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r-statistic test for time-domain cross correlation of burst candidate events.

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1 Introduction

The burst pipeline identifies time intervals when an anomaly has simultaneously occurred in the time series of multiple interferometers. In order to give ”credibility” to these events, we need at the very minimum to compare the time series at the single interferometers and verify their consistence, using a well defined criterion to quantify the ”agreement” between waveforms.

Pilot studies have already been performed on this topic (refs?), in the context of both the triggered and untriggered burst searches. This notes presents a technique based on the linear, time-domain cross correlation of burst events candidates. We show preliminary performance tests on a small set of S2 hardware injection. The tuning is still in progress, but it should be completed in time for a full-scale implementation in the S2 analysis.

2 r-statistic

Given two finite series $\{x_i\}$ and $\{y_i\}$, with $i = 1...N$, we define the *linear correlation coefficient* r (Pearson's r) as:

$$r = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2}\sqrt{\sum_i (y_i - \bar{y})^2}} \tag{1}$$

This quantity, by definition, lies between -1 and $+1$, inclusive. It is equal to $+1(-1)$ if the two series are in complete positive (negative) correlation and the (x_i, y_i) points lie on a straight line. A value of r near zero means the two series are *uncorrelated*.

We start from the **null hypothesis** that the two (finite) series $\{x_i\}$ and $\{y_i\}$ are **uncorrelated**. In this case, the $r-$statistic is normally distributed around zero, with standard deviation $\sigma = 1/\sqrt{N}$, N is being the number of points in the series ($N >> 1$). The double-sided *significance* of the null hypothesis is the probability that $|r|$ be larger than what measured if $\{x_i\}$ and $\{y_i\}$ are uncorrelated:

$$S = \text{erfc}\left(|r|\sqrt{\frac{N}{2}}\right) \tag{2}$$

The *confidence* that the null hypothesis is false (that is, the *confidence* that the two series are correlated) is:

$$C = -\log_{10}(S) \tag{3}$$

3 Implementation

Due to the non-optimal time resolution of our burst pipeline, the triggers produced by the pipeline could be as long as 1-2 seconds, even for much shorter injected signals. There are proposals on the table for the optimization of the burst trigger duration, but at this stage all we can assume is that a signal of unknown duration might be present *somewhere within the candidate trigger width.*

The most important parameter in the construction of the $r-$statistic is N, the numbers of points, or alternatively the *integration time* for the cross-correlation. We define:

- $w =$ integration window for the cross correlation;
- $\kappa =$ the sample rate;
- $N = w\kappa$.

The optimal value of w will depend on the signal. If w is too large, the signal is washed out: for this reason we cannot expect meaningful results by simply cross-correlating over the whole trigger duration. If w is too small, we risk to loose the full waveform information. We are presently considering values between 4 and 40 ms, but this is part of the tuning strategy we are addressing with simulation studies (see section 6).

The proposed strategy can be itemized as follows:

1. we divide the event trigger duration ΔT into M intervals of width w, with

$$M = ceil\left(\frac{2\Delta T}{w}\right) + 1.$$

   The intervals centers are $w/2$ apart (maximally overlapped). The first interval is centered on the trigger start time, the last one is (roughly) centered on the trigger stop time.

2. for each interval, we select the corresponding data from the whitened time series (see section 4) of two interferometers.

3. we introduce a time lag between the two data sets, up to a maximum value of τ_{max}. In order to avoid edge effects, we accomplish this by shifting one time series with respect to the other. For each lag, we calculate:

$$r_k = \frac{\sum_i (x_i - \bar{x})(y_{i+k} - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2}\sqrt{\sum_i (y_{i+k} - \bar{y})^2}} \tag{4}$$

   This provides a time series $\{r_k\}$ of linear correlations.

4. we compare the distribution $\{r_k\}$ to the **null hypothesis expectation**: a normal distribution, with zero mean and $\sigma = 1/\sqrt{N}$. The comparison is performed with a Kolmogorov-Smirnov (KS) test on the absolute value of $\{r_k\}$.

   The test succeeds if the significance of the KS-statistic is larger than a set value α[1]. A success means that the $\{r_k\}$ distribution is consistent with the hypothesis of no correlation and that we can exclude it from further analysis. If the test fails, the correlation cannot be excluded and we move on to the next step.

5. From the $\{r_k\}$ series, we calculate a time series of significances $\{S_k\}$ (eq. 2) and one of confidences $\{C_k\}$ (eq. 3), with the same sample rate κ as the original input data.

6. in order to suppress background fluctuations, we decimate the $\{C_k\}$ time series and look for its maximum, which defines, for this portion of the event trigger, the peak confidence and the lag τ between the peak confidence signals at the two interferometers.

7. we then move on to the next portion of the trigger, shifting our window of interest by $w/2$ and repeat the whole procedure.

At the end, we have a set of peak confidences C_m (with the corresponding τ_m) estimated over the M sub-intervals. The fact that the intervals are maximally overlapped ensures all data points within τ_{max} have had a chance to cross-correlate.

For a pair of interferometers, the largest C_m defines the confidence in the correlation of the two waveforms, which can be directly compared to a given threshold level. For three or more interferometers, we can require the confidence to be above threshold simultaneously for all pairs of interferometers. The threshold is directly related to the *false detection probability* β.

[1] α is the confidence level for the KS test: there is a α probability that a r_k distribution that fails the test is from uncorrelated background ("false probability").

- $C_{thr} = 1 \Rightarrow \beta = 10\%$
- $C_{thr} = 1.3 \Rightarrow \beta = 5\%$
- $C_{thr} = 2 \Rightarrow \beta = 1\%$

For an optimal tuning, this needs to be compromised with the *false rejection probability* or, alternatively, the *sensitivity*: this needs to be tuned with simulations and can, in principle, be different for the cases of double or multiple coincidence.

Ultimately, the parameters we need to tune are:

- τ_{max} = maximum lag. We chose, for the moment, to fix this parameter to 10 ms, which is the maximum delay in arrival time between the LIGO sites. This value will need to be increased by the difference in dispersion introduced by the pre-processing filters (still under study).
- w = integration window for the cross correlation calculation. The optimal value of w will depend on the signal. This is being addressed with simulation studies (see section 6).
- α = significance threshold in the Kolmogorov-Smirnov test on the r_k distribution in individual trigger portions: in principle, we want this to be generous (equivalent to a low threshold), since we will later apply a global confidence threshold. What the KS test does for us is to reduce the false rate by ensuring that obviously uncorrelated intervals do not contribute to the final confidence test. The standard KS test uses the 5% significance, but in this work we are using $\alpha = 10\%$.
- β = false detection probability (directly related to the confidence threshold, could be different for double or triple coincidences, given the different sensitivities and false rates).

Other issues that need to be addressed are: in which order to perform the shift (does it matter?), whether we can improve performance with a band-limited cross correlation, how to optimize the timing.

4 Data conditioning

The identification of correlated transients at pairs of interferometers is complicated by the presence of quasi-stationary correlations, mostly line sources of common origin. We address this problem with the adoption of *linear predictor error filters* to remove quasi-stationary predictable signal content from single interferometer data, prior to the r-statistics analysis.

4.1 Theory

The simplest linear predictive error filter is a FIR filter of the form:

$$e_n = x_n - \sum_{m=1}^{M} c_m x_{n-m} \tag{5}$$

The coefficients c_m are chosen to minimize the variance of the error signal e_n over a set of training data and constitute an auto-regressive (all-pole) model for the signal. When the filter is applied to subsequent data, the resulting error sequence contains the signal content which was not predicted by the model.

Figure 1 compares the amplitude spectral density of typical gravity wave channel data before and after application of a linear predictor error filter. The filter was trained on the one second of data ending one second prior to the test data. In both cases, the data was first high-pass filtered at 100 Hz. As is evident in Figure 1, the effect of removing of quasi-stationary predictable signal content is similar to a combination of whitening and line removal.

4.2 Implementation

For stationary stochastic signals, the least squares solution for the unknown filter coefficients is given by the following matrix equation:

$$\sum_{m=1}^{M} c_m \rho_{m-k} = \rho_k \qquad 1 \le k \le M \tag{6}$$

Here ρ_k is the auto-correlation sequence for the the input sequence, x_n, at time lag k. The resulting auto-correlation matrix is $M \times M$, but contains only M independent components.

The theory of linear predictive filters is well developed. Efficient algorithms exist for the solution of this matrix equation in order M^2. In addition, there exist algorithms for the recursive updating of the filter coefficients in order M.

The standard linear predictor error filter described above is implemented by the Matlab function *lpc*. We intend to use Matlab for the initial implementation and as a test platform for further development.

The standard linear predictor has also been implemented as a DMT class *LPFilter*, derived from the *FIRFilter* DMT base class. We plan to develop a full set of adaptive predictor algorithms for both DMT and LDAS. The development of this library is independent of this cross-correlation analysis, but will provide a platform for any future implementations of cross-correlation analysis in DMT or LDAS.

4.3 Further study

There are a number of issues which require further study, and are currently under investigation.

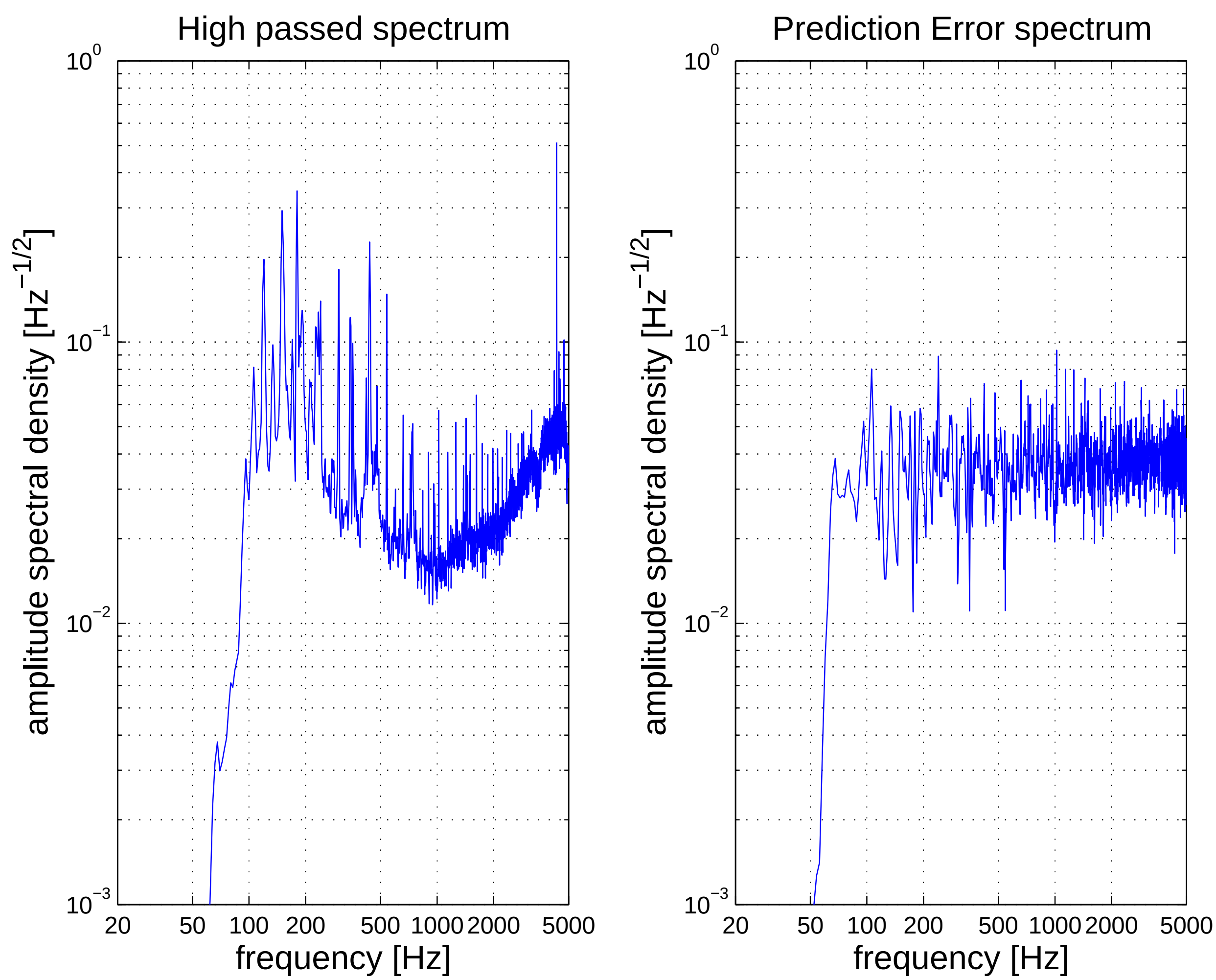


Figure 1: *Amplitude spectrum of S2 gravity wave channel data before and after whitening by a 512 point linear predictor error filter.*

Linear prediction introduces a number of parameters into the cross-correlation analysis which will require tuning. Consider a slightly more general linear predictor error filter,

$$e_n = x_n - \sum_{m=P}^{Q} c_m x_{n-m}, \tag{7}$$

where we seek to minimize the training error power,

$$\sigma_e^2 = \frac{1}{N} \sum_{n=1}^{N} e_n^2. \tag{8}$$

The filter order is now given by $Q - P$, where P is the look ahead time. In addition to P and Q, we must also consider the training length, N, and the applicability of the filter some time τ after training. We are currently investigating the effect of these parameters on the cross-correlation detection efficiency and false detection rate. A useful figure of merit for linear predictor error filters is the variance of the prediction error.

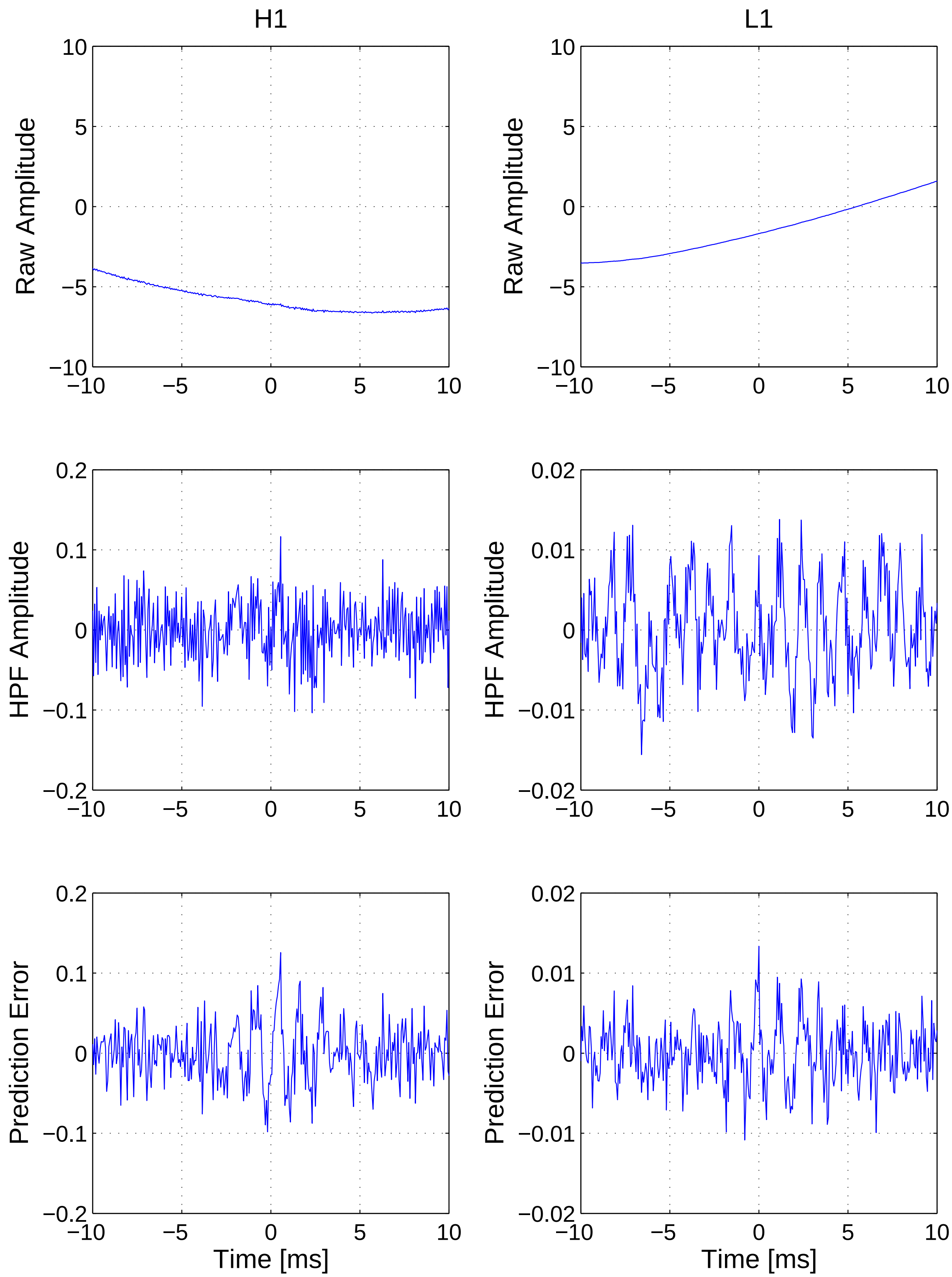


Figure 2: *H1 and L1 Time series corresponding to the sine-gaussian hardware injection 850-5 (see section 5) before and after application of a linear predictor error filter.*

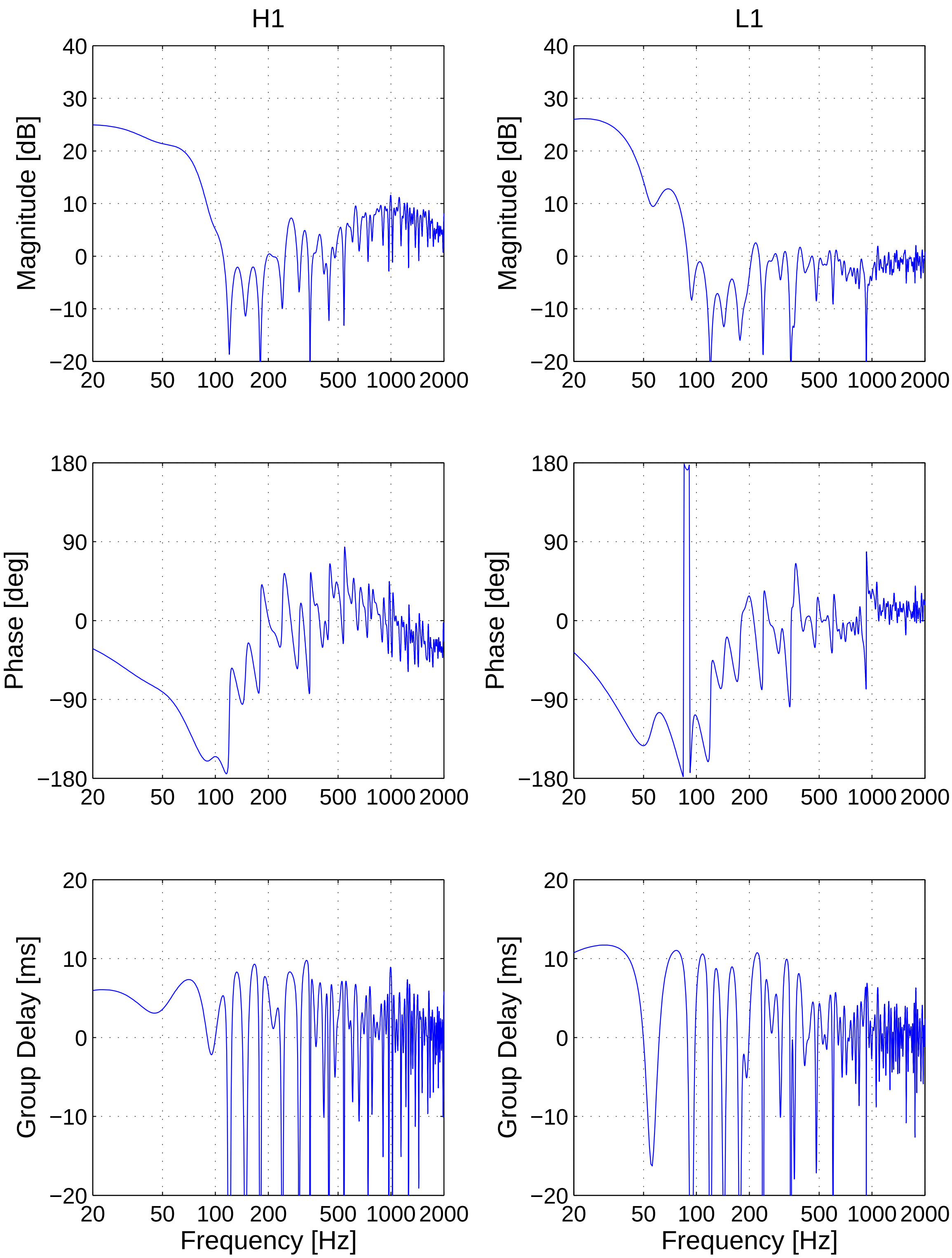


Figure 3: *Magnitude, phase, and group delay response of the linear predictor error filters applied in Figure 2.*

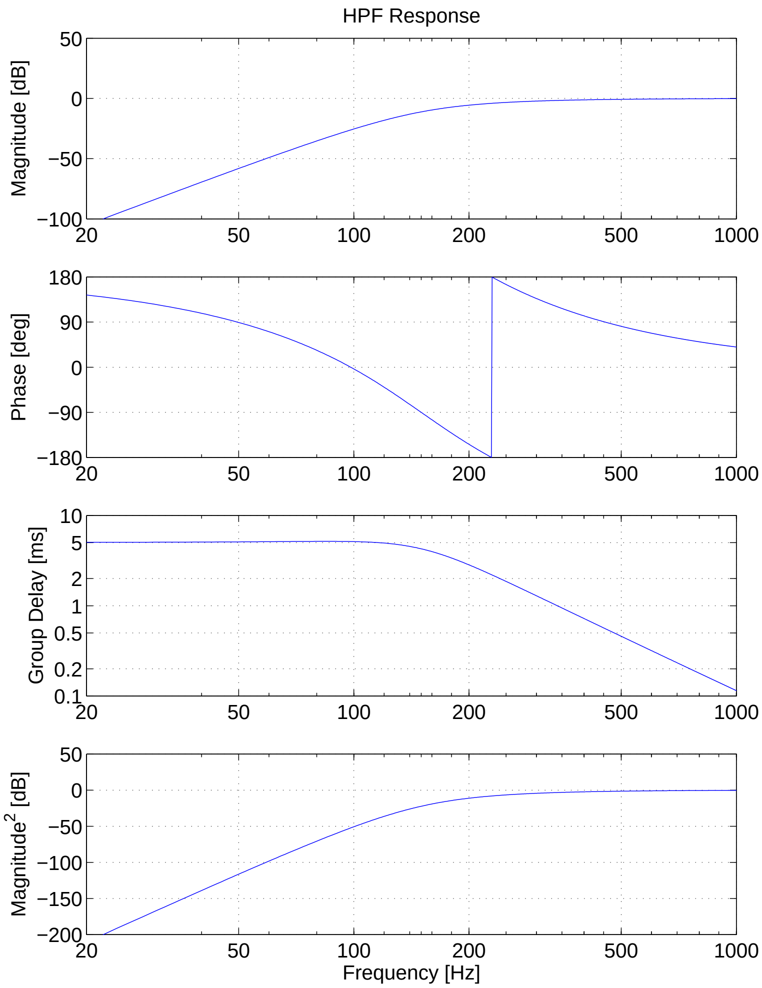


Figure 4: *The top three plots display the magnitude, phase, and group delay response of the causal high pass filter. The bottom plot displays the magnitude response of corresponding zero-phase implementation.*

One issue of concern is the effect of dispersion. Since the filters for each interferometer are trained separately, they will have different dispersion characteristics. This may effect the observed time lag associated with a correlated transient, as well as the observed correlation significance. The dispersion characteristics of standard linear predictor error filters are currently being investigated. Filters constrained to have linear phase, or other methods of reduced dispersion, are also being investigated. Figure 2 shows the result of training and applying linear predictor error filters to a sine-gaussian hardware injection at both LIGO sites. A particularly large amplitude injection was selected here to highlight the effects of filter dispersion, which is evident in the H1 error signal. Figure 3 shows the magnitude, phase, and group delay response of the linear predictor error filters which were applied in Figure 2.

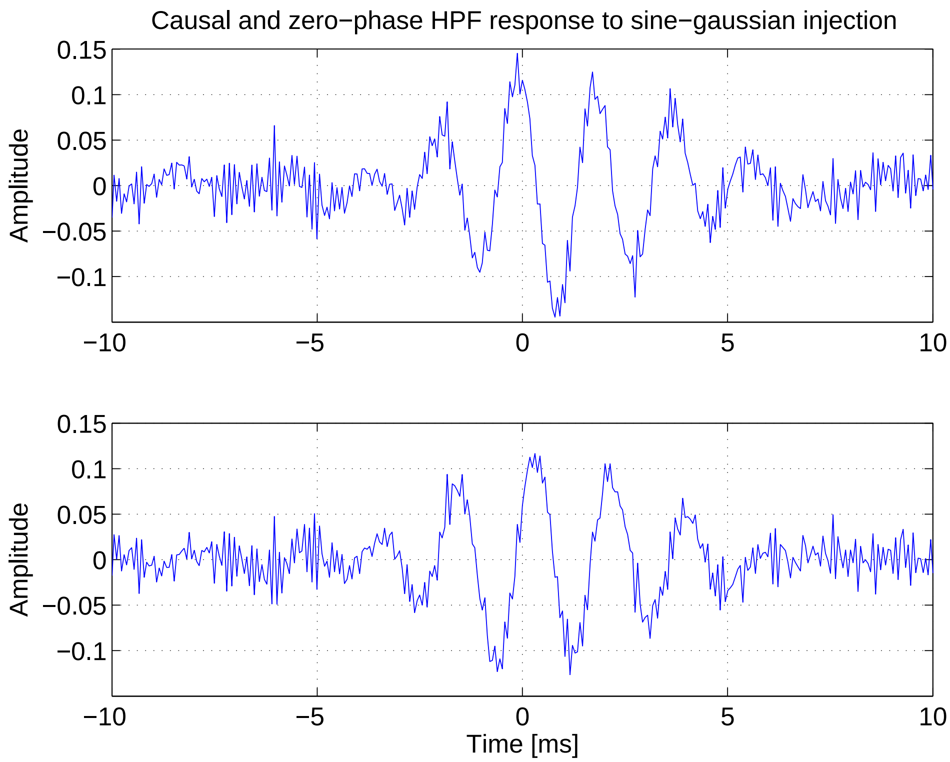


Figure 5: *Comparison of time series corresponding to the H2 sine-gaussian hardware injection 554-5 (see section 5) after filtering by causal (top) and zero-phase (bottom) high pass filters.*

4.4 High-pass filter

Another difficulty in conditioning the data prior to cross-correlation analysis is the excessive power at low frequencies. Removal of quasi-stationary signal content at low frequencies requires high order predictors, which are impractical to implement. To solve this problem, we implement a high pass filter. In order to minimize dispersion, we are investigating the use of zero-phase filtering, similar to Matlab's *filtfilt* function.

Figure 4 shows the magnitude, phase, and group delay response of the causal high pass filter under consideration. The magnitude response of the zero-phase implementation is simply the square of the magnitude response of the causal implementation. Figure 5 shows the response of the causal and zero-phase high pass filters to a sine-gaussian hardware injection. Apart from a time shift, there is little difference between the two results. Such a time shift is not problematic for correlation studies, if the same high pass filter is applied to both interferometers.

5 A practical example: 9 April 2003 hardware injections

In this section we provide practical examples of the procedure described in section 3.

We use a set of burst injections performed on April 9, 2003, toward the end of S2. The set includes sine gaussians at Q=9, 30 and 3, and ring-down waveforms. We plan to study them all, but we report here only on the very small subset in table 1.

Table 1: *A subset of hardware injections from April 9, 2003: sine gaussians, with Q=9 and central frequency 554 and 850, respectively.*

event ID	injection peak time	L1 ΔETM	H1 ΔETM	H2 ΔETM
554-1	733984971.5	0.06	0.03	0.04
554-2	733985131.5	0.21	0.09	0.13
554-3	733985291.5	0.63	0.28	0.40
554-4	733987133.5	0.63	0.28	0.40
554-5	733987293.5	2.09	0.94	1.31
850-1	733984991.5	0.18	0.08	0.12
850-2	733985151.5	0.61	0.27	0.38
850-3	733985311.5	1.83	0.82	1.15
850-4	733987153.5	1.83	0.82	1.15
850-5	733987313.5	6.08	2.74	3.84

Table 1 lists five sine gaussian signals with Q=9 and central frequency 554Hz and five at 850Hz. The injections were performed with ΔETM=ETMY-ETMX increasing in steps of 3. None of these events was detected by the SLOPE ETG; TFCLUSTERS has not yet been run over it, for technical problems that hopefully will soon be solved. On the basis of these and previous injections, we can make the qualitative statement that the injections with ID 554-3, 554-4 and 850-3, 850-4 are close to the detection threshold.

Let's first discuss figure 6: these plots refer to the L1-H1 r-statistic study for event 850-5, with $w = 20\,\mathrm{ms}$ integration time.

- The first two graphs at the top show the whitened time series for IFO1 (L1) and IFO2 (H1), centered at the time when the injected signal is expected to peak. The vertical red, dashed lines show the 20 ms interval we are cross-correlating ($\pm 10\,\mathrm{ms}$ around time zero).

- The second-left figure shows the $r-$statistic as function of the lag between the two time series. The IFO1 series was kept stationary, the IFO2 series shifted between $\pm 10\,\mathrm{ms}$, in steps of 1/16384Hz. The graph is not symmetric (likely a more pronounced ringing at one of the two sites), but the structure between -5ms and +5ms can already suggest correlation.

- The second-right plot is a MATLAB *normplot* of the r-statistic to the left. Note that the distribution of r-statistic is consistent with normal, but this alone cannot validate the null hypothesis: the slope of this line is important.

- The third-left plot shows the histogram of r-statistic fit with a normal distribution. This is consistent with the normplot result. The lower red curve in this plot is the pdf for this distribution, (it differs from the fit by the normalization factor) compared to the pdf we would have in the null hypothesis: the difference between these two is what invalidates the null hypothesis.

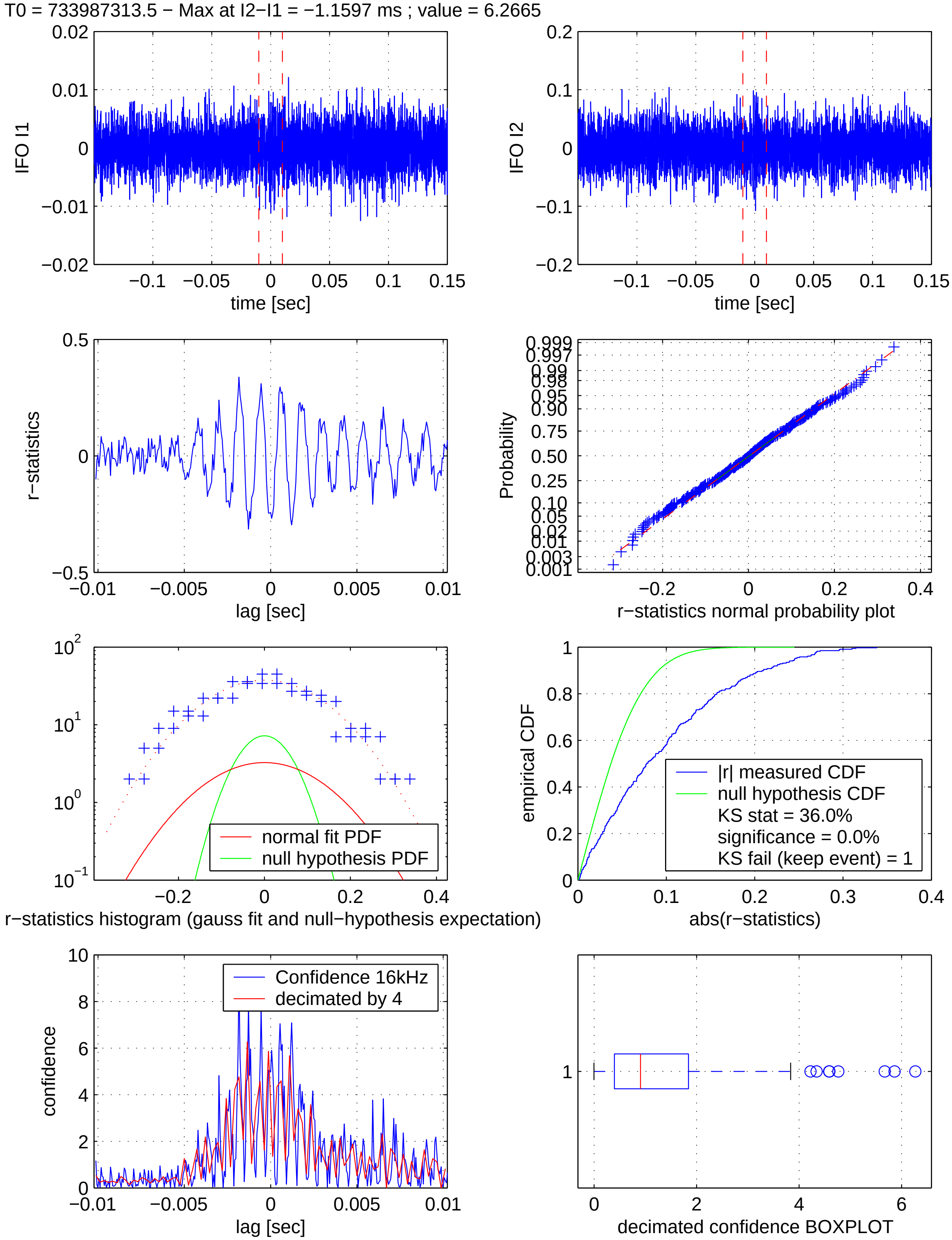


Figure 6: *Event 850-5: L1-H1 r-statistic plots with 20 ms integration time, centered at the injection time. See the text for a detailed explanation of the individual graphs.*

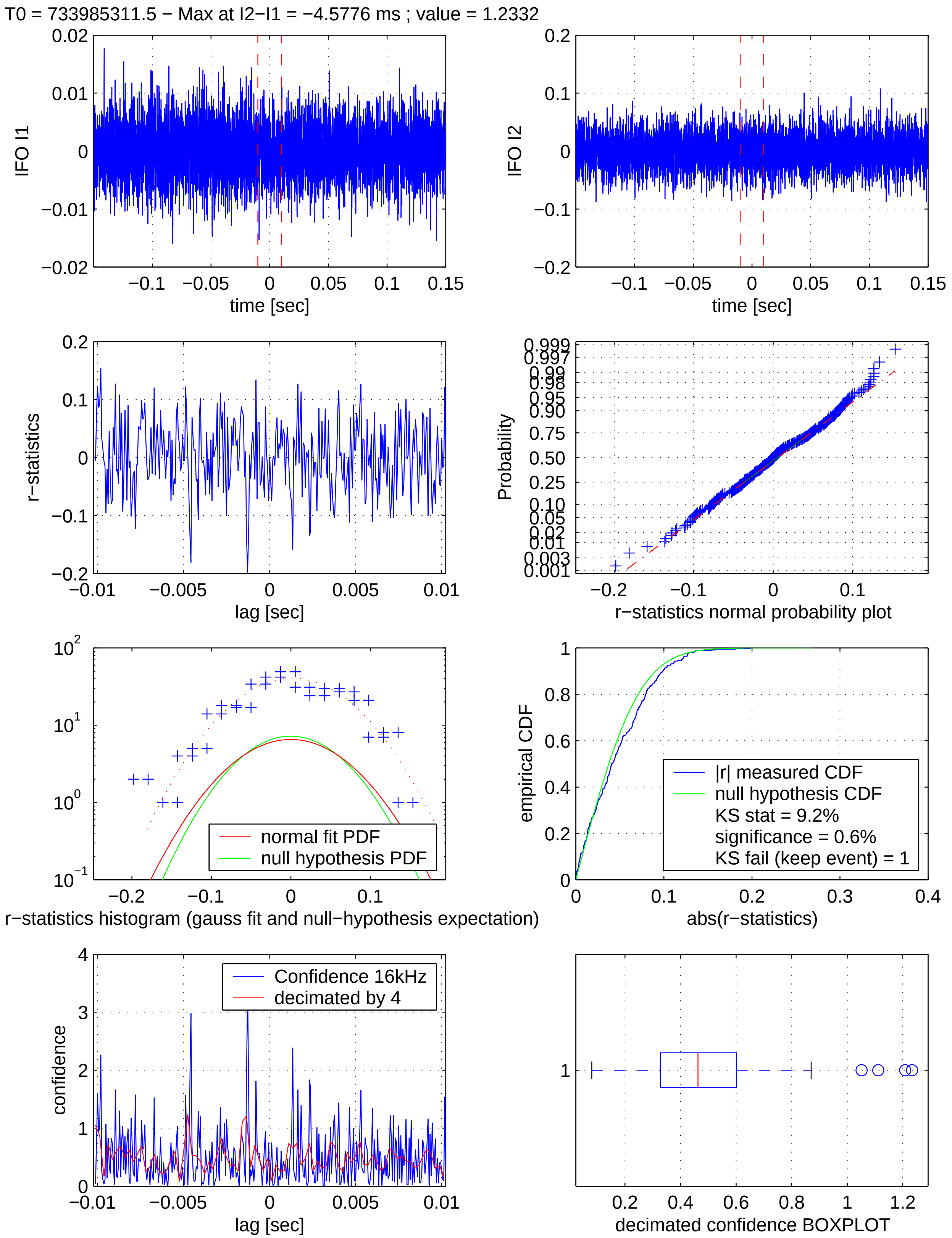


Figure 7: *Event 850-3: L1-H1 r-statistic plots with 20 ms integration time, centered at the injection time.*

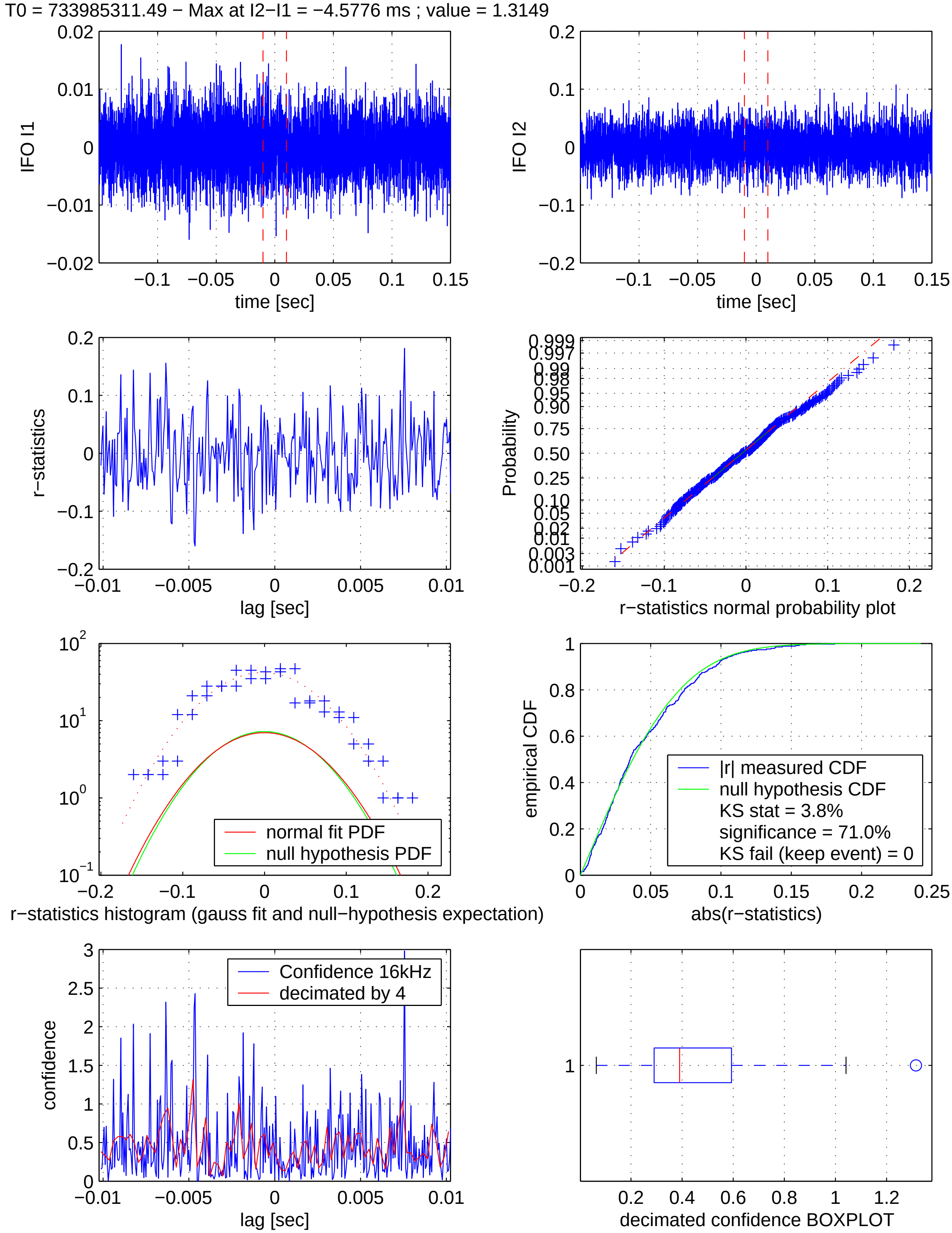


Figure 8: *Event 850-3: 20 ms integration time, centered 10 ms BEFORE the injection time (no correlation expected).*

- The third-right plot is the Kolmogorov-Smirnov (KS) test: the green curve is the cumulative distribution function (CDF) for the absolute value of the $r-$statistic we expect in the null hypothesis. The blue curve is the measured CDF for $|r|$. The KS-statistic (maximum vertical distance between the two curves) is 0.362: its significance (the probability this would happen if the distribution of $|r|$ were that of uncorrelated series) is less than 0.1%. This data set fails the KS test: there is evidence for correlation, we keep this interval as significant.

- The bottom-left graph shows the confidence vs lag plot. The blue curve, calculated from the $r-$statistic plot (second-left graph), has 16kHz sampling rate. The red curve is decimated by 4: this is not so important in the presence of correlation, but it protects from background fluctuations when the correlation is not as evident (as in figures 7 and 8) .

- Finally, the bottom-right plot is a MATLAB *boxplot* of the decimated confidence series (the red curve in the bottom-left graph). The red line marks the 50-percentile (median) of the decimated confidence distribution. The two ends of the box mark the 25-percentile and the 75 percentile. The whiskers are set at 1.5 IQR (inter-quartile range of the distribution). The circles mark the distribution outliers.

For this data segment, the maximum confidence is 6.3, at $\tau = -1.1\,\text{ms}$.

For comparison, figure 7 shows the same set of plots for event 850-3, which is close to threshold. In this case, the significance of the KS-statistic is 0.6%. If we compare this to $\alpha = 10\%$, we can state this $|r|$ distribution fails the KS test and take this data set to the next level of analysis.

Figure 8 is from the same event (850-3), but the data sample was taken 10ms before the expected event time: in this case, the significance of the KS-statistic is very high (74%) and we can claim there is no correlation in this data interval.

6 Global plots: confidence peaks and integration time

Figures 10-14 show, for the 850 Hz events and different integration times, plots of the C_m sets for the three combination of interferometers (L1-H1, L1-H2, H1-H2) with different integration times (the cases presented in section 5 and in figures 6-8, for events 850-3 and 850-5 with 20 ms integration, are in graphs 14-c and 12-c). In these plots:

- the black markers, at $C_m = 0.5$, are for trigger portions that pass the KS test with significance $\alpha = 10\%$;

- the green markers are for trigger portions whose KS significance is between 5% and 10%.

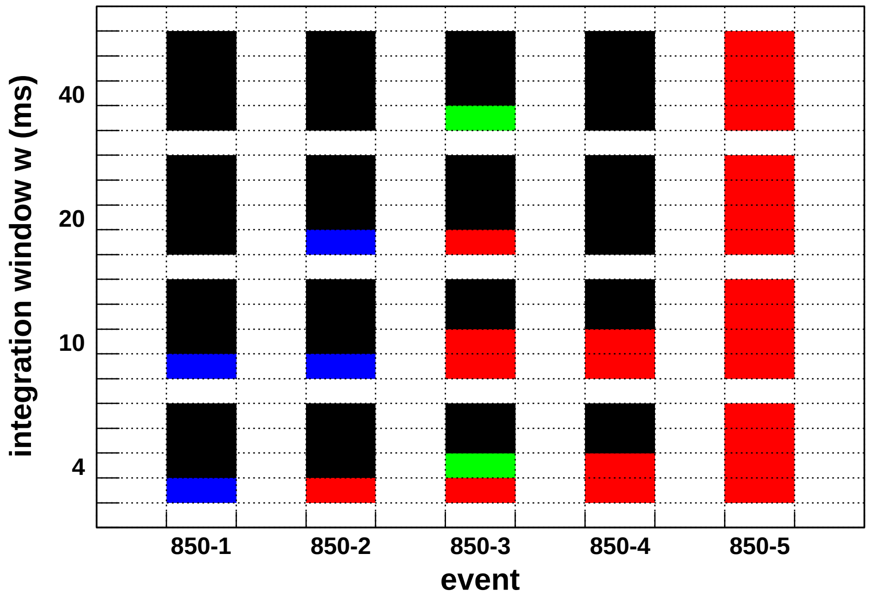
- the blue markers are for trigger portions whose KS significance is between 1% and 5%.

- the red markers are for trigger portions that fail the KS test with significance $\alpha = 1\%$.

The presence of a simultaneous, high confidence peak in all three graphs, as in figure 14, is a "smoking gun" for detection of correlated signals at three interferometers. Unfortunately, we will mostly deal with much less sharp features (as in figure 12 or 13) and we need to devise robust methods to combine the information in each of these plots.

The integration time is clearly an important issue for detectability: its optimal value is, in general, waveform dependent. For the 850 Hz sine gaussians shown here, both $w = 4\,\text{ms}$ and $w = 10\,\text{ms}$ seem to work well, the $w = 20\,\text{ms}$ and $w = 40\,\text{ms}$ are too long.

Figure 9 summarizes the effect of α, β and w on the detectability of the 10 events in table 1. For each event and 4 integration times (4, 10, 20 and 40ms), there is a block of 4 bricks.



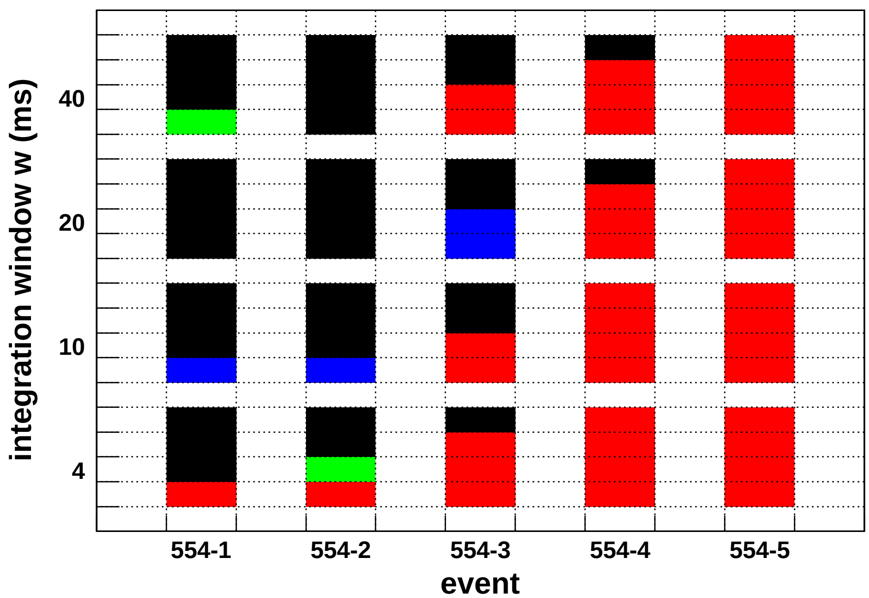


Figure 9: *Color masks for the 10 hardware injections described in this memo.*
For each event and 4 integration times (4, 10, 20 and 40ms), there is a block of 4 bricks.
A brick is black if the event does not pass the C_{thr} cut, it is colored if it passes the cut at all three pairs of IFOs simultaneously. Starting from the bottom, the first brick corresponds to $C_{thr} = 1$ ($\beta = 10\%$), the second corresponds to $C_{thr} = 1.3$ ($\beta = 5\%$), the third to $C_{thr} = 2$ ($\beta = 1\%$) and the fourth to $C_{thr} = 3$ ($\beta = 0.1\%$).
The color is set by the α value used in the KS test: green is for $\alpha = 10\%$; blue is for $\alpha = 5\%$; red is for $\alpha = 1\%$.

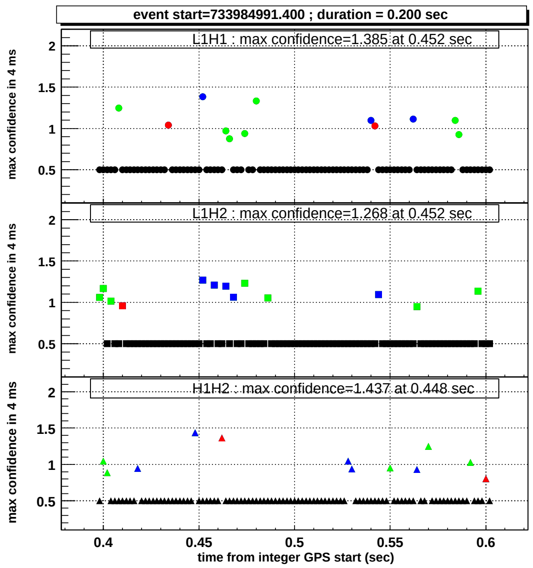
A brick is black if the event does not pass the C_{thr} cut, it is colored if it passes the cut at all three pairs of IFOs simultaneously. Starting from the bottom, the first brick corresponds to $C_{thr} = 1$ ($\beta = 10\%$), the second corresponds to $C_{thr} = 1.3$ ($\beta = 5\%$), the third to $C_{thr} = 2$ ($\beta = 1\%$) and the fourth to $C_{thr} = 3$ ($\beta = 0.1\%$).

The color is set by the α value used in the KS test: green is for $\alpha = 10\%$; blue is for $\alpha = 5\%$; red is for $\alpha = 1\%$.

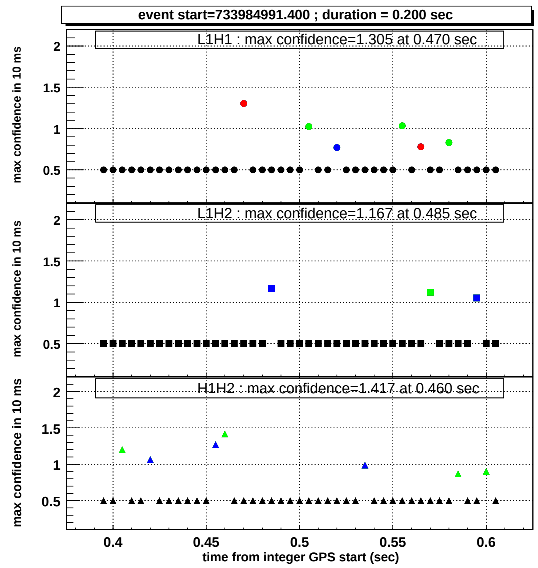
Before we set on chosen values for the three parameters α, β and w, we need to analyze more waveforms and amplitudes. However this kind of graph tells us at a glance where the detection threshold could be set. If, for instance, we decide to work with $\alpha = 10\%$ (false correlation probability in the KS test on the r_k distribution) and $\beta = 5\%$ (95% CL on the correlation in three interferometer pairs), the event is detected if there are at least two colored bricks at one of the integration times. This would set the threshold between 850-2 and 850-3 (or between 554-2 and 554-3). If we want use, instead, $\alpha = 5\%$, we treat the green bricks as black. If we want a higher threshold on the simultaneous correlation, we require 3 or more colored bricks. As far as the w integration time goes, at this point we are still considering to OR the results obtained in the 4 cases, since in general we have no handle on what signal width to expect (especially in absence of a signal).

7 Work in progress

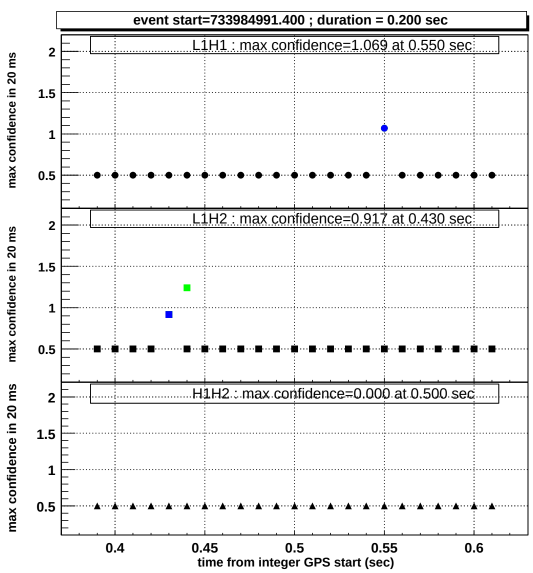
- complete HW injection studies
- study playground events for background characterization
- software injections: Gaussians, Sine-Gaussians, ZM
- do we need calibrated time series?
- optimize the time resolution: the filters described in section 4 in general induce different group delays at different interferometers. We might consider repeating the r-statistic analysis on triggers that pass all significance tests, using filters that are not as efficient at cleaning the signal but introduce less delay. Also, on these events, we might repeat the max confidence search without decimation.
- this is a broadband correlation study: we are considering band-limiting the signal (right now we only high pass it).



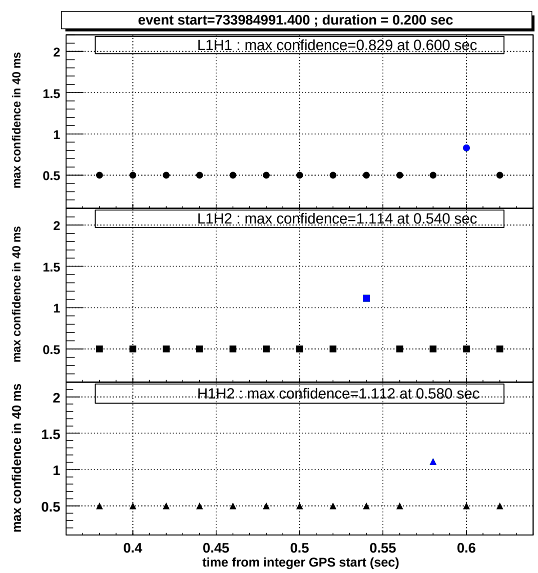
(a) $w = 4$ ms



(b) $w = 10$ ms

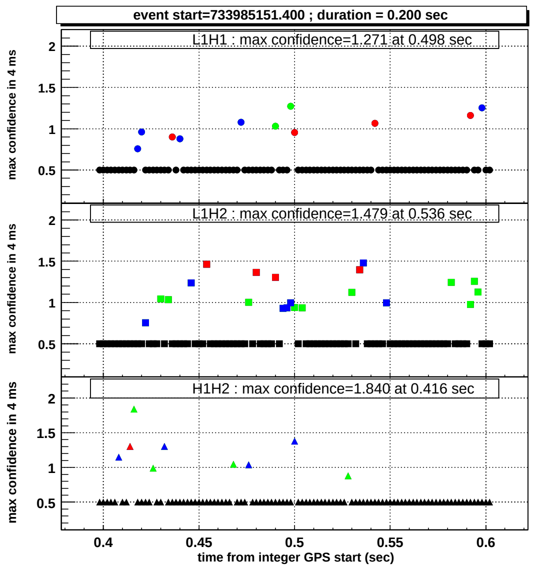


(c) $w = 20$ ms

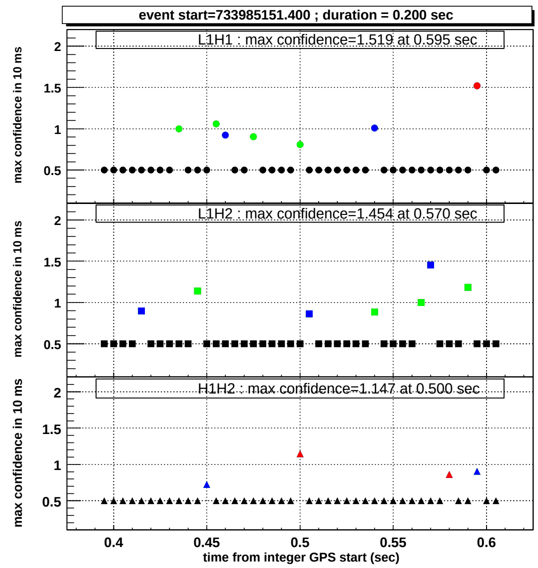


(d) $w = 40$ ms

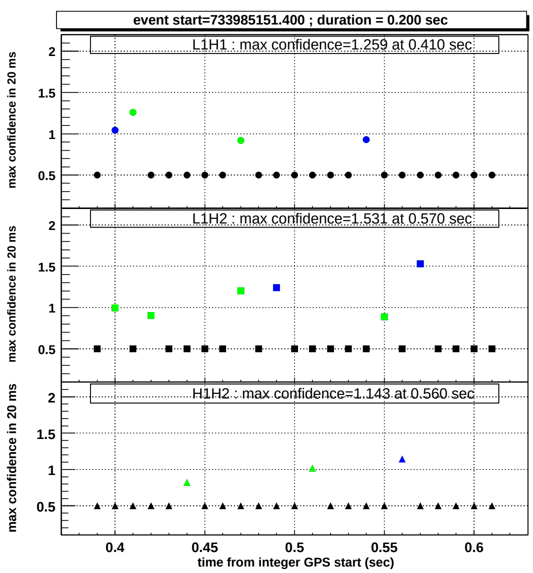
Figure 10: *Event 850-1.*
Peak confidence C_m calculated in the M subsets for all pairs of IFOs. $C_m = 0.5$ for intervals that pass the KS test of no correlation.
This event is below detection threshold. IFO pairs reach confidence values above 1.3 (5three pairs, independently from the value of w.



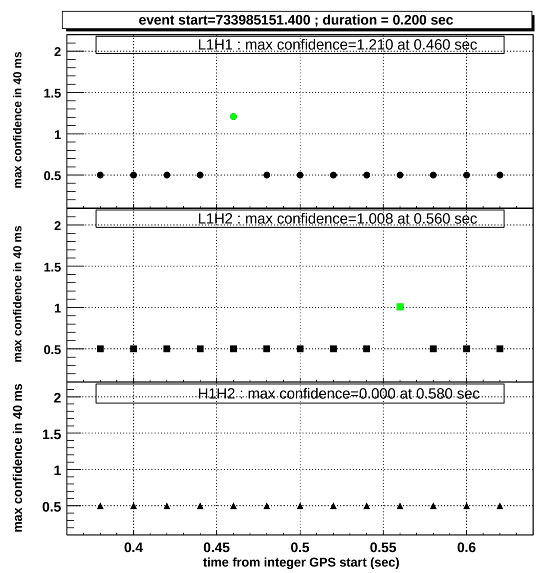
(a) $w = 4\,\mathbf{ms}$



(b) $w = 10\,\mathbf{ms}$

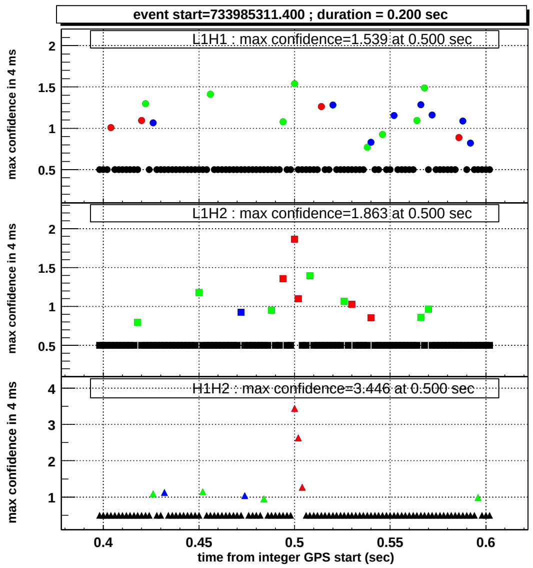


(c) $w = 20\,\mathbf{ms}$

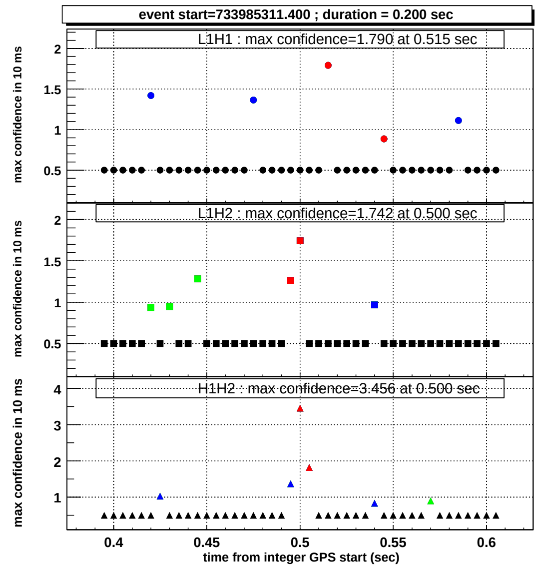


(d) $w = 40\,\mathbf{ms}$

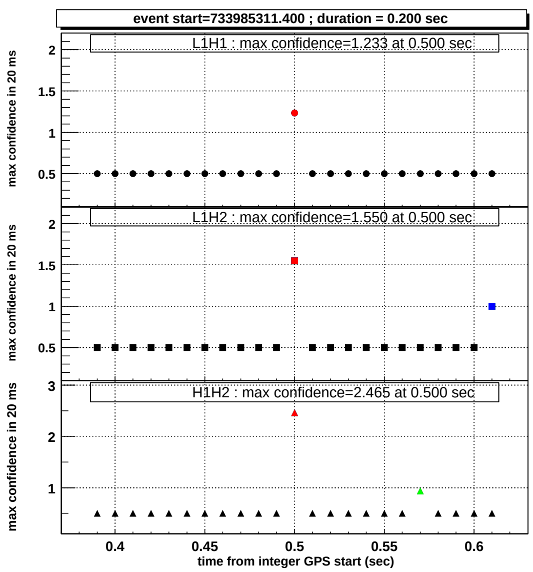
Figure 11: *Event 850-2.*
Peak confidence C_m *calculated in the* M *subsets for all pairs of IFOs.* $C_m = 0.5$ *for intervals that pass the KS test of no correlation.*
This event is below threshold as well.



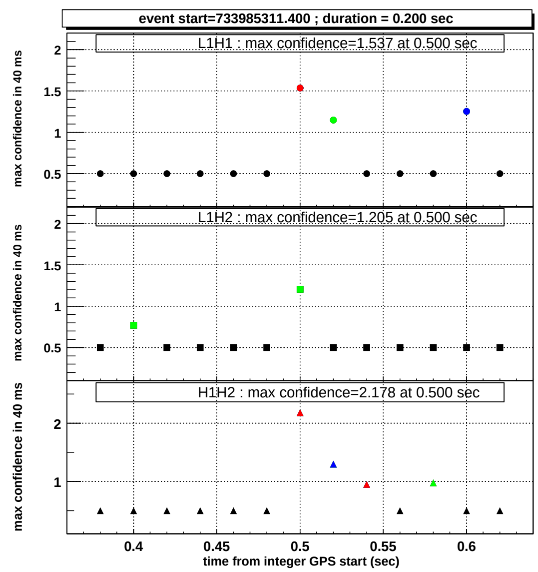
(a) $w = 4\,\mathbf{ms}$



(b) $w = 10\,\mathbf{ms}$

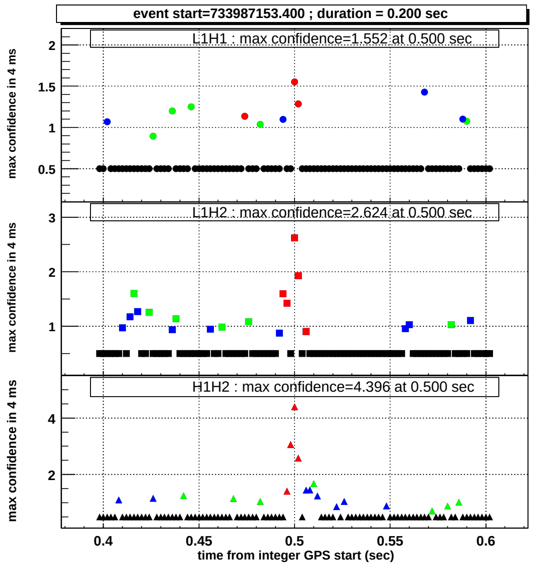


(c) $w = 20\,\mathbf{ms}$

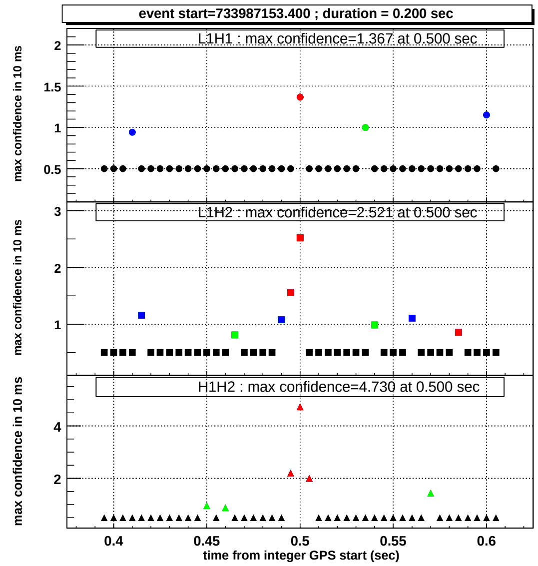


(d) $w = 40\,\mathbf{ms}$

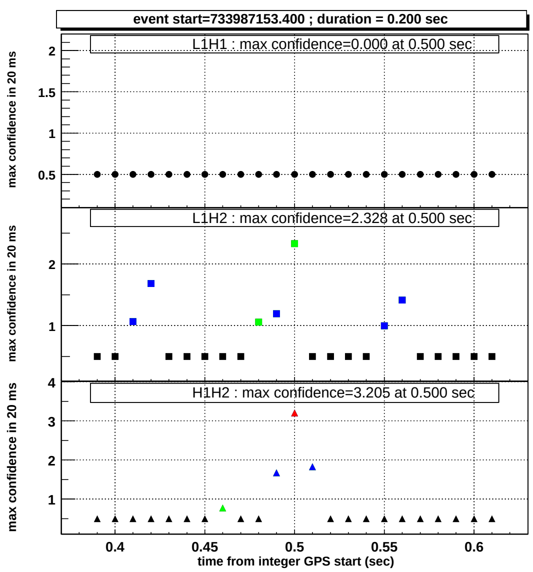
Figure 12: *Event 850-3.*
Peak confidence C_m *calculated in the* M *subsets for all pairs of IFOs.* $C_m = 0.5$ *for intervals that pass the KS test of no correlation.*
In this case, there are simultaneous peaks for $w = 4, 10, 40$ *ms. The The* β *value will establish whether this event is detected or not.*



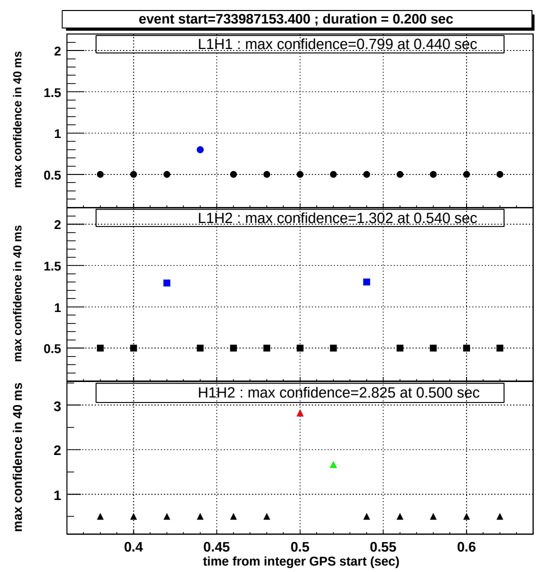
(a) $w = 4$ ms



(b) $w = 10$ ms

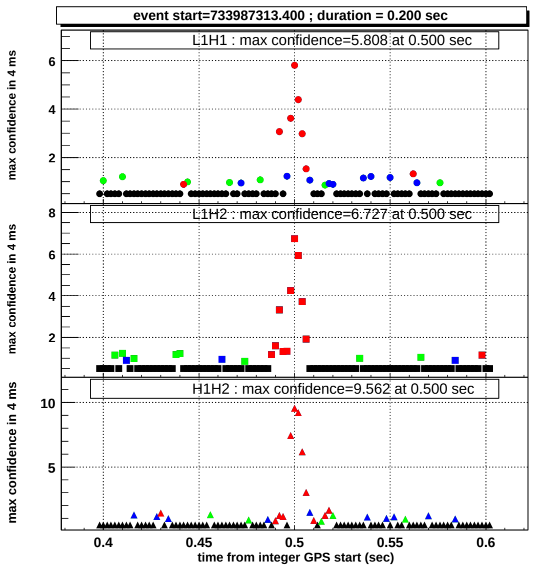


(c) $w = 20$ ms

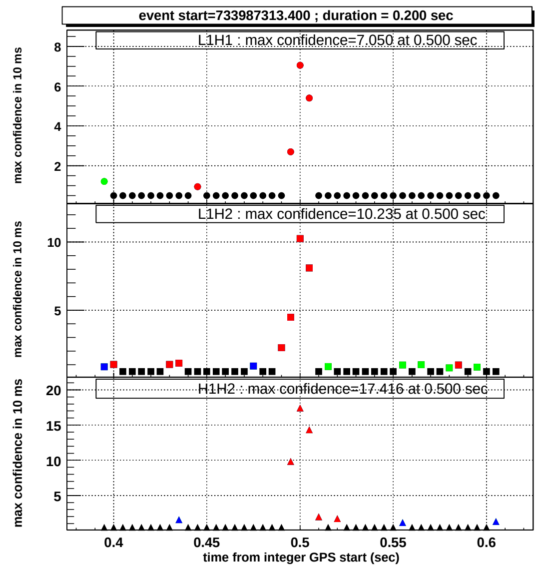


(d) $w = 40$ ms

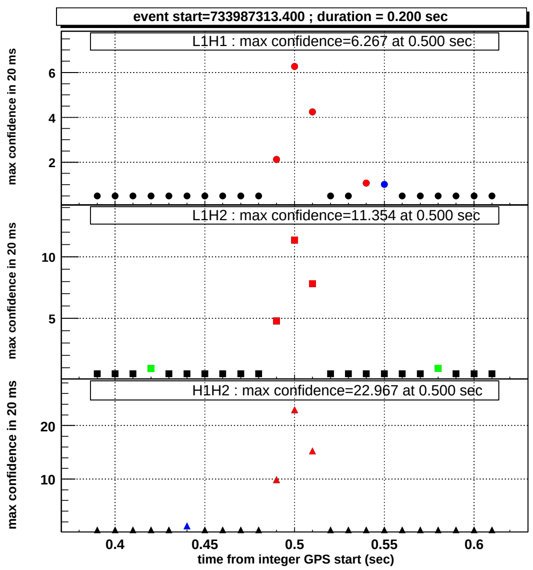
Figure 13: *Event 850-4.*
Peak confidence C_m calculated in the M subsets for all pairs of IFOs. $C_m = 0.5$ for intervals that pass the KS test of no correlation.
This event has the same injection amplitude of the previous, but it is (partially) missed with the larger integration times (20ms, 40ms).



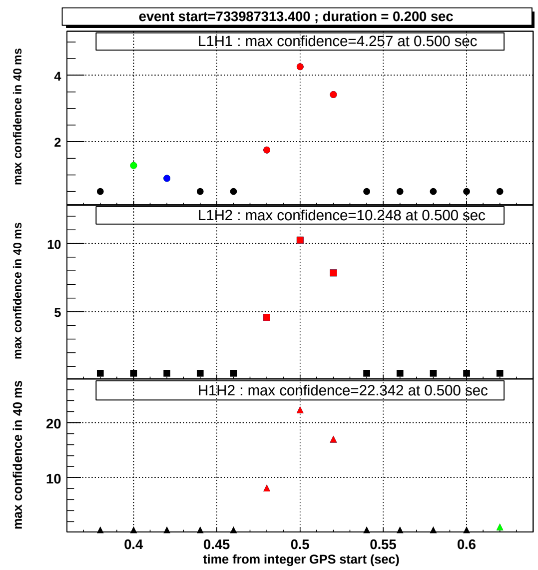
(a) $w = 4\,\mathbf{ms}$



(b) $w = 10\,\mathbf{ms}$

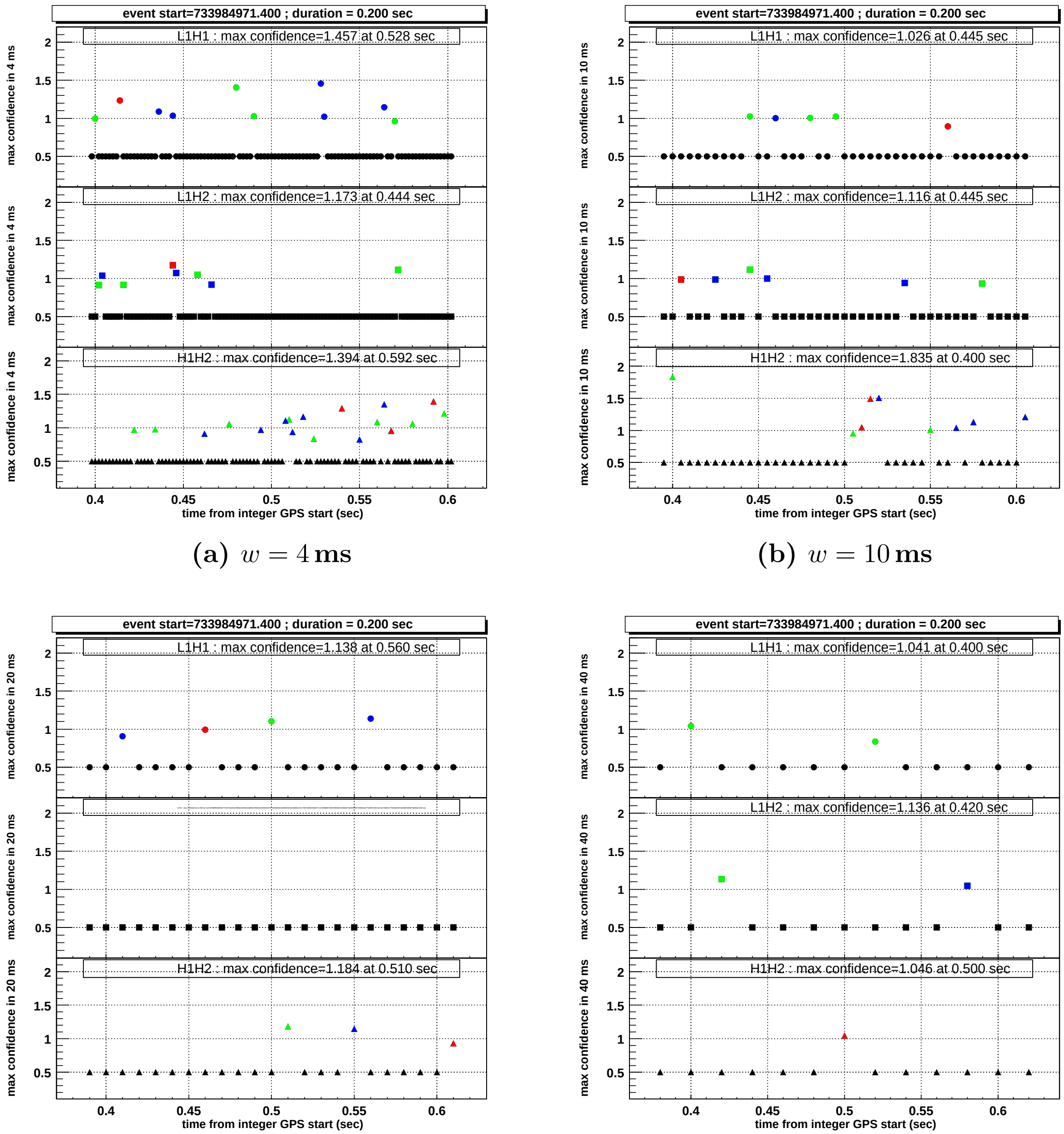


(c) $w = 20\,\mathbf{ms}$



(d) $w = 40\,\mathbf{ms}$

Figure 14: *Event 850-5.*
Peak confidence C_m calculated in the M subsets for all pairs of IFOs. $C_m = 0.5$ for intervals that pass the KS test of no correlation.
There is, in all plots, a well defined confidence peak at 733987313.500 (the expected injection time). This event would definitely pass the r-statistic fit, no matter how tight we make α and β, for all four tested integration parameters.



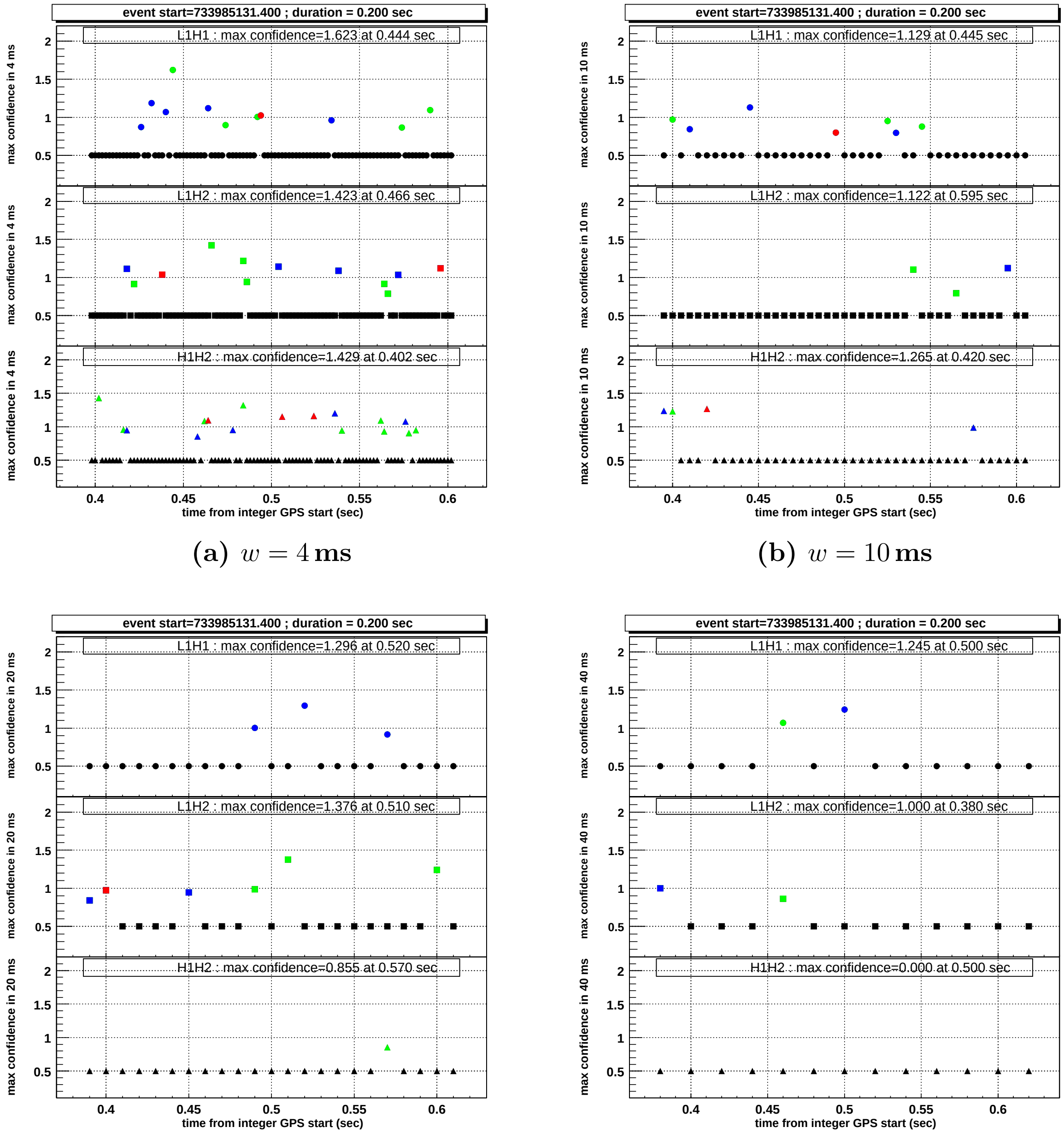
(a) $w = 4\,\mathbf{ms}$

(b) $w = 10\,\mathbf{ms}$

(c) $w = 20\,\mathbf{ms}$

(d) $w = 40\,\mathbf{ms}$

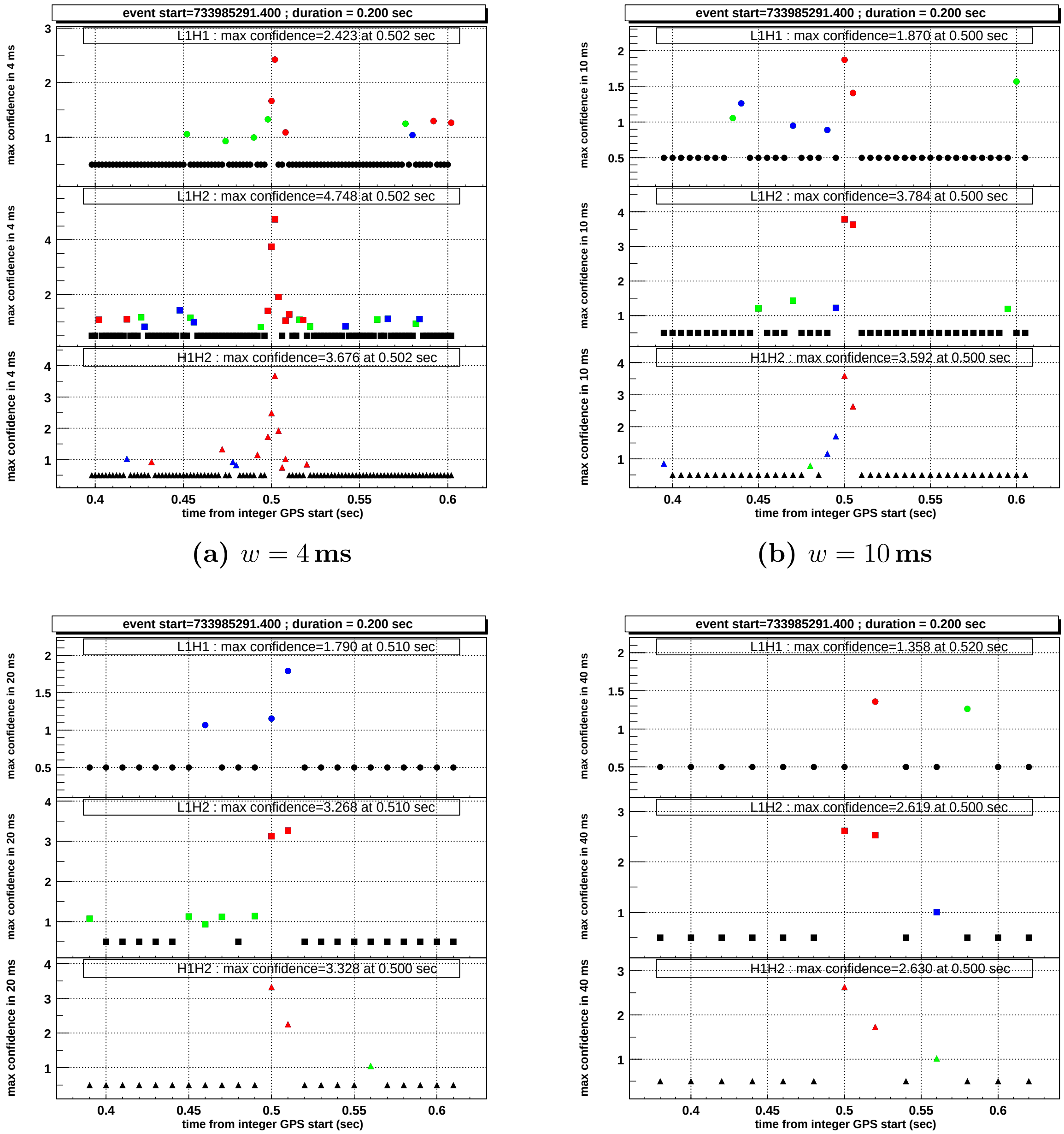
Figure 15: *Event 554-1.*
Peak confidence C_m *calculated in the* M *subsets for all pairs of IFOs.* $C_m = 0.5$ *for intervals that pass the KS test of no correlation.*
This event is below detection threshold. IFO pairs reach confidence values above 1.3 (5three pairs, independently from the value of w*.*



(a) $w = 4\,\mathbf{ms}$ **(b)** $w = 10\,\mathbf{ms}$

(c) $w = 20\,\mathbf{ms}$ **(d)** $w = 40\,\mathbf{ms}$

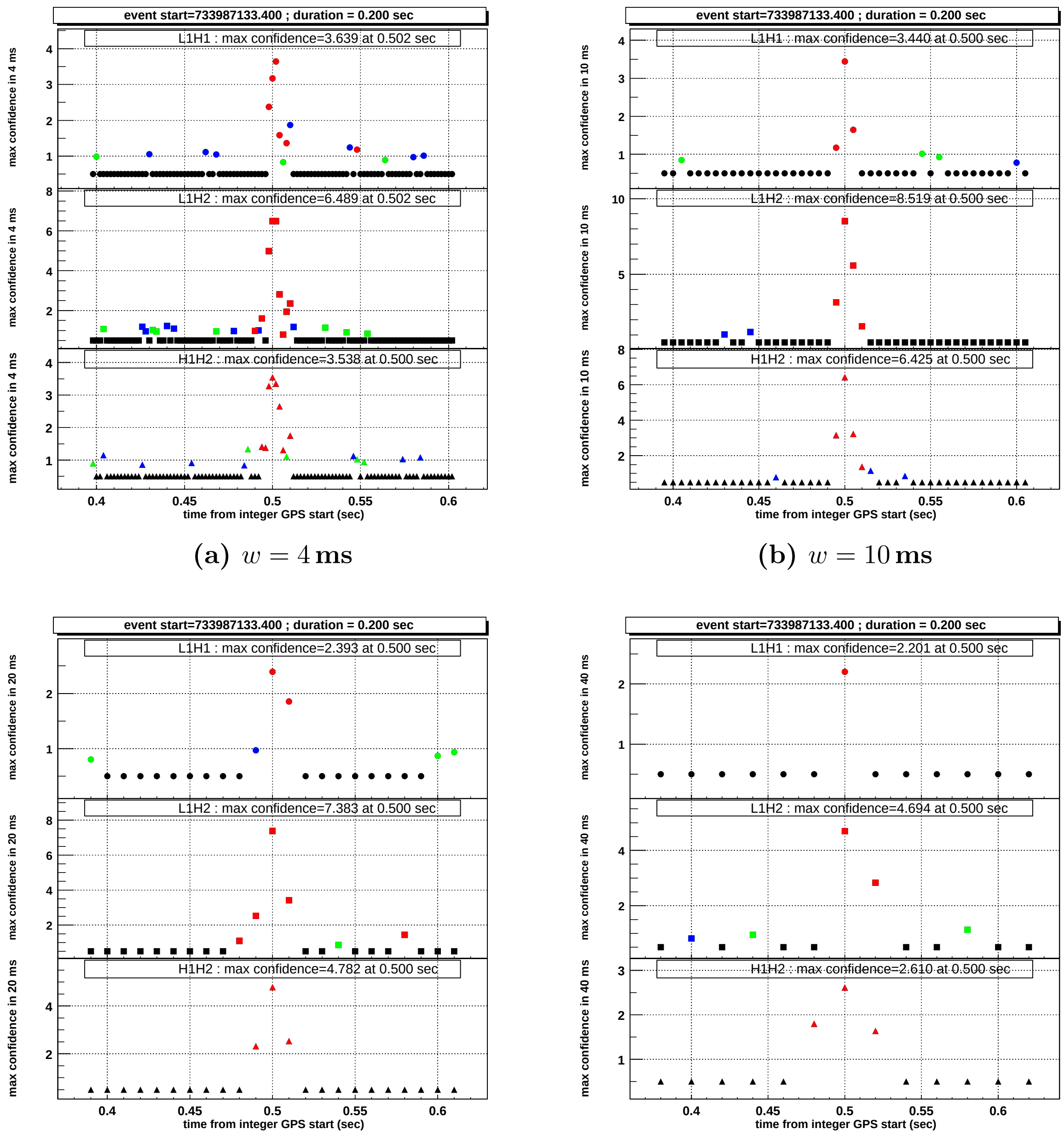
Figure 16: *Event 554-2.*
Peak confidence C_m calculated in the M subsets for all pairs of IFOs. $C_m = 0.5$ for intervals that pass the KS test of no correlation.
This event is below threshold as well.



(a) $w = 4\,\mathbf{ms}$ **(b)** $w = 10\,\mathbf{ms}$

(c) $w = 20\,\mathbf{ms}$ **(d)** $w = 40\,\mathbf{ms}$

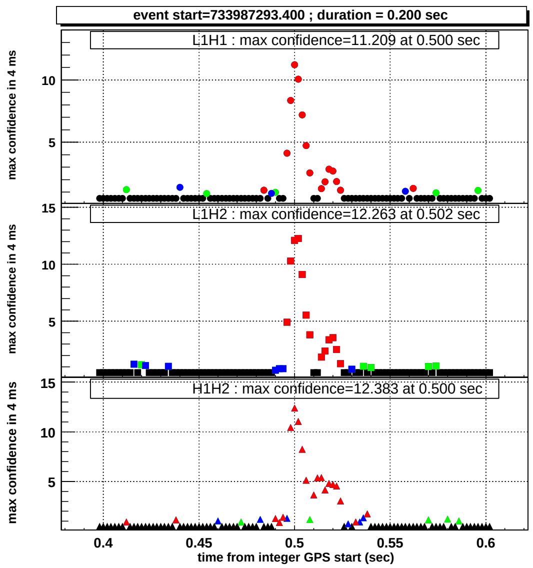
Figure 17: *Event 554-3.*
Peak confidence C_m calculated in the M subsets for all pairs of IFOs. $C_m = 0.5$ for intervals that pass the KS test of no correlation.
I



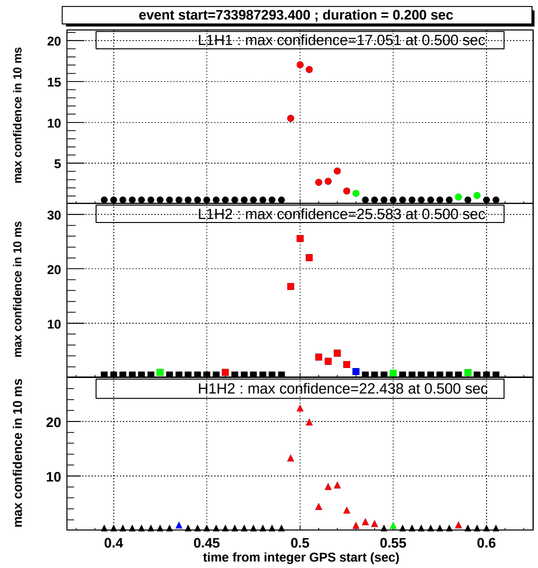
(a) $w = 4\,\mathbf{ms}$ **(b)** $w = 10\,\mathbf{ms}$

(c) $w = 20\,\mathbf{ms}$ **(d)** $w = 40\,\mathbf{ms}$

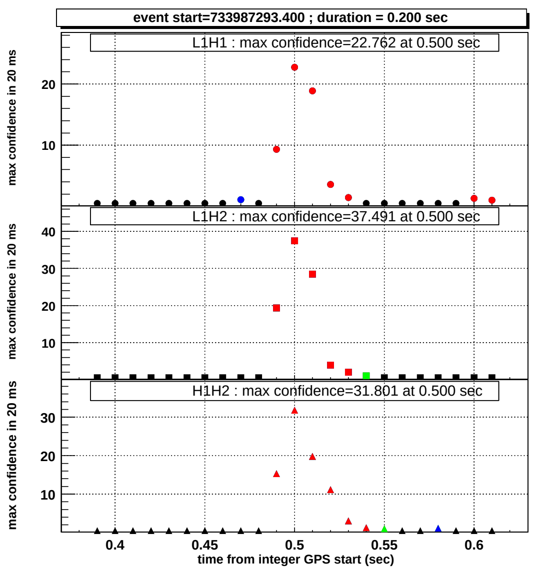
Figure 18: *Event 554-4.*
Peak confidence C_m *calculated in the* M *subsets for all pairs of IFOs.* $C_m = 0.5$ *for intervals that pass the KS test of no correlation.*



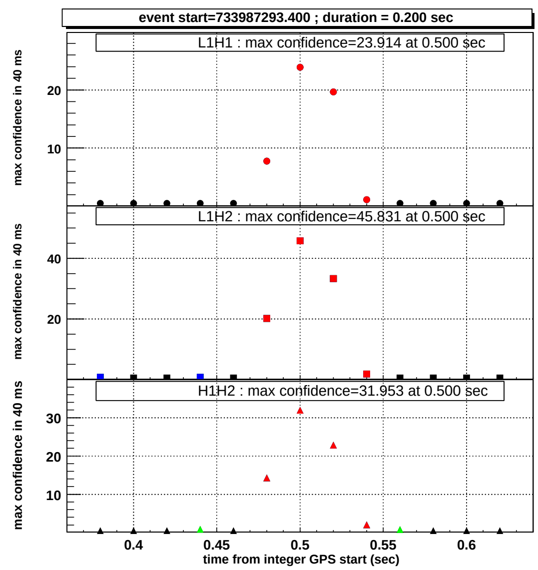
(a) $w = 4\,\mathbf{ms}$



(b) $w = 10\,\mathbf{ms}$



(c) $w = 20\,\mathbf{ms}$



(d) $w = 40\,\mathbf{ms}$

Figure 19: *Event 554-5.*
Peak confidence C_m calculated in the M subsets for all pairs of IFOs. $C_m = 0.5$ for intervals that pass the KS test of no correlation.