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E2E and FFT : How to compare results

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Objective

The objective of this note is to summarize differences between two very important simulation codes of LIGO: E2E and FFT in notation, physics implementation and in some other small, simple but crucial issues. The awareness on the part of any user of these two important codes is essential for drawing conclusions on comparison or analysis of results from these codes. The issues that we deal with in this note are very simple but these very simple things may often work together to create hell lot of confusion in an user's mind when he/she tries to compare results from these two codes.

E2E and FFT

E2E[1] is a time-domain simulation package developed at Caltech. The so-called FFT code was developed at MIT [2]. E2E can calculate field evolution due to dynamics in longitudinal or transversal perturbations in the system. The FFT code, on the other hand, performs calculations only under static condition.

E2E uses time-domain Modal Model for calculations of spatial perturbations (like rotation and shift of mirrors, mode-mismatch etc.) and so is limited by the maximum number of spatial modes used in a computation. So, in cases of relatively large (static) spatial perturbations in system, E2E cannot produce as accurate result as the FFT code.

E2E is designed for studying any optics configuration in a plane but FFT, as it currently stands, is written for studying only the fixed optics configuration of power-recycled interferometers with Fabry-Perot cavity in each arm - like the 1st generation LIGO interferometers are.

However, we can expect that, when spatial perturbations are small and the system is static, E2E results should match well with the FFT results. However, even under such a condition, the comparison might prove to be a hard and confusing job if one is not aware of differences in notation, treatment of lengths and their resonant conditions etc. In this note these differences are noted down.

Mirror Notation

To preserve the unitarity of fields (albeit upto the order to which spatial modes are considered) the following notation is adopted by e2e: Field amplitude on reflection from the HR-coated side of the mirror, gains a minus sign along with the reflectivity parameter (which could be a number for plane waves or a matrix representing perturbation for Gaussian beams). Field reflected from the opposite side does not get that phase. FFT's notation is just the opposite of E2E's notation as explained in the figure1.

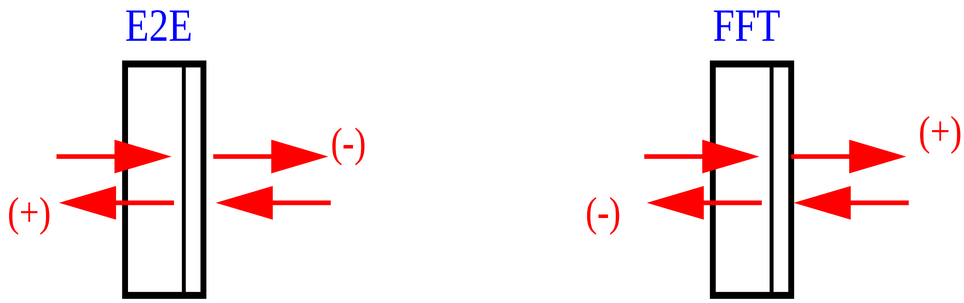

Figure 1: Difference in notation for reflected light from a mirror in FFT and E2E. The reflected light from HR-coated side gets a minus sign in E2E, while that from the opposite side does not get any sign change. In FFT the notation is just opposite.

Resonant Condition

The following equation determines the resonant condition for an exact cavity length, $L = L_0 + \delta L$:

$$-2(L_0 + \delta L) * (2\pi/\lambda) + 2G + \alpha_{\rm mis} + \phi = -2\pi N \tag{1}$$

where N is a large integer, λ is the laser wavelength, G is the propagation Gouy phase in traversing length, L, $\alpha_{\rm mis}$ represents the effect of modal mismatch that may exist between the cavity's fundamental eigenmode and the beam and ϕ is any longitudinal phase offset. L_0 and δL represent the user-supplied macroscopic length and the adjusted microscopic length respectively as explained below.

Note that, although G can be easily calculated analytically, it is impossible to accurately calculate $\alpha_{\rm mis}$ especially when mode-mismatch is large (e.g. in cold state LIGO interferometer) and so guess, by simple arithmatic, what length adjustments are needed.

Cavity length in FFT

Once the macroscopic lengths for various LIGO cavities are given to FFT by user, with notation shown below, it calculates microscopic length adjustments necessary to take LIGO to its operating condition.

```
Input data for simulation of full LIGO interferometer
=====================================================
                      -------
                        ^^^   Mrback
                      ^^ | ^^
                         |
                     L5  |
                         |      ^
                         |   E3 |
                       \ | /
                   Mfr  ---
                       -----
                         |
         Mrecyc      L3 |     Mft           Mtback
                         | /
    Ein | /      L1      |/  L2  |/   L4      \  |
------->| |--------------/-------||------------| |
   <--- | \      <-Esymm/|       |\           /  |
   Eref     E1-->      / |          E2-->
                     Mbs |
                         v Easymm
                        \_/

******************************************************
```

A file records these length adjustments like the following for the following given macroscopic values of the LIGO Hanford 4 km interferometer in its final hot state (which represents an optimally mode-matched state) 4.397, 4.9765, 4.5975, 3995.055, 3995.055, for $L1_0$, $L2_0$, $L3_0$, $L4_0$ and $L5_0$ respectively (the zero in subscripts represent the macroscopic lengths):

```
Begin storage file of variables obtained for the resonant LIGO cavity :
 -------------------------------------------
 Len2corr1, Len3corr1, Len4corr1, Len5corr1 :
  5.2721796E-08
  5.96666945E-08
  3.23538385E-07
  3.24035644E-07
 Len2corrs, Len3corrs, Len4corrs, Len5corrs :
 -7.66821475E-13
 -2.5214247E-12
  4.96258459E-13
  1.83337916E-12
```

So, a length of

```
3995.055+3.23538385E-07+4.96258459E-13
```

will represent the desired resonant length for L4=$L4_0 + \delta$L4 as represented in Eq.1. Note that G and $\alpha_{\rm mis}$ are practically independent of the microscopic length adjustments and are fully determined by $L4_0$ and radii of curvature of mirrors of the cavity.

Corresponding E2E length

To make good comparison between E2E and FFT results we must simulate the same LIGO interferometer in both. The obvious first step is to make reflectivities, losses of mirrors, modal content of input beam etc same in both codes. However, special scrutiny of lengths in these 2 codes is essential to make the whole system same in both codes.

It may be noted that, SimLIGO, an E2E model for the full LIGO system including length and alignment control and other advanced features, is capable of automatically adjusting lengths to their resonant conditions and locking the interferometer. However, these lengths may turn out to be different from FFT-adjusted lengths because, as already mentioned, E2E uses Modal Model with spatial modes upto certain order and so may suffer from inaccuracy when mode-mismatch is large. Also, SimLIGO has realistic suspension system which, together with a realistic servo, may make lengths microscopically different from FFT's locked length.

In the following we refer only to a LIGO optics model in E2E which does not include the length or angular control. We make everything in this model same as in FFT model and then compare results to see, for example, how the finite order modal model results differ from accurate results from FFT model.

Converting FFT-adjusted lengths into E2E model is somewhat tricky and there remains every chance of getting confused if the user is not careful. Refer to section 3.2 in Ref.[1] for a more elaborate discussion on resonant conditions in E2E. The discussion there is not repeated here but various ways of setting parameters are referred to.

Note that Eq.1 is treated in various ways in E2E code to allow users the flexibility of studying different kinds of problems and situations encountered in real interferometers. However, while using lengths from FFT outputs, one must choose the setting for the boolean flag *KeepGuoyOffset* in *"fi eld-gen"* (i.e., laser source) module to be 1. In that case E2E rounds off the macroscopic length, L_0 to $N\lambda$ and the effects of the physical quantities G and $\alpha_{\rm mis}$ can be taken into account by calculating necessary microscopic adjustments from FFT values. This allows the user to directly convert the lengths without bothering about the hard-to-calculate contribution of the mismatch parameter, $\alpha_{\rm mis}$ in Eq.1,

So, if the parameter *KeepGuoyOffset* in *"fi eld-gen"* module is set to 1 and *dphiGuoy* for all propagators of the system are set to zero, FFT lengths can be properly transferred to corresponding E2E lengths by setting *dphi*, the longitudinal phase offset parameter in E2E's *propagator* module to ϕ from Eq.1 :

$$\phi = -\frac{2\pi}{\lambda}\left(L_{\rm FFT} - {\rm round}\Big[\frac{L_{\rm FFT}}{\lambda}\Big]\right), \tag{2}$$

$$L_{\rm FFT} = L = L_0 + \delta L, \tag{3}$$

In case the user wishes to set a nonzero value, $G_{\rm offset}$ for *dphiGuoy* in *propagator* modules (like in some E2E-studies involving thermal lensing), the above equation simply changes to

$$\phi = -\frac{2\pi}{\lambda}\left(L_{\rm FFT} - {\rm round}\Big[\frac{L_{\rm FFT}}{\lambda}\Big]\right) + G_{\rm offset}. \tag{4}$$

Caution : Avoid adjusting the microscopic lengths for positions of 2 mirrors of the cavity by a total value of $-\phi\lambda/2\pi$ instead of setting *dphi* in *propagator* modules by the value from the above equation. Although appropriate adjustments in either *dphi* or mirror positions amounts to the same round-trip phase shift, one must remember that the field outputs of E2E at a mirror is always given at the reference (zero-shifted) position of the mirror and not at the actual mirror surface which will not be at the reference position if microscopic length adjustments are set to the mirror. This may create confusion in user's mind and he/she needs to manually add appropriate phases to E2E's field outputs to compare with FFT outputs which are always given at actual mirror surfaces. Setting *dphi* instead avoids these unnecessary complications.

Remark 1: For FFT-adjusted length in Eq.1, N could be any large odd or even integer. However, in E2E, when *KeepGuoyOffset* in *"fi eld-gen"* is set to 1, the user-supplied macroscopic length, L_0 is rounded to integer multiple of wavelength, so that N in Eq.1 is always even. So, after adjusting microscopic lengths in E2E from FFT-supplied values, the user may, in certain cases, find that corresponding adjustment in *dphi* of a *propagator* module is close to π

in recycling cavity lengths, even in cases of very good mode mismatch (i.e. $\alpha_{\rm mis}$ is small) and even though G values are small for this small cavity.

Remark 2: Last but not the least, the FFT code needs to be run for each frequency (carrier and sidebands) separately. The results of each run is normalized by the input power of 1. On the other hand, E2E simulates all frequencies together just like what happens in actual interferometer. As a result, even for a total input laser power of 1, the results need to be normalized by appropriate quantity for the corresponding frequency. For example, in E2E, if the laser light is phase modulated at radio frquency with a modulation index of 0.45 (LIGO I value), the field amplitudes in carrier run results need to be normalized by $\simeq 1 - 0.45^2/4 \simeq 0.95$ and those in 1st order positive sideband need to be normalized by $0.45/2 = 0.225$ if one wishes to compare with corresponding FFT results.

In this short note we discussed various sources of confusion that may arise in user's mind while comparing FFT results with E2E results. Although each of the differences is easy to keep track of, the awareness of all these differences is essential to avoid any confusion and wrong conclusion.

References

[1] Bhawal, B., Evans, M., Rakhmanov, M., Yamamoto, Y., *e2e primitive module - Reference Manual* -, LIGO doc. T000047-03-E

[2] Bochner, B., *Modelling the performance of Interferometric Gravitational-wave detectors with realistically imperfect optics*, Ph.D theses (MIT, 1998), LIGO Doc. P980004-00R