LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
– LIGO –

LIGO Laboratory

LIGO-T030004-00-D 7 Jan 2003

Shot noise and optical gain of the H1 interferometer

P Fritschel

Distribution of this draft:

This is an internal working note
of the LIGO Project.

California Institute of Technology
LIGO Project - MS 51-33
Pasadena CA 91125
Phone (818) 395-2129
Fax (818) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project - NW17-161
Cambridge, MA 01239
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

LIGO Hanford Observatory
P.O. Box 1970
Mail Stop S9-02
Richland, WA 99352
Phone 509-372-8106
Fax 509-372-8137

LIGO Livingston Observatory
P.O. Box 940
Livingston, LA 70754
Phone 225-686-3100
Fax 225-686-7189

http://www.ligo.caltech.edu/

1 OVERVIEW

This note describes measurements of the H1 (LHO 4km interferometer) anti-symmetric port sensing chain, specifically the AS1 photodetection channel. Included below are: the calibration of the AS1 detection chain; measurements of optical gain using the AS1 channel, and comparisons to calculation; comparisons of the interferometer noise to shot noise, and extrapolations of the shot noise sensitivity.

2 CALIBRATION OF THE AS1 CHANNEL

2.1. Block diagram

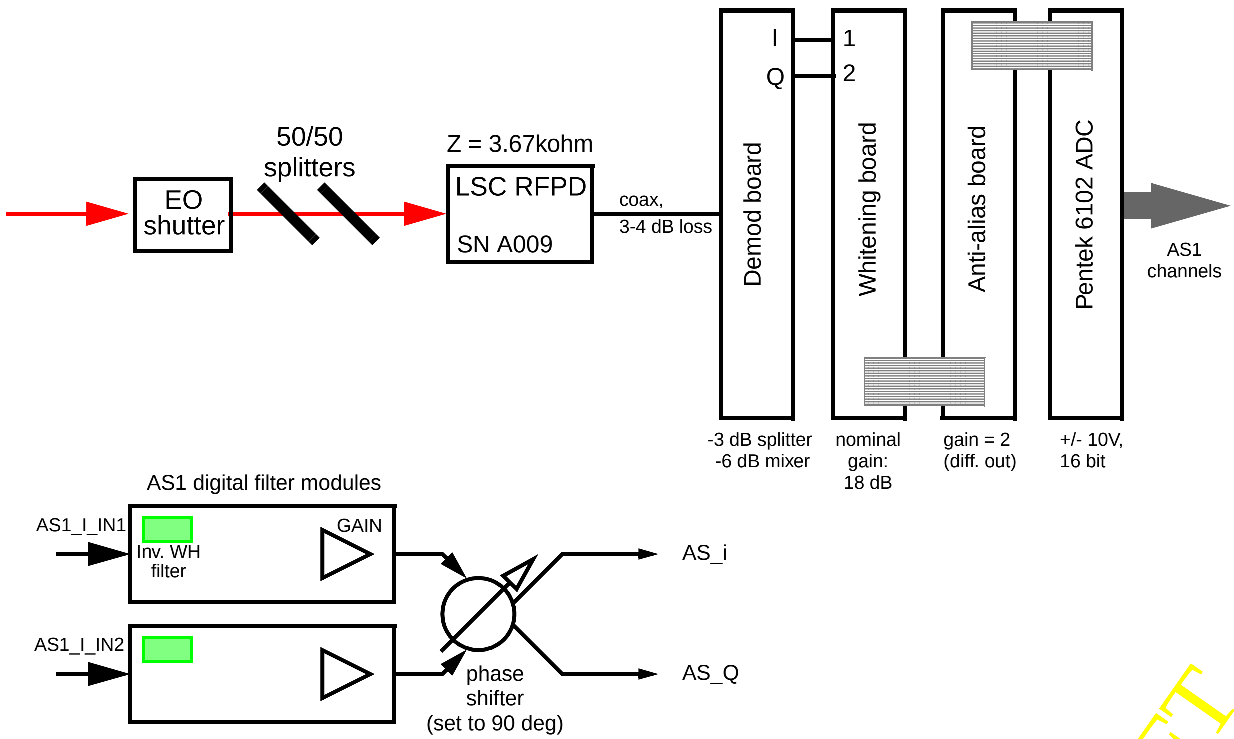

Figure 1: Block diagram of the anti-symmetric port, AS1 signal chain.

2.2. Light bulb shot noise measurements

Nergis M made measurements of the power spectrum of AS_Q when the AS1 RF PD was illuminated by a light bulb; see the LHO Detector elog, 27 November 2002. The assumption is that the

LIGO-DRAFT

light bulb produces a shot-noise limited photocurrent which we can use to calibrate the chain. The result of these measurements is shown in Figure 2.

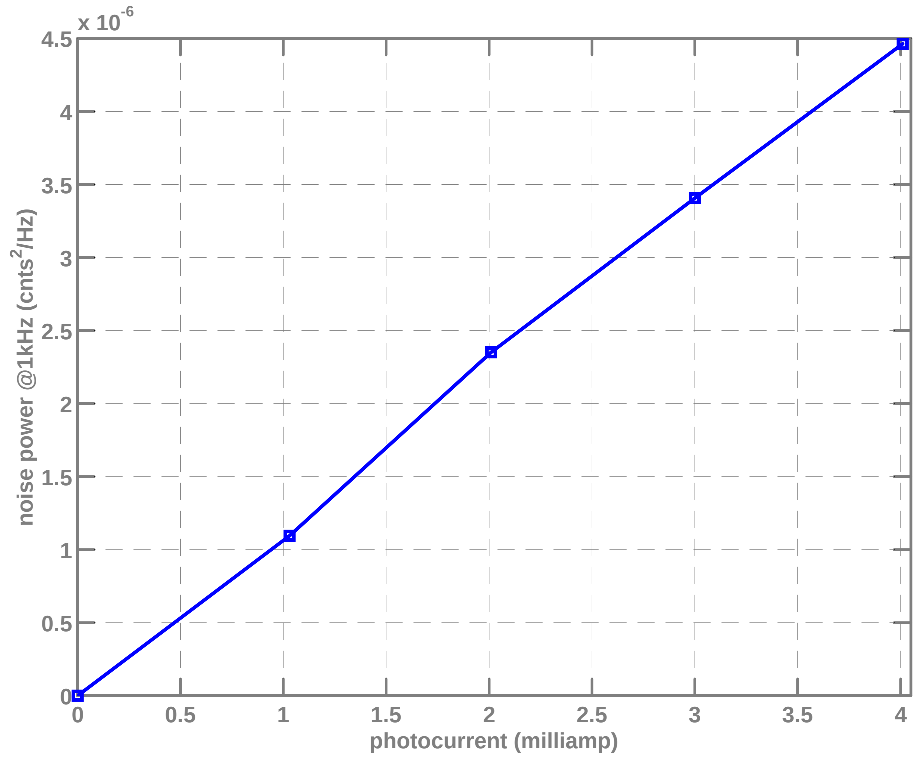

Figure 2: Light bulb shot noise calibration of AS1. Plotted is the noise power, in counts^2/Hz, measured at 1kHz on H1:LSC-AS_Q. The dark level of $(1.55\times10^{-3})^2$ counts^2/Hz has been subtracted from the ordinate values. For the measurement, the whitening filters were ON, the whitening gain was 18 dB, and the digital AS1_GAIN values were set to 1.0. The points fall on a straight line reasonably well, as would be expected for shot noise.

The calibration is made using the highest photocurrent data point, and converting the DC photocurrent into a RF photocurrent noise power using: $i_n^2 = 2eI_{dc}$; this gives a calibration for RF photocurrent to ADC counts of:

$$\text{Noise calibration: } 5.83\times10^7 \text{ counts/amp}\,. \tag{1}$$

The random noise above and below the modulation frequency added incoherently to produce the demodulated noise power. For a signal, this does not happen, and the conversion factor is reduced by sqrt(2):

$$\text{Signal calibration: } 4.12\times10^7 \text{ counts/amp} \equiv G_{AS1}\,. \tag{2}$$

LIGO-DRAFT

2.3. Comparison with expectation

The expected current-to-counts conversion for a signal is (symbols and value defined in the table below):

$$Z_{rf} \cdot T_{\text{coax}} \cdot T_{\text{demod}} \cdot G_{\text{wh}} \cdot G_{\text{aa}} \cdot C_{\text{adc}} = 4.54 \times 10^7 \text{ counts/amp},$$

which is only 10% higher than the measured value.

Symbol	*Quantity*	*Value*
Z_{rf}	RF transimpedance of RF PD, SN A009 (from E010138-03-D)	3.67 kohm
T_{coax}	transmission of coax cable between RF PD and rack (estimated from cable spec)	-3.5 dB loss, T = 0.67
T_{demod}	Demodulation board transmission (estimated from component specs)	-3 dB, 2-way splitter -6 dB mixer conversion T = 0.355
G_{wh}	Whitening board gain	18 dB = 7.94
G_{aa}	Anti-alias board gain	2
C_{adc}	ADC conversion	3276.8 cnts/V

3 OPTICAL GAIN MEASUREMENTS

3.1. Modulation depth

The optical gain of course depends on the RF phase modulation depth, so we need to know this first. Rick Savage and Luca Matone measured the carrier to first-order sideband power ratio using an optical spectrum analyzer on the PSL table; see the 18 November 2002 LHO detector elog. The (single) first-order sideband to carrier power ratio was 0.04, which corresponds to a modulation index of $\Gamma = 0.392$ rad. The optical gain depends on the product:

$$J_0(\Gamma) \cdot J_1(\Gamma) = 0.185 \tag{3}$$

3.2. Simple Michelson

The simple Michelson optical gain is measured by setting up a well-aligned Michelson (RM & ETMs misaligned) and sweeping it through several fringes by pushing, e.g., on an ITM. We can then just look at the bright fringe DC level and the peak AS_Q signal to compare the optical gain

LIGO-DRAFT

to the expected value, with no need to calibrate the mirror drive; see Figure 3 for the sweep time series.

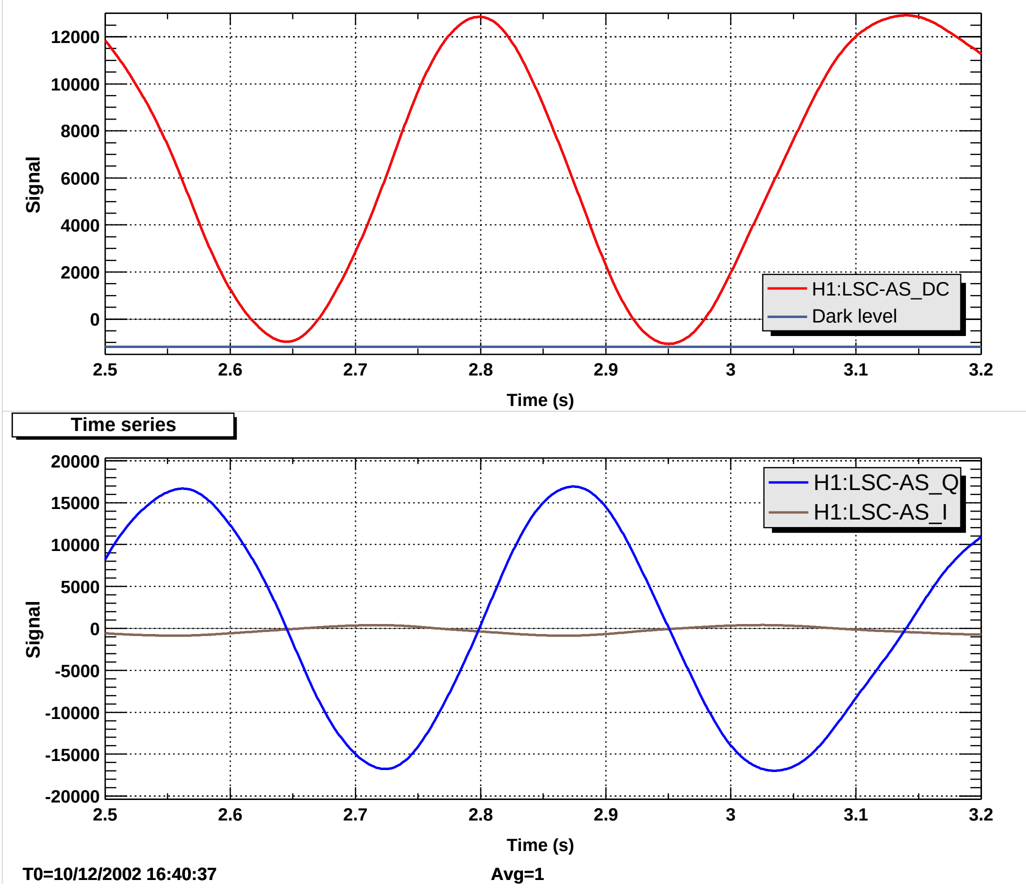

Figure 3: Michelson fringes, sweeping ITMX, used to measure the optical gain. The measurement is made with H1:LSC-AS1_I(Q)_GAIN = 4.0, and the digital phase shift is 90 deg. The dark level offset of AS_DC is –1184 counts.

The bright fringe level is (12840 + 1184 = 14024 counts). The previously calibrated slope for AS_DC is 1.603×10^{-7} amps/count, so the bright fringe photocurrent is 2.25 milliamps. In fact the RF phase modulation is on for this measurement, but the reduction in power is less than 1%, since about 97.5% of the sideband power is transmitted to the bright fringe.

The small signal in AS_I indicates the phase shift is not quite lined up with the signal, and so we add the two AS_Q and AS_I peak signals in quadrature: $16915^2 + 635^2 = (16927\ \text{cnts})^2$.

The calculated peak demodulated signal for the Michelson is:

$$2 \cdot J_0(\Gamma) \cdot J_1(\Gamma) \cdot I_b \cdot \sin\delta \cdot G_{AS1}, \qquad (4)$$

where I_b is the Michelson bright fringe photocurrent (with no phase modulation), and δ is the phase offset for the RF sidebands due to the Michelson asymmetry:

$$\delta = 2\pi \cdot f_{mod} \cdot \Delta l_a / c = 0.154, \qquad (5)$$

LIGO-DRAFT

with f_{mod} = 24.481 MHz, and the asymmetry Δl_a = 0.3 m. The conversion factor G_{AS1} is as given above in Eq. 2, but we also need to multiply by 4 to account for the increase in digital gain. Thus, the expected AS_Q peak signal is 21,045 counts. The comparison of measurement to calculation is:

$$\frac{\text{measured signal}}{\text{calculated signal}} = \frac{16927}{21045} = 0.804$$

The 20% discrepancy is larger than it ought to be, and I don't understand where it comes from. Measurements with lower amplitudes (less power) gave the same result, so it doesn't appear there is any saturation happening (even for the sweep above, the ADC signal level is only ~17000/4 counts, corresponding to about 1.25 V at the ADC input, and 0.6 V at each of the AA filter differential outputs).

3.3. Single arm cavity

The optical gain of an arm cavity is made using the M Landry's ETM drive calibrations. The measurement was made on the X arm, and a sine response was measured from ETMX to AS_Q and AS_I. The sine frequency was 397 Hz, and a digital notch filter at this frequency was included in the feedback filter bank to ensure that the loop gain at 397 Hz was much smaller than unity.

Using Landry's dc calibration for ETMX of 1.08 nm/cnt, the drive level at 397 Hz is: $1.08\ \text{nm/cnt} \cdot (0.76/397)^2 = 3.96\times10^{-15}\ \text{m/cnt}$. The sine response magnitudes for AS_I and AS_Q (scaled to AS1_I(Q)_GAIN values of 1.0) were:

AS_I = 0.0121 cnts
AS_Q = 0.00286 cnts
root-square-sum = 0.0125 cnts

The measured optical gain is thus: $3.15\times10^{12}\text{cnts/meter}$.

The expected optical gain is:

$$\frac{16F}{\lambda} \cdot J_0(\Gamma) \cdot J_1(\Gamma) \cdot I_0 \cdot G_{AS1} \cdot \sqrt{\frac{1}{1 + (397/f_p)^2}}, \tag{6}$$

where F is the cavity finesse, f_p is the cavity pole frequency, and I_0 is the equivalent input photocurrent. The cavity finesse is calculated from the ITMX transmission, plus 70 ppm per mirror loss. For the transmission, I use Bill Kells' step measurement value of T = 2.79%, which is 1.5% larger than the value on Garilynn's web page of 2.75%. Then the finesse is F = 221. The cavity pole frequency is $c/(4LF) = 84.9\ \text{Hz}$.

The AS_DC level during the measurement was 2740 + 1184_{dark} = 3924 counts. To get I_0 we use the above mentioned slope, and also divide by the resonant reflectivity of the X arm cavity, measured by B Kells to be (1 - 0.0241); thus I_0 = 0.645 milliamp.

The expected optical gain then works out to be: 3.42×10^{12} counts/meter. Comparison:

$$\frac{\text{measured signal}}{\text{calculated signal}} = 0.921$$

LIGO-DRAFT

Here we could expect some reduction in the measured signal compared to calculation, since the spatial overlap of the arm cavity mode and the RF sideband mode will not be perfect. This might account for a few percent reduction, but probably not the full 8% seen. Still, the agreement is fairly good, and makes the poorer agreement of the Michelson measurement even more curious.

3.4. Full interferometer

The full interferometer optical gain was measured also with a sine response measurement, again at 397 Hz, but this time exciting DARM_CTRL so that both ETMs are driven. The 397 Hz notch filter was included in the DARM filter bank. The common mode servo was engaged for the measurement, and the AS port EO shutter was at its nominal, nearly fully open level. Care was taken to have good optical alignment, as indicated by high levels for the arm powers and SPOB (recycling cavity RF sideband power level).

The ETM drive levels at 397 Hz were:

ETMX:	1.0552 cnt	4.176×10^{-15} m
ETMY:	0.9132 cnt	3.715×10^{-15} m
(ETMX – ETY)/2:		3.945×10^{-15} m

The AS signals, scaled to AS_I(Q)_GAIN values of 1.0, were:

AS_I:	0.9545 cnt
AS_Q:	5.609 cnt
RSS:	5.690 cnt

The measured optical gain is thus: 1.442×10^{15} cnts/meter.

The expected optical gain is (see T970084-00-D):

$$S = -8J_0(\Gamma)J_1(\Gamma)(\eta P_{in})g_{cr}t_{sb}r_c'(k\Delta L_D)\frac{1}{1+if/f_p}, \quad (7)$$

where P_{in} is the input power, η is the watts→amps conversion factor, g_{cr} is the carrier amplitude recycling gain, t_{sb} is the input→AS port amplitude transmission for the RF sidebands, r_c' is the change in carrier arm cavity reflected field per arm length change, k is the wave number, and the differential arm length change is $\Delta L_D = (\Delta L_1 - \Delta L_2)/2$.

To get the effective photocurrent ηP_{in}, we can use the simple Michelson bright fringe photocurrent (I_{MIB}) scaled by the recycling mirror transmission: $\eta P_{in} = I_{MIB}/T_{RM}$. The standard normalized power measurements give us:

$$g_{cr} = \sqrt{T_{RM}\cdot(\mathrm{NPTRR}+\mathrm{NPTRT})/2} \qquad t_{sb} = \sin\delta\sqrt{T_{RM}\cdot\mathrm{NSPOB}}. \quad (8)$$

The factor r_c' is: $r_c' = r_{ETM}(1-r_{ITM}^2)/(1-r_{ITM}r_{ETM})^2 \approx 4/T_{ITM}$. The optical gain is then:

LIGO-DRAFT

$$|S| = -32 J_0(\Gamma) J_1(\Gamma) I_{\mathrm{MIB}} \left(\frac{\sin\delta}{T_{\mathrm{ITM}}}\right) \sqrt{\mathrm{NSPOB} \cdot \frac{(\mathrm{NPTRR} + \mathrm{NPTRT})}{2}} \cdot k \Delta L_D \left|\frac{1}{1 + if/f_p}\right| . \quad (9)$$

Values for the parameters are:

I_{MIB}	(12835+1184$_{\mathrm{dark}}$ =14019 cnts): 2.25 ma
$\sin\delta$	0.155
T_{ITM}	2.775% (average of ITMX & Y)
NSPOB	342
NPTRR + NPTRT	1545 + 1525 = 1535·2
f_p	84.4 Hz (average of X & Y arms)
ΔL_D	3.945×10^{-15} m

Including the conversion factor G_{AS1} gives: $|S| G_{\mathrm{AS1}} = 10.76$ counts, and

$$\frac{\text{measured signal}}{\text{calculatd signal}} = 0.529$$

Presumably this is a measure of the spatial overlap of the carrier mode (determined by the arm cavities) and the RF sideband mode (determined by the unstable power recycling cavity). The implication is that only 28% of the RF sideband power is in the arm cavity TEM_{00} mode. Some comments on RF sideband efficiency:

- SB recycling gain: $342 \times T_{\mathrm{RM}} = 9.2$
- Input SB power transmission to AS port: $9.2 \times (\sin\delta)^2 = 0.22$
- Implied SB TEM_{00} mode power transmission to AS port: 0.22×0.28 = 6.2%

This last number (6.2%) is quite small; the intention of course is to achieve a level of SB TEM_{00} mode power at the AS port, ~75% or more of input SB power.

4 SHOT NOISE

4.1. Shot noise measurements

The first question is: is the spectrum shot-noise limited? To compare the interferometer noise level with the light bulb shot noise calibration, I use data from UTC time: 30 Dec 2002, 05:19:47 (see Daniel's LHO detector elog of 29 Dec 2002 on '4k Coherence Measurements'). For this data the AS2 detector was being used to form AS_Q, but the AS1 channel was active up to the AS1 filter module output switch (which was off). That is, the channel H1:LSC-AS1_I_OUT contains the

LIGO-DRAFT

desired data. In the band 800-1000 Hz, the noise level on this channel, scaled to a GAIN value of 1.0 (actual gain was 0.044), is 3.75×10^{-3} cnts/$\sqrt{\text{Hz}}$. This value is corrected for two things:

- *Non-zero coherence.* If the noises in AS1 and AS2 were independent of each other, the coherence between them would be zero (actually it would be 1/(# of avgs)). The actual coherence between AS1_I_OUT and AS_Q was a minimum in the 800-1000 Hz band at a level of 0.13 (100 avgs), indicating some correlated noise in the two channels (likely from frequency noise coupling). We're interested only in the uncorrelated component of the two channels. The coherence of 0.13 indicates that the noise power of the correlated component is 0.565 times that of the uncorrelated component (see Appendix 1 for the derivation): $N_{meas} = N_{unc} + 0.565N_{unc}$. So to get the uncorrelated noise amplitude, we correct the measured noise amplitude n_{meas} by: $n_{unc} = n_{meas}/\sqrt{1.565} = 0.8n_{meas}$.
- *Dark noise.* We subtract the dark noise from the uncorrelated noise. This is because the light bulb noise level must be scaled for the interferometer's AS_DC level, and also because the dark noise was seen to increase a little from the time the light bulb measurements were made. The dark noise for the interferometer measurement was 1.65×10^{-3} cnts/$\sqrt{\text{Hz}}$.

So we find the shot-noise component (more properly, the uncorrelated component) of AS1, in the 800-1000 Hz band, to be:

$$n_{AS1} = 10^{-3}[(3.75\cdot0.8)^2 - 1.65^2]^{1/2} = \underline{2.51\times10^{-3}\ \text{cnts}/\sqrt{\text{Hz}}}$$

For comparison to the light bulb measurements, we make three corrections to the light bulb data:

- *Subtract dark noise.* The dark noise level for the light bulb measurements was 1.55×10^{-3} cnts/$\sqrt{\text{Hz}}$.
- *Scale for the AS_DC level.* With the interferometer, the average level of AS_DC was 27,000 counts, whereas the highest current light bulb data was taken with AS_DC = 23877 cnts (neither includes the dark offset).
- *Non-stationary noise factor.* The non-stationary (periodic) character of the interferometer AS port photocurrent increases the noise power due to shot noise by a factor: $[(3+R_{csb})/(2+R_{csb})]$, where R_{csb} is the ratio of carrier power to single RF sideband power at the AS port. Previous measurements indicated roughly equal levels of carrier and (total) RF sideband power at the AS port; i.e., $R_{csb} = 2$.

So, based on the light bulb calibration, we would expect a shot noise component level of:

$$10^{-3}[2.62^2 - 1.55^2]^{1/2}\left(\frac{28184}{25061}\right)^{1/2}\sqrt{\frac{5}{4}} = \underline{2.50\times10^{-3}\ \text{cnts}/\sqrt{\text{Hz}}}$$

The agreement with the measured value is very good, supporting the conclusion that the uncorrelated component of the interferometer noise is due to shot noise.

LIGO-DRAFT

The calibrated displacement noise at 1 kHz is $9.0\times10^{-18}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$, of which approximately 67%, or $6.0\times10^{-18}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$ is due to shot noise.[1] The corresponding effective bright fringe photocurrent is $I_{\mathrm{MIB}}\cdot(\mathrm{NPTRR}+\mathrm{NPTRT})/2 = 2.25\ \mathrm{ma}\times 1535 = 3.45\ \mathrm{amps}$.

4.2. Contrast defect

We can estimate the carrier contrast defect using the preceding numbers. The estimated bright fringe (BS power) photocurrent is 3.45 amps. The dark fringe current is 28184 cts → 4.5 ma, approximately 50% of which is due to the carrier. Thus the ratio of carrier dark port power to carrier beamsplitter power is estimated to be (2.25 ma/3.45 amp) = 6.5×10^{-4}.

4.3. Shot noise extrapolations

The SRD displacement noise at 1 kHz is $5.0\times10^{-19}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$, a factor of 12 below the present shot noise component. If the sideband efficiency were not to improve, we would need 144× more effective power to meet the SRD noise level (this neglects the present dark noise, which will become less important as more power is detected per photodetector; furthermore, the present dark noise level is higher than it should be, and can be reduced by careful engineering). There are some factors at our disposable to increase the power:

Detect all AS port EO shutter transmitted power: currently beam is split into 4 beams, and only one is included in AS_Q	4×
Operate at maximum EO shutter transmission: current transmission is about 90%	1.1×
Fix poor MC transmission: currently MC seems to have about 65% transmission; should be more like 95%	1.5×
Send maximum available power into MC: currently inject 0.84 W, can go to about 4.5 W	5.36×
Total	35.4×

These power increases would theoretically put the shot noise at an equivalent displacement noise level of $1.0\times10^{-18}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$.

Next consider fixing the RF sideband efficiency. Ideally, thermal lensing in the ITMs would lead to a high TEM_{00} mode buildup of the SB's in the recycling cavity. Let's assume we achieve 75% TEM_{00} mode transmission of the SB's from input to AS port, and that essentially all the AS port SB power is in the TEM_{00} mode (the required SB recycling gain would be 32). Then the signal strength would increase by a factor: $\sqrt{75/6.2} = 3.48$. The noise power scales according to: $N \sim (P_c + P_{sb})[(3+R_{\mathrm{csb}})/(2+R_{\mathrm{csb}})] = (P_{sb}/2)(3+R_{\mathrm{csb}})$. The sideband power at the AS port (P_{sb}) would increase, and the carrier-to-sideband power ratio R_{csb} would decrease, by a factor (75/

1. This is approximate, because the AS2 detector was used to produce the calibrated displacement noise spectrum, and the dark noise level may not be the same as for AS1.

LIGO-DRAFT

22) = 3.4×, increasing the noise amplitude by a factor of 1.56. The signal-to-noise ratio would thus increase by a factor of 2.23×.

Another approach is to increase the *S/N* ratio by passing the AS port beam through an output mode cleaner. This would reduce the detected power, and thus the noise, but leave the signal unchanged (ideally at least, assuming the output mode cleaner had 100% transmission for the carrier and sideband TEM_{00} modes). Assuming that the mode cleaner would reject all non-TEM_{00} sideband power, and that it would reduce the AS port carrier power to a negligible level, the noise power would be reduced by a factor: $(6.2/22)[3/(3+2)] = 0.17$. The *S/N* ratio would thus increase by a factor of 2.4×. graphically displays the shot noise extrapolations based on the preceding estimates.

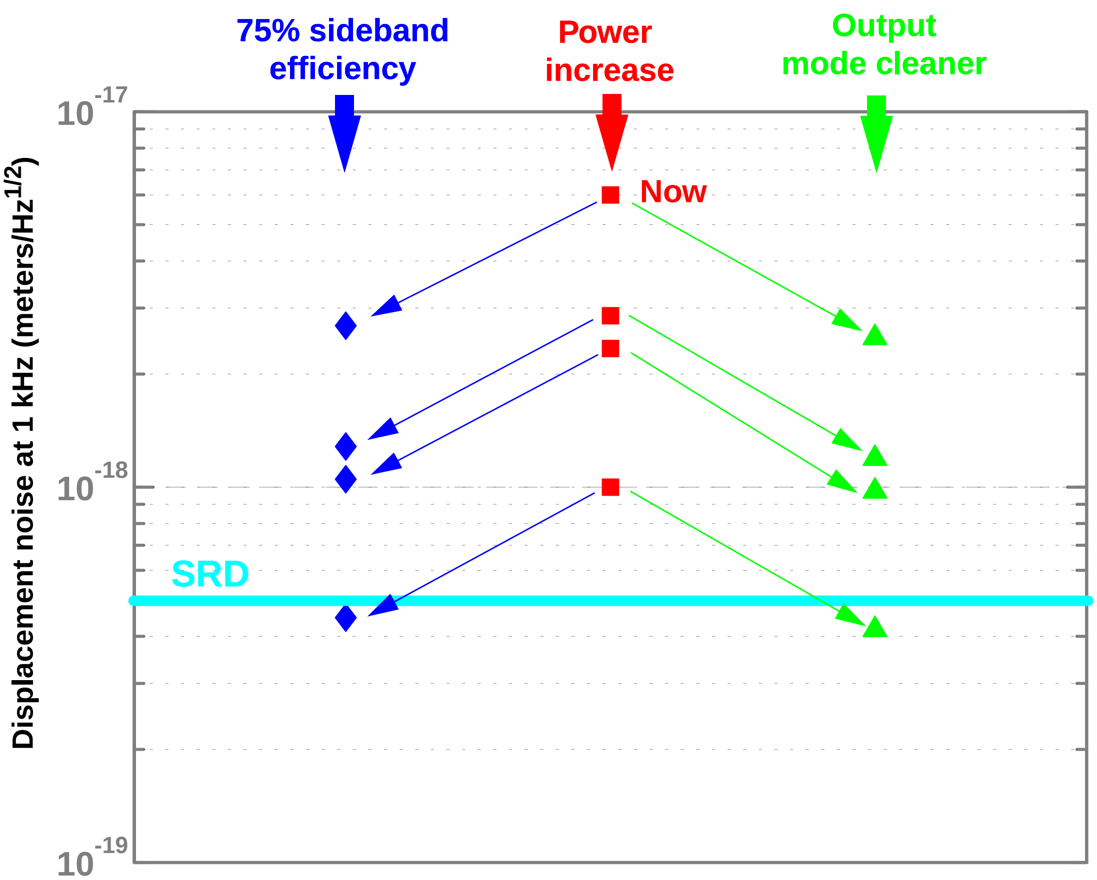

Figure 4: Shot noise projections for three cases: simply increasing the effective power (with no assumed change in RF sideband efficiency due to thermal lensing); increasing the power along with improving the sideband efficiency to 75% (TEM_{00} transmission to AS port); increasing the power along with implementing an output mode cleaner. The four power levels, from top to bottom, correspond to: the current detected power level; detecting all the currently available AS port power; improving the MC transmission efficiency to 95%; increasing the input power to the MC to 4.5 W. Electronics noise (dark noise) is neglected for these numbers.

APPENDIX 1 COHERENCE WITH CORRELATED NOISE

The coherence function is calculated as:

$$C = \frac{|\langle P_{xy} \rangle|^2}{\langle P_{xx} \rangle \cdot \langle P_{yy} \rangle} \tag{10}$$

where P_{xx} and P_{xy} are the power spectrum of signal x, and the cross-spectrum of signals x and y, respectively; the $\langle\ \rangle$ denote averaging over spectra. We wish to know how the coherence scales for two signals which have known uncorrelated and uncorrelatd components:

$$x(t) = z(t) + as(t)$$
$$y(t) = w(t) + as(t)$$

where $P_{zz} = P_{ww} = P_{ss}$, and $P_{zw} = P_{zs} = P_{ws} = 0$. Then:

$$C = \frac{a^4 \langle P_{ss} \rangle^2}{\langle P_{zz} + a^2 P_{ss} \rangle \cdot \langle P_{ww} + a^2 P_{ss} \rangle} = \left(\frac{a^2}{1 + a^2} \right)^2$$

LIGO-DRAFT