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Neutrino Mass Matrix and Hierarchy

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Abstract

We build a model to describe neutrinos based on strict hierarchy, incorporating as much as possible, the known data, Δ_{sol} and Δ_{atm}, and the mixing angles determined from neutrino oscillation experiments. Since the hierarchy assumption is a statement about mass ratios, it lets us obtain all three neutrino masses. We obtain a mass matrix, M_ν and a mixing matrix, U, where both M_ν and U are given in terms of powers of Λ, the analog of the Cabibbo angle λ in the Wolfenstein representation, and two parameters, ρ and κ, each of order one. The expansion parameter, Λ, is defined by $\Lambda^2 = m_2/m_3 = \sqrt{(\Delta_{sol}/\Delta_{atm})} \approx 0.14$, and ρ expresses our ignorance of the lightest neutrino mass m_1, ($m_1 = \rho\Lambda^4 m_3 \approx \rho 10^{-3} eV$), while κ scales s_{13} to the experimental upper limit, $s_{13} = \kappa\Lambda^2 \approx 0.14\kappa$. These matrices are similar in structure to those for the quark and lepton families, but with Λ about $\sqrt{2}$ times larger than the λ for the quarks and charged leptons. We obtain an upper limit of $4 \times 10^{-3} eV$ for the effective neutrino mass in double β-decay experiments. The model, which is fairly unique, given the hierarchy assumption and the data, is compared to supersymmetric extension and texture zero models of mass generation.

1 Introduction

The hierarchical model has been very successful in describing the mass patterns and mixing matrices for quarks and charged leptons [1]. Both the mass patterns and mixing angles are dominated by an expansion parameter, which for each family is given by $\lambda = \sqrt{(m_2/m_3)}$. Furthermore the λ's for the three families are roughly equal, $0.22 < \lambda < 0.25$. Here, we will try to see whether neutrinos can be brought simply into the standard fold. We have a fair handle on the mixing matrix, but as far as the masses are concerned, we only know the two mass-squared differences, Δ_{sol} and Δ_{atm}. This allows the mass ratio (m_2/m_3) to range from about 1 (degeneracy) to small, ~ 0.1, (hierarchy). To determine all three masses one more equation is needed and it is provided by the hierarchy assumption.

The three neutrino mass eigenvalues, m_1, m_2, m_3 give a diagonal mass matrix. This matrix can be undiagonalised by the mixing matrix, U, and the results classified [2] according to possible mass assignments consistent with Δ_{sol} and Δ_{atm}. In this paper, we try to build a model based on strict hierarchy, incorporating the known data as much as possible. Since the hierarchy assumption is a statement about mass ratios, it lets us obtain all three masses from the two Δ's. This leads to a mass matrix and mixing matrix, almost entirely in terms of powers of Λ, the analog of the Cabibbo angle λ in the Wolfenstein representation. These matrices are similar in structure to the quark and lepton families, but with Λ about $\sqrt{2}$ larger than λ for the other families. The effective mass, $< m >$, measured in $\beta\beta_{0\nu}$ decay [3], which could be a real deal breaker, is then obtained and found not to be in contradiction with experiment (yet). The model, which is fairly unique given the hierarchy assumption and the data, is compared to models of mass generation [4, 5].

2 Determination of the neutrino mass matrix and mixing matrix

The mixing matrix, U, [6] which rotates mass (Majorana) eigenstates $\Psi_{1,2,3}$ into flavor eigenstates $\Psi_{\nu_e,\nu_\mu,\nu_\tau}$ is parameterized as usual

$$U = \begin{bmatrix} c_{12}c_{13} & -s_{12}c_{13} & s_{13}e^{-i\delta} \\ s_{12}c_{23} + c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & -c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & c_{12}s_{23} + s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{bmatrix} \quad (1)$$

Letting $\delta = 0$ (no CP violation), and assuming maximal mixing for the atmospheric oscillation [7], i.e., $s_{23} = c_{23} = 1/\sqrt{2}$, we have for U:

$$U = \begin{bmatrix} Cc_{13} & -Sc_{13} & s_{13} \\ \frac{1}{\sqrt{2}}S + \frac{1}{\sqrt{2}}Cs_{13} & \frac{1}{\sqrt{2}}C - \frac{1}{\sqrt{2}}Ss_{13} & -\frac{1}{\sqrt{2}}c_{13} \\ \frac{1}{\sqrt{2}}S - \frac{1}{\sqrt{2}}Cs_{13} & \frac{1}{\sqrt{2}}C + \frac{1}{\sqrt{2}}Ss_{13} & \frac{1}{\sqrt{2}}c_{13} \end{bmatrix} \quad (2)$$

The angle θ_{13} is known to be small [8, 9], with an upper limit $s_{13} < 0.16$ and at present no lower limit. We start with the two rotations known not to vanish, θ_{12} and θ_{23}. With $\theta_{13} = 0$, we have:

$$U = \begin{bmatrix} C & -S & 0 \\ S/\sqrt{2} & C/\sqrt{2} & -1/\sqrt{2} \\ S/\sqrt{2} & C/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \tag{3}$$

where $S = \sin(\theta_{12}), C = \cos(\theta_{12})$ and we have set the rotation angle $\theta_{23} = \pi/4$ (maximal mixing) and the angle $\theta_{13} = 0$, no CP violation.

To lowest order in S (expanding C in Eq. (3) in terms of S), U is given by

$$U_1 = \begin{bmatrix} 1 & -S & 0 \\ S/\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} \\ S/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \tag{4}$$

which, except for the extreme θ_{23} mixing, is much like the quark mixing matrix. In fact, if we consider U_1 to be the result of two successive rotations, $U_1 = v_1 v_0$, we get

$$v_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \tag{4a}$$

and

$$v_1 = \begin{bmatrix} 1 & -S/\sqrt{2} & -S/\sqrt{2} \\ S/\sqrt{2} & 1 & 0 \\ S/\sqrt{2} & 0 & 1 \end{bmatrix} \tag{4b}$$

This suggests that the appropriate expansion parameter for U is given by:

$$\varepsilon = S/\sqrt{2} = \sin(\theta_{12})/\sqrt{2}. \tag{5}$$

There are now measurements of θ_{12} [10, 6] and it is roughly 30^o, i.e., $\tan^2(\theta_{12}) \approx 0.4$.

Turning to the diagonal mass matrix, we define the conventional hierarchical mass pattern by:

$$m_3; m_2 : m_1 = 1 : \Lambda^2 : \rho\Lambda^4. \tag{6}$$

($\rho = 1$ would correspond to strict hierarchy). The three eigenvalues are then $m_3, m_2 = \Lambda^2 m_3$ and $m_1 = \rho\Lambda^4 m_3$.

The hierarchy expansion is in terms of the traditional hierarchy parameter, Λ.

$$\Lambda = \sqrt{(m_2/m_3)}. \tag{7}$$

For quarks and charged leptons, the mass matrices, parametrized by λ, and the mixing matrices, given in terms of θ_{ij} are related. The observed mixing angles of the mixing matrices are given as powers of λ, the Cabibbo angle, as seen in the Wolfenstein [11] representation of the V_{CKM}. We will try to determine the analogous relationship for neutrinos from the data. We note that both expansion parameters, ε and Λ, can be evaluated, independently, from experimental data. With $\tan^2(\theta_{12}) = T^2 \approx 0.4$, we have:

$$S^2 \approx 0.28 \tag{8}$$

and therefore

$$\varepsilon^2 = S^2/2 \approx 0.14 \tag{9}$$

On the other hand $\Lambda^2 = m_2/m_3$, which can be evaluated in the hierarchical expansion, using

$$\Delta_{sol} = m_2^2 - m_1^2 \approx 5 \times 10^{-5}(eV)^2 \tag{10}$$

and

$$\Delta_{atm} = m_3^2 - m_2^2 \approx 2.5 \times 10^{-3}(eV)^2 \tag{11}$$

forms the ratio

$$\begin{aligned}\sqrt{(\Delta_{sol}/\Delta_{atm})} &= \sqrt{[(m_2^2 - m_1^2)/(m_3^2 - m_2^2)]}\\ &= \sqrt{[(\Lambda^4 m_3^2 - \rho^2\Lambda^8 m_3^2)/(m_3^2 - \Lambda^4 m_3^2)]}.\end{aligned} \tag{12}$$

Expanding in Λ^2 we obtain

$$\sqrt{(\Delta_{sol}/\Delta_{atm})} = \Lambda^2 + (1/2)\Lambda^6(1-\rho^2). \tag{13}$$

Thus to order Λ^4 we have

$$\Lambda^2 = \sqrt{(\Delta_{sol}/\Delta_{atm})} = 0.14, \tag{14}$$

so that

$$\varepsilon^2 = \Lambda^2 = 0.14 \tag{15}$$

or

$$\sin(\theta_{12})/\sqrt{2} \approx \Lambda \tag{16}$$

Phenomenologically, at least, there is a close relationship between θ_{12} and Λ.

We will use equation 16 as an equality. Another fit to the solar data [12] gives for the Large Mixing Angle (LMA) solution $\Delta_{sol} = 3 \times 10^{-5}(eV)^2$ and $\tan^2(\theta) = 0.3$. With these values the equality of equation 16 still holds, but $S^2/2 = \Lambda^2 = 0.11$, more in conformity with the quark and charged lepton values for λ^2. (If we use these values, all expressions would remain unchanged, but all masses would become correspondingly smaller.) As the data on $\tan^2(\theta_{12})$ and the Δ's crystallizes, there will probably be a correction factor of order unity. We can now express both the mass matrix, M, as well as the rotation matrix U, in terms of one parameter, defined in equation 7 as $\Lambda = \sqrt{(m_2/m_3)}$, in analogy with the Cabibbo angle for quarks and charged leptons.

The mixing matrix, U, is now

$$U = \begin{bmatrix} \sqrt{(1-2\Lambda^2)} & -\sqrt{2}\Lambda & 0 \\ \Lambda & \sqrt{(1/2)(1-2\Lambda^2)} & -1/\sqrt{2} \\ \Lambda & \sqrt{(1/2)(1-2\Lambda^2)} & -1/\sqrt{2} \end{bmatrix} \tag{17}$$

While $\Lambda = 0.37$ is a rather large number, the expansions made in equation 17 are of the square roots and thus, effectively, the expansion parameter is $\Lambda^2 = 0.14$.

The mass matrix M_ν is given by

$$M_\nu = UMU^{-1} \tag{18}$$

with

$$M = m_3 \begin{bmatrix} \Lambda^4\rho & 0 & 0 \\ 0 & \Lambda^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{19}$$

To order Λ^4, M_ν, is given by

$$M_{\nu_4} = m_3 \begin{bmatrix} \Lambda^4(\rho+2) & -\Lambda^3 & -\Lambda^3 \\ -\Lambda^3 & -\Lambda^4 + (1/2)\Lambda^2 + 1/2 & -\Lambda^4 + (1/2)\Lambda^2 - 1/2 \\ -\Lambda^3 & -\Lambda^4 + (1/2)\Lambda^2 - 1/2 & -\Lambda^4 + (1/2)\Lambda^2 + 1/2 \end{bmatrix} \tag{20}$$

where, using eq. (7), $\Lambda = \sqrt{(m_2/m_3)} = \sqrt{.14} = 0.37$ and

$$m_3 = \sqrt{(\Delta_{atm} + \Delta_{sol})} = 5 \times 10^{-2} eV. \tag{21}$$

While the analytic expressions are to the orders in Λ indicated, the data, unfortunately, are not. Thus, all results will be given to one significant figure only.

We now can determine the masses, m_1, m_2 and m_3 to order Λ^4:

$$m_3 = \sqrt{(\Delta_{atm} + \Delta_{sol})} = 5 \times 10^{-2} eV \tag{22}$$

$$m_2 = \Lambda^2 m_3 = \sqrt{(\Delta_{sol}/\Delta_{atm})} m_3 = 7 \times 10^{-3} eV \tag{23}$$

and

$$m_1 = \rho\Lambda^4 m_3 = \rho(\Delta_{sol}/\Delta_{atm}) m_3 = \rho \times 10^{-3} eV \tag{24}$$

We may let ρ range from, say, 2 to -2 and still consider $|m_1|$ to be of order Λ^4 or smaller. These masses follow directly from the hierarchy assumption and the experimental values: $\Delta_{sol} = m_2^2 - m_1^2 \approx 5 \times 10^{-5} (eV)^2$ and $\Delta_{atm} = m_3^2 - m_2^2 \approx 2.5 \times 10^{-3} (eV)^2$ and are independent of the mixing matrix U. The effective neutrino mass $< m >$ measured in $\beta\beta_{0\nu}$ decay, [3] however, does depend on the neutrino mass matrix, M_ν and is given by $M_{\nu_{e1}}$, the 1, 1 matrix element. From eq. (20) we have

$$< m > = M_{\nu_{e1}} = m_3 \Lambda^4 (\rho + 2) = (\rho + 2) 10^{-3} eV \tag{25}$$

Taking $\rho = 2 (m_1 = 2 \times 10^{-3} eV)$ as an extreme case, we have as an upper limit:

$$< m > \ \leq 4 \times 10^{-3} eV \tag{26}$$

On the other hand, m_1 and therefore ρ, may be negative. For $\rho = -2 (m_1 = -2 \times 10^{-3} eV)$, we have $M_{\nu_{e1}} = 0$ and $< m > = 0$. Thus, the limits on $< m >$ in this model are

$$0 < \ < m > \ < 4 \times 10^{-3} eV \tag{27}$$

which is consistent with a recent result [3], which gives as an upper limit

$$< m > \ \leq 8.2 \times 10^{-3} eV \tag{28}$$

If $< m >$ is found to be significantly larger than $5 \times 10^{-3} eV$, our simple hierarchical model fails.

3 Inclusion of $s_{13} = \sin(\theta_{13})$

While the present data do not demand a non-vanishing θ_{13}, several models of hierarchy generation do [2, 13]. We want to investigate the effect of a finite s_{13}.

Since we know that $s_{13} < 0.16$ [8, 9] and there is at present no lower limit, we will scale s_{13}:

$$s_{13} = \kappa\Lambda^2 \tag{29}$$

where $1 \geq |\kappa| \geq 0$.

Substituting for s_{13} in eq. (20) and forming the mass matrix M_ν, we obtain to order Λ^4:

$$M_{\nu_4} = m_3 \begin{bmatrix} \Lambda^4(\kappa^2+\rho+2) & -\Lambda^3 - \frac{1}{\sqrt{2}}\Lambda^2\kappa & -\Lambda^3 + \frac{1}{\sqrt{2}}\Lambda^2\kappa \\ -\Lambda^3 - \frac{1}{\sqrt{2}}\Lambda^2\kappa & -\frac{1}{2}\Lambda^4(\kappa^2+1) + \frac{1}{2}\Lambda^2 + \frac{1}{2} & \frac{1}{2}\Lambda^4(\kappa^2+1) + \frac{1}{2}\Lambda^2 - \frac{1}{2} \\ -\Lambda^3 + \frac{1}{\sqrt{2}}\Lambda^2\kappa & \frac{1}{2}\Lambda^4(\kappa^2-1) + \frac{1}{2}\Lambda^2 - \frac{1}{2} & -\frac{1}{2}\Lambda^4(\kappa^2+1) + \frac{1}{2}\Lambda^2 + \frac{1}{2} \end{bmatrix} \tag{30}$$

We now have two parameters, ρ and κ, where ρ is defined by $m_1 = \rho\Lambda^4 m_3$ and $s_{13} = \kappa\Lambda^2, |\kappa| \leq 1$. There are three special regimes for κ which are interesting.

1. $\kappa = 0, s_{13} = 0$ This is the case which was discussed earlier. We give here the leading elements of M_ν which depend on κ.

$$M_{\nu_{e1}} = \langle m \rangle = \Lambda^4(\rho+2) \tag{31}$$

$$M_{\nu_{e2}} = M_{\nu_{e3}} = -\Lambda^3 \tag{32}$$

2. $\kappa \approx 1$ (upper limit), $s_{13} = \Lambda^2$

$$M_{\nu_{e1}} = \langle m \rangle = \Lambda^4(\rho+3) \tag{33}$$

$$M_{\nu_{e2}} = -M_{\nu_{e3}} = \pm(1/\sqrt{2})\Lambda^2 \tag{34}$$

and the most interesting possibility,

3. $\kappa = \kappa'\Lambda$ i.e., $s_{13} = \kappa'\Lambda^3$. For case (3) the mass matrix is:

$$M_{\nu_4} = m_3 \begin{bmatrix} \Lambda^4(\rho+2) & -\Lambda^3\left(1+\frac{\kappa'}{\sqrt{2}}\right) & -\Lambda^3\left(1-\frac{\kappa'}{\sqrt{2}}\right) \\ -\Lambda^3\left(1+\frac{\kappa'}{\sqrt{2}}\right) & -\Lambda^4 + \Lambda^2/2 + \frac{1}{2} & -\Lambda^4 + \Lambda^2/2 - \frac{1}{2} \\ -\Lambda^3\left(1-\frac{\kappa'}{\sqrt{2}}\right) & -\Lambda^4 + \Lambda^2/2 - \frac{1}{2} & -\Lambda^4 + \Lambda^2/2 + \frac{1}{2} \end{bmatrix} \tag{35}$$

and

$$M_{\nu_{e2}} = -\Lambda^3[1 + (1/\sqrt{2})\kappa'] \tag{36}$$

$$M_{\nu_{e3}} = -\Lambda^3[1 - (1/\sqrt{2})\kappa'] \tag{37}$$

Case (3) is the only case which allows a zero in an off diagonal element. Texture zeros have been considered as a possible source of hierarchies and mixing angles [4, 5, 14, 15]. From Eq. (35), we see that only $\kappa = \kappa'\Lambda$ provides the possibility of having two texture zeroes.

Taking $\rho = -2$ and $\kappa' = \pm\sqrt{2}$ will make $M_{\nu_{e1}}$ and either $M_{\nu_{e2}}$ or $M_{\nu_{e3}}$ vanish to order Λ^4. With $\kappa' = -\sqrt{2}, M_{\nu_4}$ becomes

$$M_{\nu_4} = m_3 \begin{bmatrix} 0 & 0 & -2\Lambda^3 \\ 0 & -\Lambda^4 + \Lambda^2/2 + \frac{1}{2} & -\Lambda^4 + \Lambda^2/2 - \frac{1}{2} \\ -2\Lambda^3 & -\Lambda^4 + \Lambda^2/2 - \frac{1}{2} & -\Lambda^4 + \Lambda^2/2 + \frac{1}{2} \end{bmatrix} \tag{38}$$

In terms of masses we substitute $m_2 = \Lambda^2 m_3, m_1 = \rho\Lambda^4 m_3 = -2\Lambda^4 m_3$ and get

$$M_{\nu_4} = \begin{bmatrix} 0 & 0 & -\sqrt{(-2m_1m_2)} \\ 0 & \frac{1}{2}(m_1 + m_2 + m_3) & \frac{1}{2}(m_1 + m_2 - m_3) \\ -\sqrt{(-2m_1m_2)} & \frac{1}{2}(m_1 + m_2 - m_3) & \frac{1}{2}(m_1 + m_2 + m_3) \end{bmatrix} \tag{39}$$

Eq. (39) is identical to the matrix derived for the hierarchical case by B. R. Desai et. al.[15], who systematically categorize the neutrino mass matrices, consistent with experimental constraints, with two texture zeros. Note that for this model $< m >= M_{\nu_{e1}}$ vanishes. In order to compare with a recent model for hierarchy generation,[4] we continue with $\rho = -2$, but do not specify κ' in $s_{13} = \kappa'\Lambda^3$. In that case , keeping only the leading order in Λ in each matrix element of M_{ν_6}, we obtain :

$$M_{\nu_6} = m_3 \begin{bmatrix} \Lambda^6(4 + \kappa'^2) & -\Lambda^3(1 + \frac{1}{\sqrt{2}}\kappa') & -\Lambda^3(1 - \frac{1}{\sqrt{2}}\kappa') \\ -\Lambda^3(1 + \frac{1}{\sqrt{2}}\kappa') & \frac{1}{2} & -\frac{1}{2} \\ -\Lambda^3(1 - \frac{1}{\sqrt{2}}\kappa') & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{40}$$

The leading orders of Λ in each matrix element are the orders indicated in the work of Ramond, et al.,[4] (and 'tuned' by Fishbane and Kaus [16]). This model suggests , within a super symmetric extension of the standard model, that the existence of mass hierarchies within fermionic sectors imply at least one additional $U(1)$ family symmetry one of which must be anomalous, with a cancellation of its anomaly through the Green-Schwarz mechanism then implying relations across fermionic sectors. This has the additional property of predicting Λ, which should be the same for all families. However, the data for neutrinos, Δ_{atm} and Δ_{sol}, suggest that $\Lambda \approx 0.38$, while for the other family sectors, we have the traditional $\lambda \approx 0.25$; the data may still change to bring Λ in line with λ.

4 Summary

We have shown that the assumed hierarchy pattern and the present data imply that a mixing matrix, U, and mass matrix, M_ν may be expressed in terms of powers of the expansion parameter Λ and two parameters ρ and κ of order one, where ρ expresses our ignorance of the lightest neutrino mass, m_1, where $m_1 = \rho\Lambda^4 m3 \approx \rho 10^{-3}eV$ and κ scales s_{13} to the experimental upper limit, $s_{13} = \kappa\Lambda^2 \approx 0.14\kappa$. The expansion parameter, Λ, where $\Lambda^2 = m_2/m_3 = \sqrt{(}\Delta_{sol}/\Delta_{atm} \approx 0.14$ is identical in spirit, though not in value, to the Wolfenstein parameter [11], λ, in the quark V_{CKM} and is measured by solar and atmospheric oscillation experiments. We obtain an upper limit of $4 \times 10^{-3}eV$ for the effective neutrino mass that may be obtained in double β-decay experiments.

The models of hierarchical mass generation that we compared to, supersymmetric extension [4] and texture zeroes [4, 5, 14, 15] each want $M_{\nu_{e1}}$ to be of order Λ^6 or even vanish. This implies $\rho = -2$ from Eq. (30), i.e., $m_1 \approx -2 \times 10^{-3} eV$ and $< m > \approx 0$. Both of these models require the $M_{\nu_{e2}}$ and $M_{\nu_{e3}}$ terms to be of order Λ^3 or smaller. Therefore s_{13} in Eq. (30) has to be of order Λ^3, i.e., $s_{13} = \kappa' \Lambda^3 = 0.05\kappa'$, where κ' is of order unity or smaller. More specifically, for a texture zero in $M_{\nu_{e2}}$ or $M_{\nu_{e3}}$ one must have $\kappa' = \pm\sqrt{2}$ or $s_{13} \approx 0.07$. All these demands are well below the present upper experimental upper limits of $< m >$ and s_{13}.

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