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Quite Hydraulic Actuation Bellows Design Considerations

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Xyz

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Quite Hydraulic Actuation Bellows Design Considerations

Over the course of the last three weeks, we have been testing the quite hydraulic actuation bellows for conformance as well as suitability. We have had the luxury of testing three different bellows: one that was as designed, on that was annealed as a result of a vacuum brazing procedure, and one that was manufactured to physical dimensions that were incorrect as well as unfavorable.

I have gathered together a set of design calculations to compare and qualify them with experimental data that Joe Lacour (Kineoptics) has afforded us.

Spring Constant

The *Standards of The Expansion Joint Manufactures Association, Inc. (EJMA)* defines the bellows theoretical initial elastic spring rate f_i as:

$$f_i = 1.7 \frac{D_m E_b t_p^3 n}{w^3 C_f}$$

Where:

D_b = Inside diameter of cylindrical tangent and bellows convolution (in)

E_b = Modulus of Elasticity for the bellows (psi)

t_p = Bellows thickness for one ply corrected for thinning during forming(in)

n = Number of bellows material ply thickness

w = Convolution height minus the material thickness nt (in)

C_f = Factor used to relate U-shaped convolution segments to a simple strip beam

I calculated the spring rate for each of the three types of bellows, two of which had different geometries and on which had a different material property

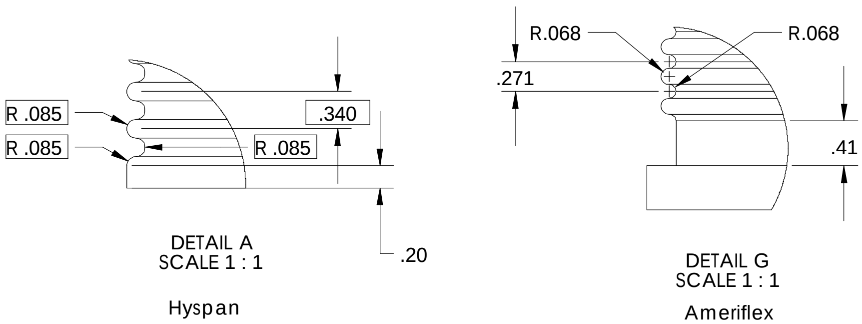

Figure 1 Hyspan vs. Ameriflex Bellows Geometry

Bellows	Spring Rate	% difference from the "as designed" bellows
As designed (304L)	55.15 lb/in	0
As designed (17-7PH)	57.91 lb/in	5.004%
Ameriflex as built (304L)	63.15 lb/in	14.506%

The effect of using a slightly different material was small as compared with the effect of changing the physical dimensions. Of primary interest was the effect of changing the material. The spring constant is directly proportional to Young's Modulus which is reported as being 29,000 psi for 304L stainless steel and 30,450 psi for 17-7 PH stainless steel (a precipitation hardened steel). Since the effect of changing the material is so small (Young's modulus is 30,000 for all carbon steels) a different material that has a higher strength might become an attractive alternative.

Bellows Circumferential Membrane Stress due to Pressure

EJMA states, "Excessive hoop stress in the straight cylindrical end tangent of a bellows will cause circumferential yielding" and defines it as:

$$S_1 = \frac{P(D_b + nt)^2 L_t E_b k}{2(ntE_b L_t (D_b + nt) + t_c k E_c L_c D_c)}$$

Where:

S_1 = Bellows Tangent Circumferential Membrane Stress due to Pressure
P = Pressure (psig)
D_b = Inside diameter of cylindrical tangent and bellows convolution (in)

L_t = Bellows Tangent Length (in)

Dc = Mean diameter of bellows reinforcing tangent collar (in)

k = Stiffening Effects Factor

t = bellows nominal material thickness for 1 ply (in)

t_c = Bellows tangent reinforcing collar material thickness (in)

L_c = Bellows Tangent Collar Length (in)

This is a modification of the Barlow equation, which defines hoop stress as:

$$s_t = \frac{PD}{2t}$$

For the sake of comparison, I omitted the collar effect to better pronounce the different bellows geometries:

$$S_1 = \frac{P(D_b + nt)^2 L_t E_b k}{2ntE_b L_t (D_b + nt)}$$

Or more simply

$$S_1 = \frac{P(D_b + nt)\cancel{^2}\cancel{L_t E_b}k}{2nt\cancel{E_b L_t}\cancel{(D_b + nt)}} \qquad S_1 = \frac{P(D_b + nt)k}{2nt}$$

Which looks much more like the hoop stress equation.

Results of the equations yield:

Bellows Tangent Circumferential Membrane Stress due to Pressure

S_1	304L	17-7 PH	As built by Ameriflex
	P x (321.135) psi	P x (321.135) psi	P x (873.57) psi
@70 psi	22,479.45 psi	22,479.45 psi	61,150 psi
@90 psi	28,902.15 psi	28,902.15 psi	78,621 psi
@140 psi	44,958.90 psi	44,958.90 psi	122,300 psi

It's obvious that material has no effect on the circumferential tangential membrane stress as all of the Young's modulus affects cancel out. However the change in the geometry, especially the change in the geometry of the tangential tab yields a huge difference in the resulting stress. The results for the "as built" bellows are consistent with the test that Joe Lacour performed on the as built bellows. At 90 psi, the bellows end buckled along the straight cylindrical end tangents due to excessive hoop stress. The main effect was dominated by the length of the tangent section, L_t and is effectively squared in the numerator because it is also present in the stiffening factor “k”.

Circumferential Membrane Stress

EJMA also supplies another modification of the Barlow equation that takes into account the effect for the bellows convolution geometry. The equation is defined as:

$$S_2 = \frac{PD_m}{2nt_p}\left(\frac{1}{0.571 + 2w/q}\right)$$

And takes into account the convolution height (*w*) and the bellows pitch (*q*).
The results of these calculations are as follows:

S_2	304L & 17-7 PH
	P x (291.48) psi
@70 psi	20,403.5 psi
@90 psi	26,233.08 psi
@140 psi	40,807.01 psi

Substantial pressures do result from these convolution geometry effects and the two circumferential pressures S_1 and S_2 are combined and compared against the allowable stress by:

$$S_1 + S_2 \leq C_{wb} S_{ab}$$

Where C_{wb} is a longitudinal weld joint efficiency factor, which will be assumed to be 1 for the practicality of this exercise and S_{ab}, is the allowable stress for the bellows.

	$S_1 + S_2$	$C_{wb}S_{ab}$ (304L typical Values)	$C_{wb}S_{ab}$ (17-7 PH typical values)
@70 psi	42,882.95	29,700 psi (annealed)	50,000 psi
@90 psi	55,135.23	80,000 psi (work hardened)	229,875 psi
@140 psi	85,165.9	100,000 psi (work hardened high value)	

It’s obvious that an annealed 304L stainless steel could never be an attractive alternative as our experimentation has proven via Joe Lacour (the annealed bellows failed between 90 psi to 130 psi, which correlates to about 70,000 to 90,000 psi for failure stress). The work-hardened 304L (the present solution) does exceed the combined circumferential stresses. However several caveats apply:

1) I am unsure about the longitudinal weld joint efficiency factor. I feel comfortable stating that it would probably be higher than .75, but that I have no empirical data to back this up nor do I have any values afforded by the fabricator at this time.

2) The reported yield strength values are reported as functions of the amount of cold work applied to the annealed metal (i.e. 10%, 40% etc.). With the limited quantity of bellows that we have produced, we have not been able to quantify the hardness due to the work hardening and are not able to quantify it against historical data. A reasonable solution is to ask the vendor for any historical data that they have produced to help us better understand the work-hardening process.

While the none of the 304L conditions provide attractive safety factors, (safety factor range is 1.86 to 2.33 at the design pressure and .93 to 1.17 for 2x the working pressure) the 17-7 PH has a very attractive range and which entertains safety factors as high as 5.36 at working pressure and 2.68 for 2x working pressure.

Additionally, these stresses are going to be fluctuating so the possibility of fatigue should be at least entertained if not seriously considered. After the proceeding section on Meridional Stresses and life expectancy, I will further explore fatigue doe to fluctuating hoop stress (circumferential stress)

Bellows Meridional Membrane and Bending Stress due to Pressure

Typically, Meridional stresses due to internal pressure and deflection is responsible for the bellows life expectancy. The membrane Meridional stress due to pressure appears to be a local hoop stress that includes the effects of any one convolution. The bellows Meridional membrane stress is defined as follows:

$$S_3 = \frac{Pw}{2nt_p}$$

Which yields the following results:

	304L & 17-7 PH
S_3	P x (13.381) psi
@70 psi	963.67 psi
@90 psi	1,204.29 psi
@140 psi	1,605.72 psi

These stresses are combined with the meridional bending stress due to pressure, which is defined as:

$$S_4 = \frac{PD_m}{2nt_p}\left(\frac{w}{t_p}\right)^2 C_p$$

And yields the following results:

	304L & 17-7 PH
S_4	P x (236.35) psi
@70 psi	16,544 psi
@90 psi	21,181.5 psi
@140 psi	32,949 psi

The two meridional pressures S_3 and S_4 are combined and compared against the allowable stress by:

$$S_3 + S_4 \leq C_m S_{ab}$$

Where C_m is a material strength factor for austenitic metals and is 1.5 for annealed stainless steel bellows and 3.0 for bellows in the as-formed condition (work hardened).

	$S_3 + S_4$	C_mS_{ab} (annealed)	C_mS_{ab} (cold worked)
@70 psi	17,507.67	44550 psi (annealed)	89,100 psi
@90 psi	22,385.79	120000 psi (work hardened)	240,000 psi
@140 psi	34,554.72	150000 psi (work hardened high value)	300,000
		75,000 psi (17-7 PH low values)	150,000 psi (17-7 PH low values)
		344,812.5 psi (17-7 PH high values)	689,625 psi (17-7 PH high values)

None of the combined meridional stresses are in jeopardy of overcoming the factored allowable stress and really doesn't merit much discussion unless the bellows pressure was substantially increased. Of particular note is the fact that the work hardening of the bellows has the affect of increasing the bellows strength by at least a factor of two. This will become quite important in the fatigue portion of the analysis, which will be illustrated shortly.

Bellows Meridional Membrane & Bending Stress due to Deflection

The stresses that arise as a result of deflection are so small that they are hardly worth mentioning. However, for the sake of completion of this exercise (and the fact that they are represented in the fatigue life equation), I will include their evaluation with a short discussion.

S_5 the meridional membrane stress due to deflection is defined as follows:

$$S_5 = \frac{E_b t_p^2 e}{2w^3 C_f}$$

And S_6 ,the meridional bending stress due to deflection is defined as follows:

$$S_5 = \frac{5E_b t_p e}{3w^2 C_d}$$

Where e is the total equivalent axial movement per convolution and C_f and C_d are factors used to relate U-shaped convolution segments to a simple strip beam.

For a deflection of 180 microns (which is a nominal value for operation) yields the following membrane stresses:

S_5 (meridional membrane stress due to deflection)	304L	17-7 PH
@ 180 microns	.045 psi	.0467 psi
@ 1.5 mm	.375 psi	.389 psi
@ 150 mm	375 psi	389 psi

Although the stresses that result from meridional bending stresses are 100 times greater than the corresponding membrane stress. They are still quite small as compared to their corresponding pressure stress:

S_6 (meridional bending stresses due to deflection)	304L	17-7 PH
@ 180 microns	2.532 psi	2.65 psi
@ 1.5 mm	21.025 psi	22.08 psi
@ 150 mm	2102.5 psi	2208.33 psi

Of course the only difference in the material deflections are due to the difference in Young's Modulus.

Fatigue Life

EJMA has derived an expression for fatigue life for unreinforced bellows:

$$N_c = \left(\frac{c}{S_t - b} \right)^a$$

Where S_t is a combination of all meridional stresses and is combined by the following:

$S_t = 0.7(S_3 + S_4) + (S_5 + S_6)$

And a = 3.4, b = 54,000, and c = 1.86 x 10^6 for a unreinforced bellows (per *EJMA*)

	$S_t = 0.7(S_3 + S_4) + (S_5 + S_6)$	N
@70 psi	17,510.238 psi	$\leq 54{,}000 \therefore \infty$
@90 psi	22,388.358 psi	$\leq 54{,}000 \therefore \infty$
@140 psi	34557.288 psi	$\leq 54{,}000 \therefore \infty$

In each case, the combined stresses are less than 54,000 so the bellows should have an infinite life.

It is quite clear that effects of the meridional stress in the bellows are small and not worth much mention. However, the hoop stresses still are large enough to be a concern. Although EJMA only defines fatigue life analysis in terms of meridional stress, they do mention that other types of stresses, if significant, can cause fatigue failures. I have applied classic fatigue failure analysis to the resulting hoop (circumferential) stresses by using the Modified Goodman diagram and comparing it to results from the Circumferential Stresses (hoop stress) and to results from Finite Element Analysis (FEA) by comparing Von Mises stress and the Maximum principal stress.

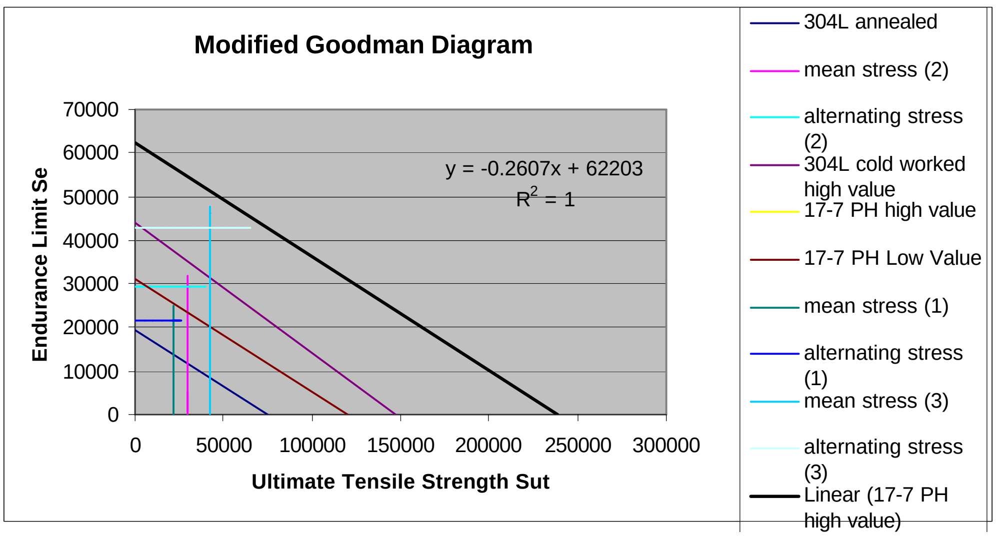

Figure 2 Modified Goodman Diagram for Several Endurance Limits

Typically, the mean and alternating values of the principal stresses are plot along with line that relate the ultimate tensile strength S_{ut} and the endurance limit S_e (the endurance limit is found by multiplying the ultimate tensile strength by .5 for most carbon steels and then derating that value by a series of factors that take into account fabrication, stress concentration, temperature effects, etc. See Mechanical Element Design by Shigley for a more in-depth explanation). The alternating principal stress is plotted along the horizontal axis and the mean stress is plotted along the vertical. The point at which the two lines intersect is the evaluation point (or operating point). If this point is above the endurance line, then the object exceeds its endurance limit and is prone to failure by fatigue. Of course, even evaluation points below the endurance line can fail if the stress is marginal. It is good engineering practice to introduce a safety factor in to the endurance line or directly in to the principal stress.

The Goodman diagram does not give any indication of life expectancy. Most of the techniques that that estimate maximum cycles to failure are experimental in nature and exceed the scope of this evaluation.

This particular Goodman Diagram relates two different stainless steels (304L and 17-7 PH) and uses high and low values for both. 304 L Stainless Steel in its annealed state has and endurance limit of 19,500 psi and is below all of the intersecting stress lines so it can quickly be dismissed. The next endurance line is for 17-7 PH and illustrates the low endurance value of the precipitation-hardened stainless steel. The endurance limit for this line is 31, 290 psi and the first intersection point, which is a Von Mises Stress at 90 psi and .180 microns of deflection is at least marginally satisfied. It, however does not satisfy the next two evaluation points which are maximum principal stresses at 2x the operating pressure (140 psi) and the third is the results of the computed circumferential pressure at 2x the operating pressure. The only line that satisfies all three of the conditions is the last line which is the upper limit of 17-7 PH which has an ultimate strength of 238,554 psi and an endurance limit of 62,202.95 psi. The evaluation intersection point intersects at x= 42,83 and y=42,883. The equivalent stress s_e for the circumferential stress has a slope equal to the 17-7 PH high value endurance line and can easily be found by simple algebra to be 59, 999 psi. Comparing this to the endurance limit of 17-7 PH stainless steel, the factor of safety is 2.3 against fatigue and 4.25 as compared to the yield strength (229,875 psi).

While both of these factors of safety are reasonable, I should comment on two issues:

1) That the upper limit of the 17-7 PH ultimate and tensile strengths are predicated by the final heat treat recommendation as prescribed by the vendor, which we have not had the luxury to evaluate yet. All this means is that we are unfamiliar with the process and this uncertainty makes me slightly apprehensive about advertising such high strengths for 17-7 PH, which justifies a high safety factor.

2) That there are certain factors that could easily diminish of increase the endurance limit that I calculated. I was quite conservative in my factor selection and feel that

they would only increase with more empirical data. However, the numbers are still somewhat arbitrary and gathered with only heuristics.

My recommendation would be to use 17-7 PH and try to approach the higher strength limits through recommended heat treat as opposed to relying upon the cold working process for 304L stainless steel. Another added benefit of this approach is the collar and the bellows can be vacuum braised together and then heat treated to achieve the desired mechanical properties. The 17-7 PH stainless steel yields a provocative approach to our current bellows concerns and while I think that the 304L stainless steel bellows are adequate for LASTI in the spirit of technology evaluation, I would feel that 17-7 PH bellows would be a more robust solution for LIGO Seismic Retrofit implementation.

I would also recommend the procurement of a 17-7 PH bellows pathfinder to gain experience with the fabrication and joining process as well as the performance.