{-10}\,\mathrm{m/s}$ due to the SEI.)
- **Now for horizontal**.

- With b = 27 kg/s on ECD and active gain 20 the settling time is 15s - the controller should be able to be optimized to improve this (later it got worse after some other aspects were improved). Note that there is no simple relationship between the gain numbers in vertical and longitudinal, they should not be compared. The feed-through of sensor noise does not exceed 5.5×10^{-10} at this gain.
- In this state the control band *rms* velocity contributions above 0.4Hz are $\sim10^{-8}$ m/s from the sensor and 10^{-8} m/s from the SEI platform. With the integral down to low frequency being 10^{-7} m/s from the platform.
- The if the active gain is reduced by a factor of 5.5, to tolerate a $10^{-10}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$ sensor, the settling time becomes 30s, the sensor and SEI *rms* velocity contributions are now 3×10^{-9} m/s and 3×10^{-8} m/s, respectively. (Note the integral over long times is 10^{-7} m/s due to the SEI.)
- The next figure shows how the *rms* velocities arise in the gain-reduced case. (velocity - m/s - against frequency - Hz) One set of curves represent sensor noise and the other (higher at 0.1 Hz represent SEI motion)



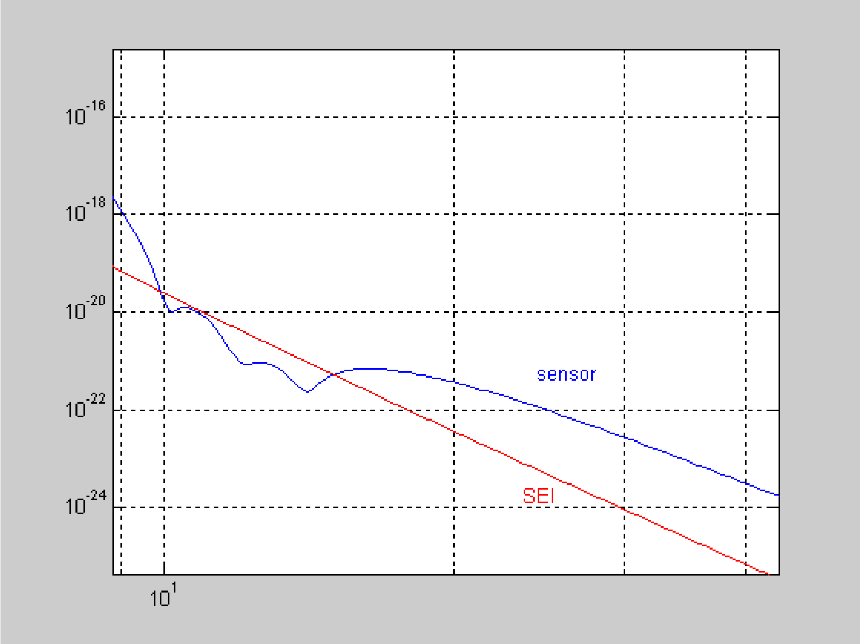
Green curves show the input noise (SEI falls steeper) red curves show mirror velocity amplitude spectral density, and blue curves show that integrated from the right to the left.

- Note that although the sensor curves exceed the SEI curves at >0.5 Hz, the motion in the sub 1 Hz band is dominated by the SEI contribution.
- So how does this look when weighted for acceleration (next figure)?

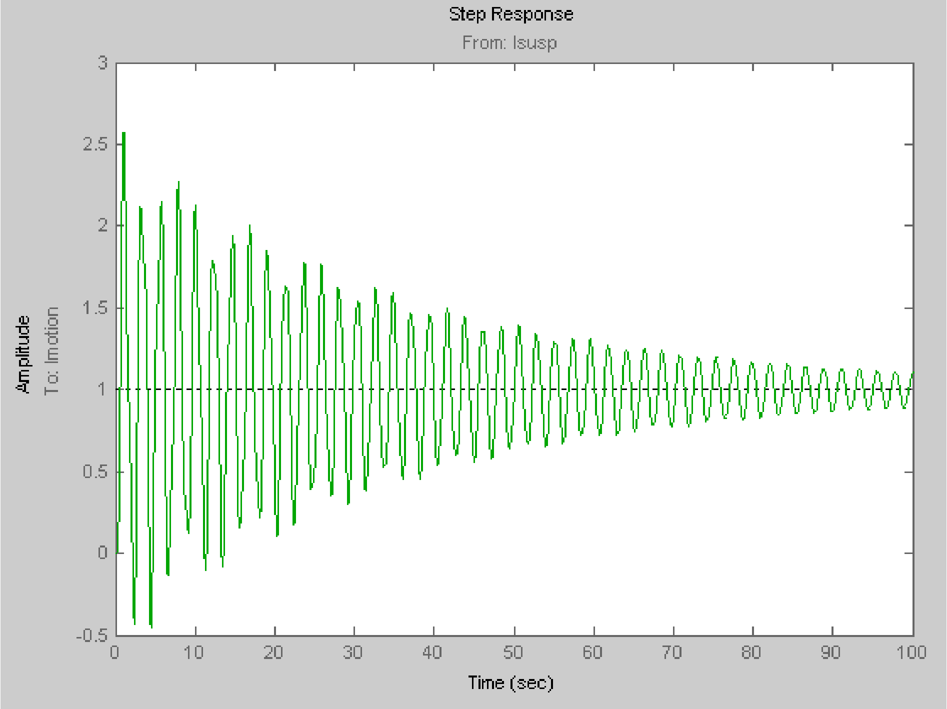


- Really not very different. There would have to be a very strong selection of 1 Hz noise over 0.5 Hz noise to make the sensor noise significant. The total acceleration to be dealt with by the penultimate mass actuators is less than 10^{-7} ms^{-2} equivalent to a force of around 10^{-5} N, which is within the capabilities of the actuators (according to G010086 there is one order of magnitude to spare). It should not be necessary to develop some very complicated hierarchical feedback system to redistribute this force (very long term drifts should, of course, be passed back to the SEI to deal with).

- Adding an extra filter stage to the longitudinal controller and plotting the displacement noise around 10 Hz gives the following result

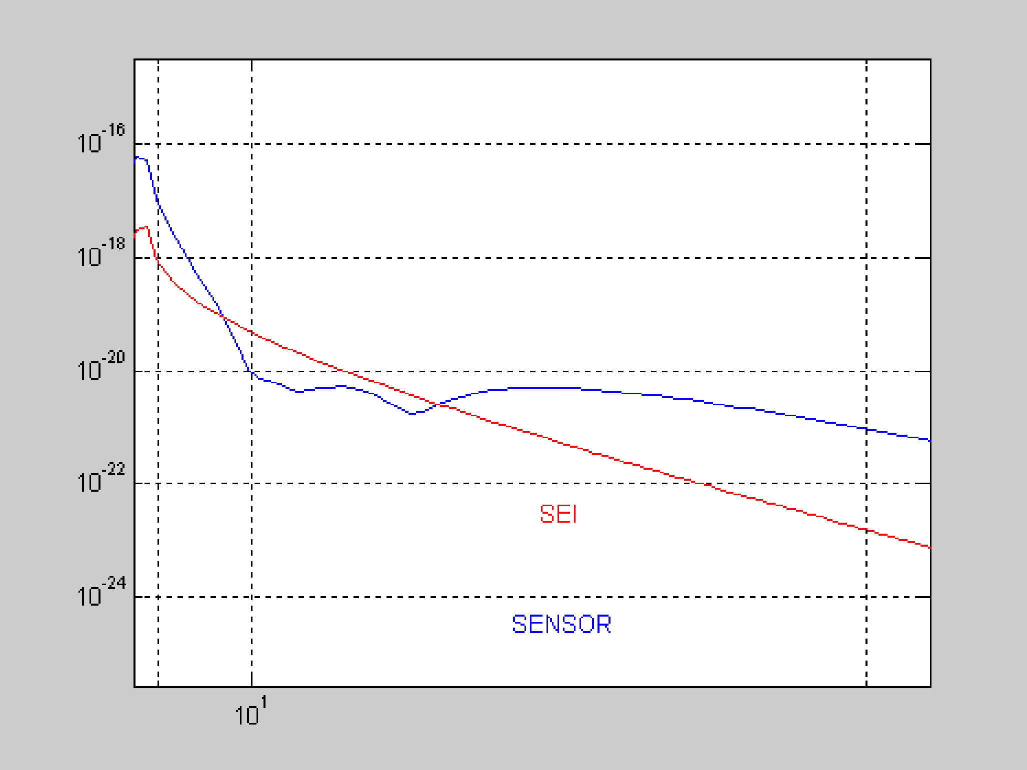


- The accompanying settling behavior looks like the following. Although it does not meet the 10s settling time requirement, it still provides reasonably strong damping together with low-noise operation. Switching the filter and increasing the gain brings better damping at the expense of more noise (although it was not possible to meet the 10s requirement with this controller due to non-optimum phase at 0.45 and 1 Hz).
-

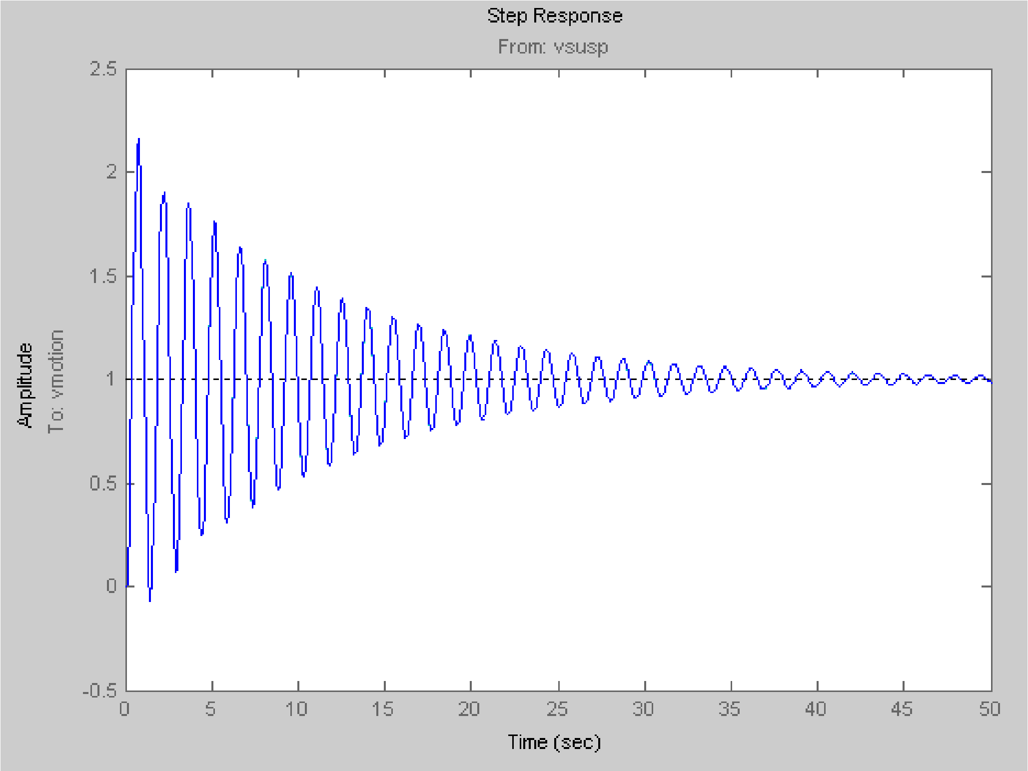


- This is encouraging, as the example noisy sensor can be used in detection mode without turning the gain down. Extra filtering would allow the sensor curve to be strictly below the seismic curve above 10 Hz. This is probably not the optimum approach, but it is interesting to see that it can be done.

- All we need now are the true sensor noise measurements (pause to write up where we've got to so far).
- **Back to vertical**
- Doing the same trick with an extra filter stage allows vertical damping close to full strength (gain adjustment to 20, settling time about 12s) while giving the following noise feed-through. Again this is far from optimized but shows promise (no switching of local damping according to detector mode for vertical – in this case the technique might actually be useful!)
-



Right hand edge of plot is just over 20 Hz. Here is the damping behavior as an example



6.1.4 Some notes

The vertical and longitudinal controllers are not yet optimized for damping; it would probably be possible to obtain the same damping for about half the gain in a better-designed controller. An automated generating method would be helpful (if we can define the cost function clearly enough).

The low-pass filter design follows an analog method tried in GEO (for several purposes) it should be easy to implement as IIR 2 poles/stage filters, not requiring much CPU time. An approach using FIR filters (or equivalent) was not considered (but should give very good performance with sufficient taps, probably requiring much more CPU time, but I'm not very experienced at optimizing FIR designs for a real task).

6.1.5 Results from reanalysis using revised noise model

Not required.

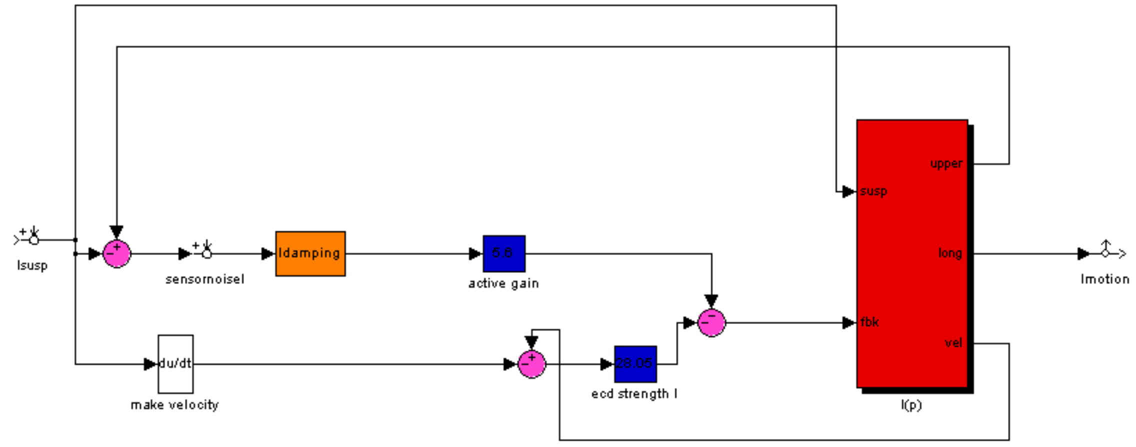
6.2 Documenting the model

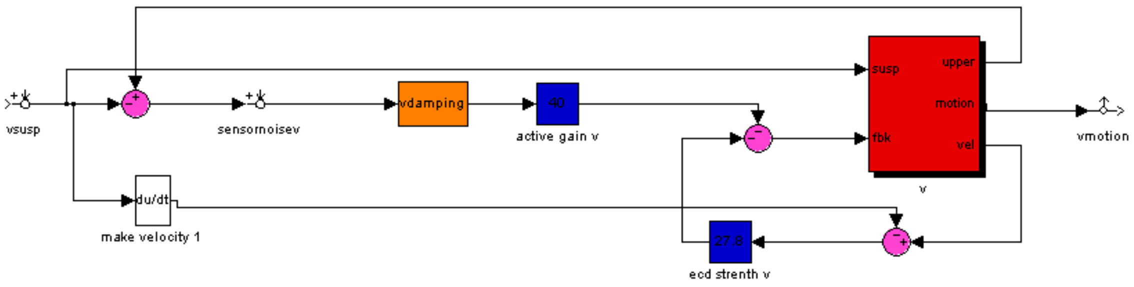
There is a new "generate_simulink" script

```
%script to allow investigation of advanced local controls

close all

clear all

global pend

pend.title = 'Pendulum parameters and derived properties';

%**********************************
```

```
ssmake4pv2eMB;           %updated  files  with  errors  corrected  (found  from
%MATHEMATICA program, Mark Barton)
%*********************************
ldamping = local4l(1);
sdamping =local4s(1);
vdamping = local4v(1);
pend      % print pendulum parameters
open pende2.mdl
```

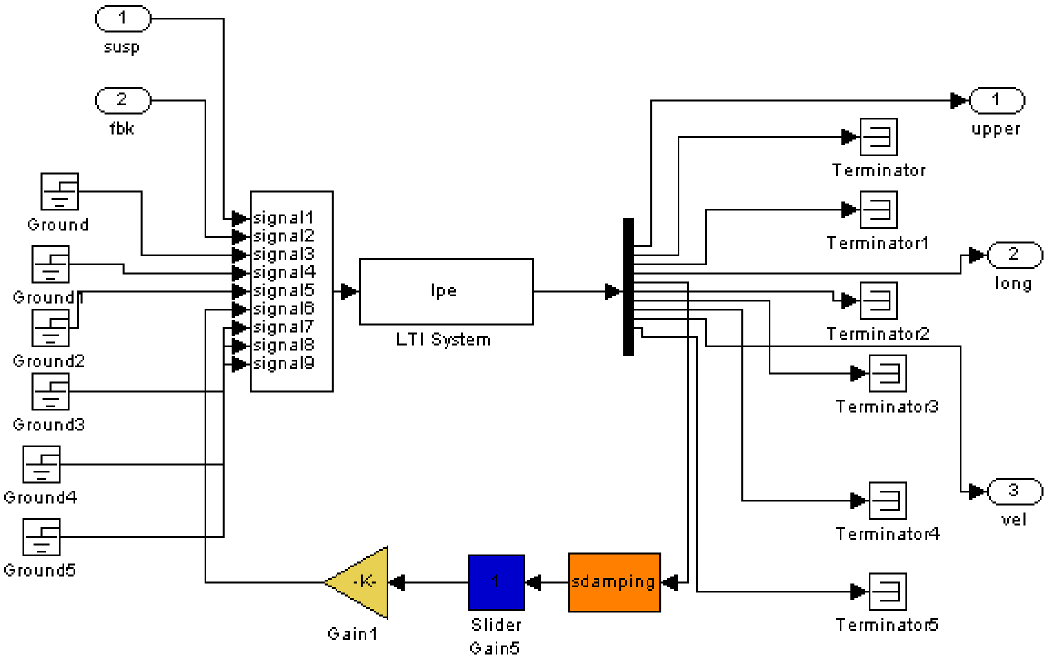
This refers to pende2.mdl (see below) and also the local4l and local4v scripts (below). No changes were made to ssmake4pv2eMB.m or quadopt.m (called therein).





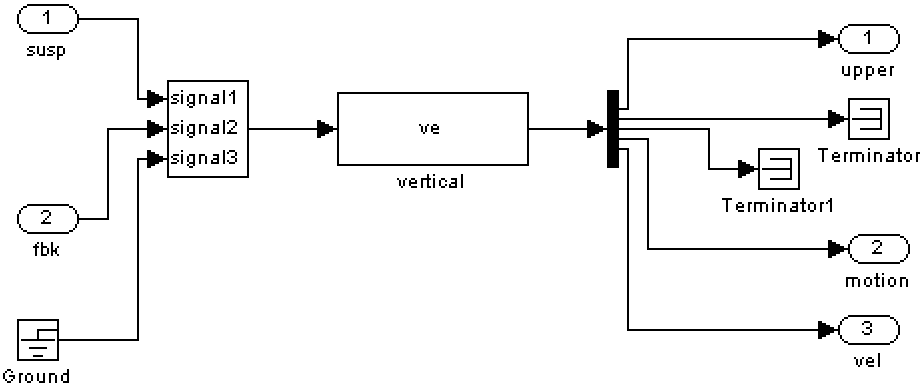
The elements of the model are defined below; anything not stated is unchanged from the standard suspension model. The blue gain boxes are multiplicative factors.

- The l(p) box contains the following



The orange box "sdamping" is the standard local4s.m damping used in earlier models, just to keep pitch modes from complicating the analysis (I don't think this is cheating).

- The v box contains the following:



- The ldamping box is a state space system defined by

```
function [damper] = local(gain)

%local to be combined with ECD kas 04/04
gain = gain * 10;  %was intended to normalize but not actually done yet
[ad1,bd1,cd1,dd1] = transdif(0.2,6,1);  %phase lead

[ad2,bd2,cd2,dd2] = transdif(1,3,1);    %more phase lead 1 to 2 Hz

[alp,blp,clp,dlp] = sculte(4.5,1,10.1,40,1); %low pass first part
```

```
[alp2,blp2,clp2,dlp2] = sculte(6,10,12,20,1); %low pass second part
[alp3,blp3,clp3,dlp3] = sculte(7.5,15,14,20,1); %optional low pass third part
[an1,bn1,cn1,dn1] = notch(2,10,1); %avoid affecting third mode
[an2,bn2,cn2,dn2] = notch(3.5,10,1); %avoid affecting fourth mode
%start with just gain and then add on other stages
al = 0; bl = 0; cl = 0; dl = gain;
[al,bl,cl,dl] = series(al,bl,cl,dl,ad1,bd1,cd1,dd1);
[al,bl,cl,dl] = series(al,bl,cl,dl,ad2,bd2,cd2,dd2);
[al,bl,cl,dl] = series(al,bl,cl,dl,alp,blp,clp,dlp);
[al,bl,cl,dl] = series(al,bl,cl,dl,alp2,blp2,clp2,dlp2);
[al,bl,cl,dl] = series(al,bl,cl,dl,alp3,blp3,clp3,dlp3);
[al,bl,cl,dl] = series(al,bl,cl,dl,an1,bn1,cn1,dn1);
[al,bl,cl,dl] = series(al,bl,cl,dl,an2,bn2,cn2,dn2);
damper = ss(al,bl,cl,dl);
```

This version reflects the extra filtering stage discussed in 6.1, it was not yet added to vertical at the time of recording the model.

- The vdamping box is a state space system defined by
-

```
function [damper] = local(gain)
%local to be combined with ECD kas 04/04
gain = gain * 10;  %was intended to normalize but not actually done yet
[ad1,bd1,cd1,dd1] = transdif(0.2,6,1); %general phase lead
[ad2,bd2,cd2,dd2] = transdif(1,3,1);   %more phase lead near 1-2 Hz
[alp,blp,clp,dlp] = sculte(4.5,1,10,40,1); %low pass, first part
[alp2,blp2,clp2,dlp2] = sculte(6,10,11,20,1); %low pass second part
[an1,bn1,cn1,dn1] = notch(2.5,10,1);   %avoid affecting second mode
[an2,bn2,cn2,dn2] = notch(4.1,10,1);   %avoid affecting third mode
%start with just gain and then add on other stages
al = 0; bl = 0; cl = 0; dl = gain;
[al,bl,cl,dl] = series(al,bl,cl,dl,ad1,bd1,cd1,dd1);
[al,bl,cl,dl] = series(al,bl,cl,dl,ad2,bd2,cd2,dd2);
[al,bl,cl,dl] = series(al,bl,cl,dl,alp,blp,clp,dlp);
[al,bl,cl,dl] = series(al,bl,cl,dl,alp2,blp2,clp2,dlp2);
[al,bl,cl,dl] = series(al,bl,cl,dl,an1,bn1,cn1,dn1);
[al,bl,cl,dl] = series(al,bl,cl,dl,an2,bn2,cn2,dn2);
damper = ss(al,bl,cl,dl);
```

- These make use of the same old Killbourn package of functions, *highpass* and *lowpass*, are first order filters with the first argument being the corner frequency (Hz) and the second the pass band gain. *Transdif* is a transitional differentiation from (1st argument) to (2nd argument) with gain below the first corner of (3rd arg). *Sculte* is a complicated low pass 2-pole filter with stop-band zeros, as defined below (it allows excellent early stop-band rejection at the expense of higher frequencies, just what we require)

```
function [a,b,c,d] = scuIte(peak,qp,notch,qn,dcGain)
%resonant 2-pole low pass filter in state space representaion
%[a,b,c,d] = scuIte(peak,qp,notch,qn,dcGain);
%peak            frequency cut (Hz)
%qp              Q factor of resonance
%notch                 notch frequency (above peak)
%qn              Q factor of notch
%dcGain                dc gain
%
%Stuart Killbourn (October 95)
z     = pi*notch*(-1/qn + i*sqrt(4 - 1/(qn^2)));
z     = [conj(z) z]';
p     = pi*peak*(-1/qp + i*sqrt(4 - 1/(qp^2)));
p     = [conj(p) p]';
k     = dcGain*(peak/notch)^2;
 [a,b,c,d] = zp2ss(z,p,k);
```

6.3 Documenting the analysis code

Much of the analysis was done using LTIviewer directly on the model shown above. Integrated displacement/velocity/acceleration estimates were made using the following code (it is not very pretty)

```
%script to analyse an output from pende2 model  KAS version 0.a
mode = 0;  %0-no weight (displacement) 1-velocity 2-acceleration
input = 4; %1=vsusp,2=vsens,3=lsens,4=lsusp (watch may vary in other versions
%of LTIviewer)
output = 2; %2-longitudinal  1-vertical
clear('factor')
%close all
%set up a frequency vector (not very critical)
f = logspace(-1,2,600);
w = 2*pi*f;
sys = pende2_14(output,input); %change to reflect system under test
%analyse
[mag,phase] = bode(sys,w);
mag = squeeze(mag); phase = squeeze(phase);
%make SEI model
if (input ==1|input ==4)  % SEI model  E440303-03 fig 2
    for a = 1:length(f);
```

```
        if f(a)<=0.2
            factor(a)=2e-7;
        end
        if (f(a)>0.2 & f(a)<=1)
            factor(a)=1e-11./f(a)^6;
        end
        if f(a)>1
            factor(a) =1e-11./f(a)^1.7;
        end
    end
    factor = factor';
%or sensor model
else  %sensor model
    factor = 1e-10 + 1e-9./f'; %dummy
    %factor = 1e-10 + 0./f';%alternative
end
%weighting function and noise curve
mag = mag.*factor;  %multiply tf with input
fom = w.^mode;      %weighting with frequency
if output == 1
  mag=mag/1000; %cross coupling for vertical->longitudinal if needed
end
mag= mag.*fom';
factor = factor.*fom';
%estimate rms from right to left
rms = zeros(1,length(f));
rms(length(f))=mag(length(f));
for k = (length(f)-1):-1:1;
    rms(k)=sqrt( rms(k+1)^2 + (f(k+1)-f(k))*mag(k)^2 );
end;
 total = rms(1)
figure(1)
loglog(f,factor,'g',f,mag,'r',f,rms,'b')
legend('input noise asd','output noise asd','output noise rms')
grid on
```

The following, optional, additional filter stage works, but with reduced stability margin, to give simultaneously short settling (if not quite meeting the 10s requirement) and sufficiently low detection-band noise for science mode.

```
[alp3,blp3,clp3,dlp3] = sculte(7,15,12,20,1); %optional low pass third part
[al,bl,cl,dl] = series(al,bl,cl,dl,alp3,blp3,clp3,dlp3);
```

(See longitudinal example for where to put these lines.)

6.4 “pend” variable, for reference

Reanalysis (and redesign of the controllers) will be required if the following numbers are changed in any respect (hence I don’t think it is worth producing more sophisticated controllers at present).

```
title: 'Pendulum parameters and derived properties'
    g: 9.8100
   nx: 0.1300
   ny: 0.5000
   nz: 0.0840
 denn: 4000
   mn: 21.8400
  Inx: 0.4678
  Iny: 0.0436
  Inz: 0.4858
   ux: 0.1300
   uy: 0.5000
   uz: 0.0840
 den1: 4000
   m1: 21.8400
  I1x: 0.4678
  I1y: 0.0436
  I1z: 0.4858
   ix: 0.1300
   ir: 0.1570
 den2: 3860
   m2: 38.4000
  I2x: 0.4733
  I2y: 0.2907
  I2z: 0.2907
   tx: 0.1300
   tr: 0.1570
 den3: 3980
   m3: 39.6100
  I3x: 0.4830
  I3y: 0.3020
  I3z: 0.2920
   ln: 0.4450
   l1: 0.3040
   l2: 0.3400
```

```
         l3: 0.6000
        nwn: 2
        nw1: 4
        nw2: 4
        nw3: 4
         rn: 5.4000e-004
         r1: 3.5000e-004
         r2: 3.1000e-004
         r3: 2.0000e-004
         Yn: 2.2000e+011
         Y1: 2.2000e+011
         Y2: 2.2000e+011
         Y3: 7.0000e+010
        lnb: 0.4800
        anb: 0.0950
        hnb: 0.0043
       ufcn: 2.3300
        stn: 9.7865e+008
  intmode_n: 70.2623
        l1b: 0.4200
        a1b: 0.0583
        h1b: 0.0046
       ufc1: 2.4800
        st1: 9.9990e+008
  intmode_1: 98.1738
        l2b: 0.3700
        a2b: 0.0489
        h2b: 0.0042
       ufc2: 1.8100
        st2: 9.8499e+008
  intmode_2: 115.5000
         dm: 0.0010
         dn: 0.0010
         d0: 0.0010
         d1: 0.0010
         d2: 0.0010
         d3: 0.0010
         d4: 0.0010
twistlength: 0
       d3tr: 0.0010
```

```
                          d4tr: 0.0010
                            sn: 0
                            su: 0.0030
                            si: 0.0030
                            sl: 0.0150
                           nn0: 0.2500
                           nn1: 0.0900
                            n0: 0.2000
                            n1: 0.0700
                            n2: 0.1400
                            n3: 0.1635
                            n4: 0.1585
                            n5: 0.1585
                           tln: 0.4162
                           tl1: 0.2768
                           tl2: 0.3412
                           tl3: 0.6020
l_suspoint_to_centreofoptic: 1.6362
l_suspoint_to_bottomofoptic: 1.7932
                            bd: 0
```