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Thermal Noise in a Violin Mode Polarized Along the Optic Axis for a Single-Loop Suspension

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Thermal Noise in Violin Mode Along Optic Axis for a Single Loop Suspension

FJR
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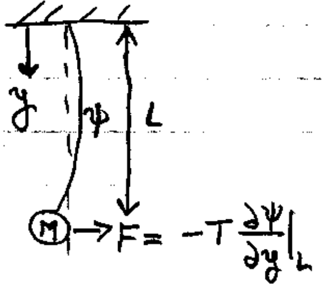
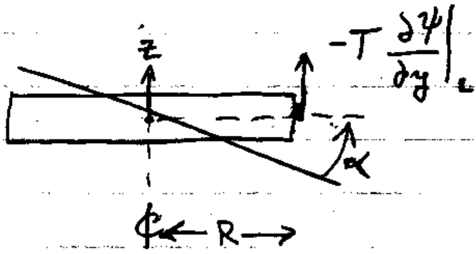




Center of Mass Motion } $-T\frac{\partial\psi}{\partial y}\Big|_L = M\ddot{z}$

Rotation About Center of Mass } $-RT\frac{\partial\psi}{\partial y}\Big|_L = I\ddot{\alpha} = \xi M R^2 \ddot{\alpha}$

Solution for Suspension Wire } $\psi_n(y,t) = A_n \underbrace{\sin(k_n y)}_{\text{clamped at } y=0} e^{i\omega_n t}$

$$\frac{\partial\psi_n}{\partial y}\Big|_L = k_n A_n \cos(k_n L) e^{i\omega_n t}$$

Boundary Condition } $z_n + \alpha_n R = A_n \sin(k_n L)$

Let $z = z_n e^{i\omega_n t}$

$\alpha = \alpha_n e^{i\omega_n t}$

$$-T\frac{\partial\psi}{\partial y}\Big|_L = -M\omega_n^2 z_n e^{i\omega t}$$

$$-T\frac{\partial\psi}{\partial y}\Big|_L = -\xi M R \omega_n^2 \alpha_n e^{i\omega t}$$

$$\Rightarrow \alpha_n = \frac{z_n}{\xi R}$$

① $$\Rightarrow z_n\left(1 + \frac{1}{\xi}\right) = A_n \sin(k_n L) = z_n \wp$$

$$-T\frac{\partial\Psi}{\partial z}\Big|_L = -M\omega_n^2\,\frac{A_n}{\zeta}\,\sin(k_nL)$$

$$T = \frac{1}{2}Mg$$

$$\frac{1}{2}\cancel{M}g\cdot k_n\,\cancel{A_n}\cos(k_nL) = \cancel{M}\omega_n^2\,\frac{\cancel{A_n}}{\zeta}\,\sin(k_nL)$$

$k_nL = n\pi + \delta k_n L$

$$\frac{\zeta}{2}\,\frac{g\,k_n}{\omega_n^2}\,\cancel{\cos(n\pi)} \simeq \cancel{\cos(n\pi)}\cdot\delta k_n\cdot L$$

$$\frac{\delta k_n}{k_n} = \frac{\zeta}{2}\cdot\frac{g}{L}\cdot\frac{1}{\omega_n^2} = \frac{\delta\omega_n}{\omega_n}$$

$$\Rightarrow \boxed{\left(\frac{\delta\omega_n}{\delta k_n}\right) = \frac{\zeta}{2}\cdot\frac{\omega_p^2}{\omega_n^2}\cdot\left(\frac{\omega_n}{k_n}\right)}$$

Frequency shift due to finite-mass termination

$$z_n\zeta = A_n\sin(k_nL) = A_n\cdot\cos(n\pi)\,\delta k_n L$$

$$= -A_n\cdot\frac{\zeta}{2}\cdot\frac{\omega_p^2}{\omega_n^2}\cdot k_n L$$

Recoil of Mirror from Vibrating Wire

$$\Rightarrow \boxed{z_n = -A_n\cdot\frac{n\pi}{2}\cdot\left(\frac{\omega_p}{\omega_n}\right)^2}$$

RMS Thermal Noise in Wire Violin Mode

$$\frac{1}{2}m_w\,\omega_n^2\,\langle A_n^2\rangle = \frac{1}{2}kT$$

where m_w is mass of free segment of wire

$$\Rightarrow \langle A_n^2 \rangle^{1/2} = \left(\frac{kT}{m_w}\right)^{1/2} \frac{1}{\omega_n}$$

$$\boxed{\langle z_n^2 \rangle^{1/2} = \frac{n\pi}{2\omega_n}\left(\frac{\omega_p}{\omega_n}\right)^2\left(\frac{kT}{m_w}\right)^{1/2} = \frac{n}{f_n}\left(\frac{f_p}{f_n}\right)^2\left(\frac{kT}{m_w}\right)^{1/2}}$$

For LIGO Arm-Cavity Mirrors

wire density $\rho = 7.9\ g/cm^2$

$d = 0.012\ in \cdot 2.54\ cm/in$

$L = 45\ cm$

$$m_w = \rho \cdot \frac{\pi}{4} \cdot d^2 \cdot L = 0.26g = 2.6 \times 10^{-4}\ kg$$

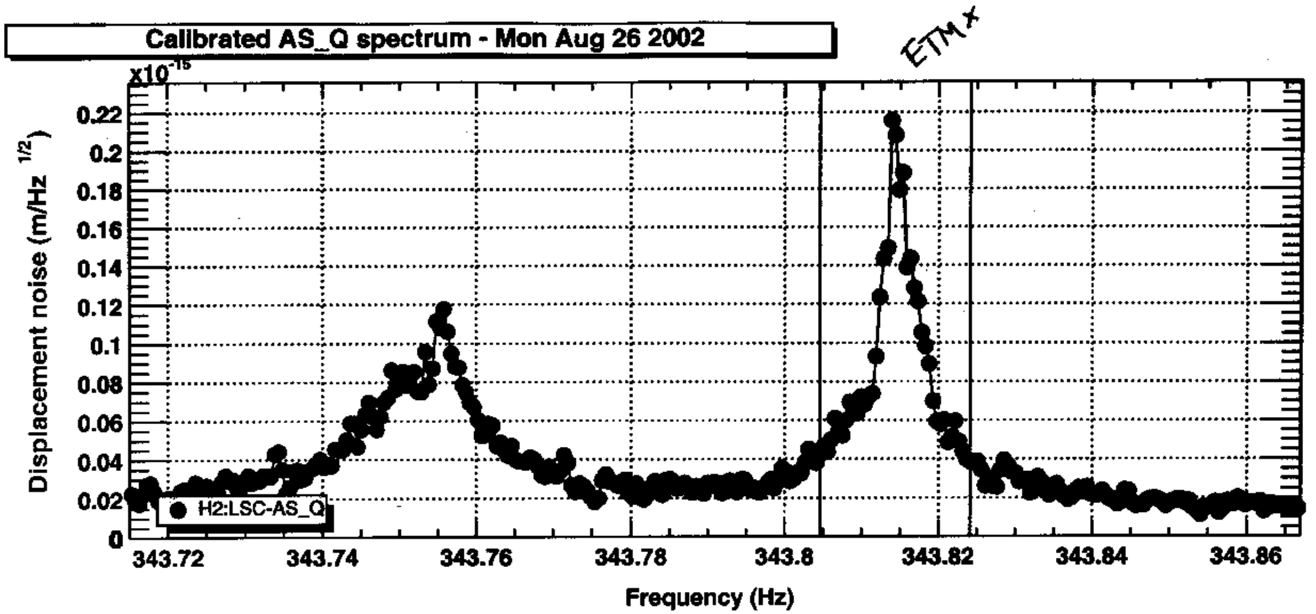
for 1st violin mode $n = 1$, $f_1 = 344\ Hz$, $f_p = 0.74 Hz$

$$\Rightarrow \boxed{\langle z_1^2 \rangle^{1/2} = 1.34 \times 10^{-17}\ m}$$

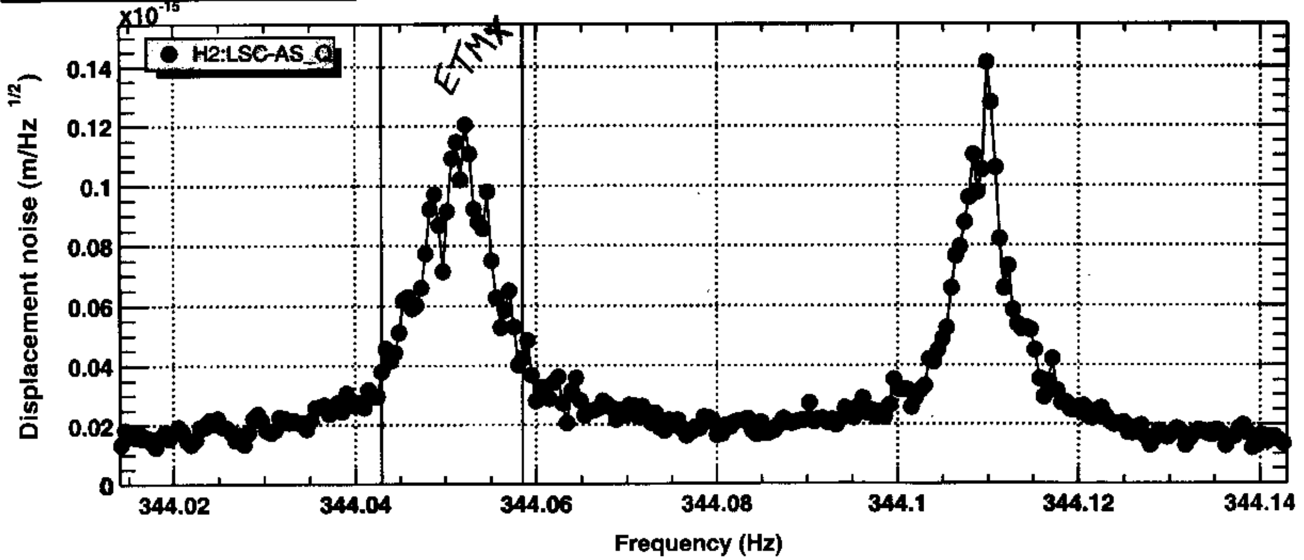
<u>H2: ETMX</u> <u>Observations</u>* (Identification by Raab/Vorvick, LHO e-log, 10/16/01)

f_1 (Hz)	$\langle z_1^2 \rangle^{1/2}$ (m)	Q
343.814	1.4×10^{-17}	~80,000
344.051	1.0×10^{-17}	~ 70,000

* Calibrated AS_Q spectrum; T_0 = 26 Aug 2002 02:05:00 UTC; Channel Separation = 0.0005 Hz; BW = 0.000732 Hz; 16 Averages; Refer to LHO e-log Mon, 26 Aug 02, 17:21:23 PDT.


Calibrated AS_Q spectrum - Mon Aug 26 2002

Power spectrum



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