LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

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IO POSITION AND ANGLE SENSING SYSTEM 40M IFO

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Abstract

1 Introduction

The 40m interferometer IO beam diagnostics optical system provides instrumentation to continuously monitor the following characteristics of the beam incident on the input mode cleaner: power level, beam profile, rf side-band frequencies, beam pointing angle, and beam position.

The purpose of this technical note is to describe the design approach and to show the performance characteristics of the IO beam angle and position sensors. An alignment method and calibration procedure will be described.

This document will present a matrix-formalism for calculating Gaussian beam properties and Gaussian beam transformations by arbitrary optical systems. The formalism was used to design the IO beam angle and position sensors

2 IO Position and Angle Sensing System

2.1 40m Interferometer IO Diagnostic System Optical Layout

The optical layout of the 40m interferometer IO diagnostic system is shown in Figure 1; it includes the Position and Angle Monitoring Subsystems.

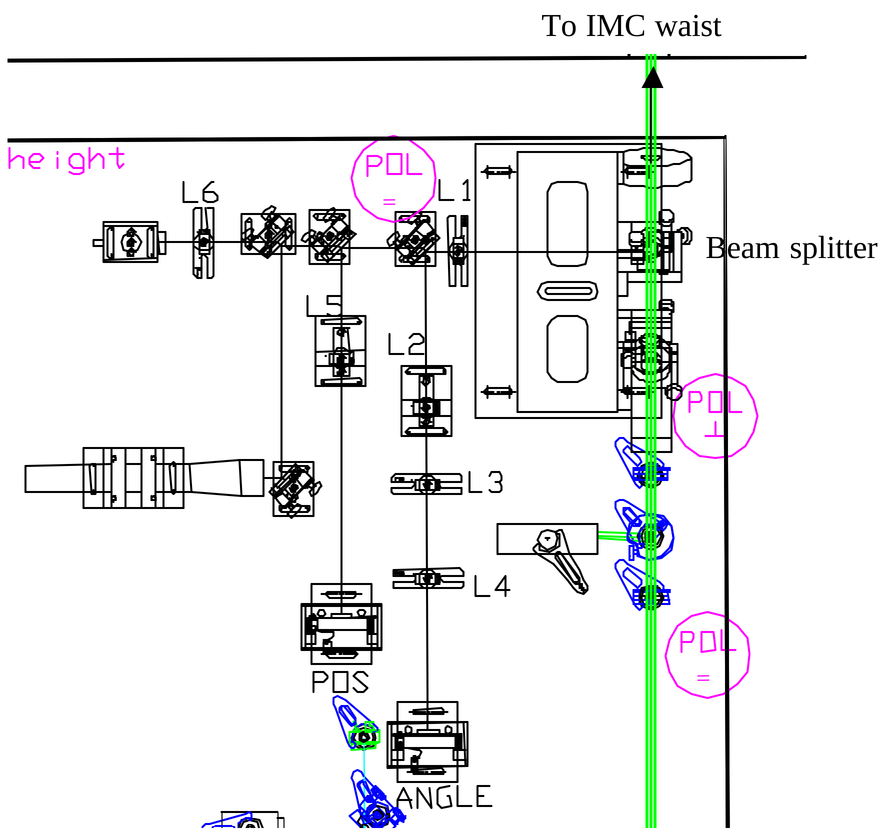

Figure 1: IO Position and Angle Monitoring System

2.2 Angle Sensor

The principle of the angle sensor is based on the property that the position of a focused laser beam at the focal plane of a lens depends only on the angle of the beam with respect to the optical axis, and is independent of the displacement of the beam from the optical axis.

The angle sensing system consists of the following elements: 1) a beam splitter to sample a portion of the input beam incident on the input mode cleaner (IMC), 2) a beam-reducing, afocal telescope formed by lenses L1 and L2 to magnify the angular deviation of the input beam, 3) a telecentric focusing lens, L3, located a focal length away from the output of the telescope, to form a spot at a virtual detection plane located at the focal plane of L3, and 4) an imaging lens, L4, that forms a magnified image of the spot at the virtual detection plane on the angle quad photo diode (QPD) to match the size of the QPD.

The following parameters were used in the final design:

focal length lens1, mm	$f_1 := 229.1$
focal length of lens2, mm	$f_2 := 11.6$
focal length of lens3, mm	$f_3 := 85.8$
focal length of lens4, mm	$f_4 := 29.1$
distance from IMC to beam splitter, mm	$l_1 := 1871 - 113$
height difference between IMC and IO beam	$\Delta h := h_{imc} - h_{io}$
distance from beam splitter to lens1, mm	$l_2 := \Delta h + 197$
distance from lens1 to lens2, mm	$l_5(f_2, \delta 2) := f_1 + f_2 + \delta 2$
distance from lens2 to lens3	$l_{23}(f_2, f_3) := f_2 + f_3$
distance from lens3 to angle QPD image, mm	$l_8(f_3, \delta 3) := f_3 + \delta 3$
magnification of angle sensing signal	$m_{ang} := \dfrac{2}{0.352}$
distance from QPD image to L4, mm	$l_{ol4}(m_{ang}) := \dfrac{(m_{ang} + 1) \cdot f_4}{m_{ang}}$
diatance from L3 to L4, mm	$d_{l3l4} := l_8(f_3, \delta 3) + l_{ol4}(m_{ang})$
distance from L4 to ang QPD, mm	$l_{l4qpd}(m_{ang}) := m_{ang} \cdot l_{ol4}(m_{ang})$

2.2.1 Calculation of Spot Size at the Angle Sensor QPD

system matrix at focus of L3

$$M_{l3}(\delta2,\delta3,f_2,f_3) := M_{s8}(f_3,\delta3)\cdot ML3(f_3)\cdot M_{s6}(f_2,f_3)\cdot ML2(f_2)\cdot M_{s5}(f_2,\delta2)\cdot ML1(f_1)\cdot M_{imc1}$$

$$M_{l3}(\delta2,\delta3,f_2,f_3) = \begin{pmatrix} 0 & -1.695\times10^{3} \\ 5.901\times10^{-4} & 1.135 \end{pmatrix}$$

Spot size at focus of L3

$$A(f_3) := M_{l3}(\delta2,\delta3,f_2,f_3)_{0,0} \qquad B(f_3) := M_{l3}(\delta2,\delta3,f_2,f_3)_{0,1}$$

$$C(f_3) := M_{l3}(\delta2,\delta3,f_2,f_3)_{1,0} \qquad D(f_3) := M_{l3}(\delta2,\delta3,f_2,f_3)_{1,1}$$

$$w_{f3}(f_3) := \left[\frac{\lambda}{\pi}\cdot\frac{-(B(f_3))^2-(A(f_3))^2\cdot\left(\pi\cdot\frac{w_{01}{}^2}{\lambda}\right)^2}{\pi\cdot\frac{w_{01}{}^2}{\lambda}\cdot\left(B(f_3)\cdot C(f_3)-A(f_3)\cdot D(f_3)\right)}\right]^{0.5}$$

$$f_3 := 85.8 \qquad w_{f3}(f_3) = 0.352$$

system matrix at QPD

$$M_a(f_3) := M_{l4qpd}(f_3)\cdot ML4(f_4)\cdot M_{l4}(f_3)\cdot M_{l3}(\delta2,\delta3,f_2,f_3)$$

$$M_a(f_3) = \begin{pmatrix} 0 & 9.62\times10^{3} \\ -1.04\times10^{-4} & 58.032 \end{pmatrix}$$

Spot size at focus of angle QPD

$$A(f_3) := M_a(f_3)_{0,0} \qquad B(f_3) := M_a(f_3)_{0,1}$$

$$C(f_3) := M_a(f_3)_{1,0} \qquad D(f_3) := M_a(f_3)_{1,1}$$

$$w_a(f_3) := \left[\frac{\lambda}{\pi}\cdot\frac{-(B(f_3))^2-(A(f_3))^2\cdot\left(\pi\cdot\frac{w_{01}{}^2}{\lambda}\right)^2}{\pi\cdot\frac{w_{01}{}^2}{\lambda}\cdot\left(B(f_3)\cdot C(f_3)-A(f_3)\cdot D(f_3)\right)}\right]^{0.5}$$

spot size at angle QPD, mm $\quad w_a(f_3) = 2$

2.2.2 Height Versus Beam Angle and Displacement at Angle Sensor QPD

The beam height at the QPD is a function of the beam angle and displacement at the position of the IMC; it can be calculated using the ABCD matrix.

$$\begin{pmatrix} h_a \\ \alpha_a \end{pmatrix} := \begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} h_1 \\ \alpha_1 \end{pmatrix}$$

$$h_a(h_1, \alpha_1) := M_a(f_3)_{0,0} \cdot h_1 + M_a(f_3)_{0,1} \cdot \alpha_1$$

The linear response of spot height versus beam angle for a perfectly aligned sensor is shown in Figure 2. Note that three graphs with different beam displacements of h_1 = -1mm, h_1 = 0mm, h_1 = 1mm are plotted on the graph and the curves are exactly on top of each other, indicating that the angle sensor is not sensitive to beam displacement. The angle sensor is capable of measuring a minimum angle of approximately $1\text{x}10^{-6}$ rad, which is limited by the minimum measurable displacement of approximately 0.01mm on the QPD.

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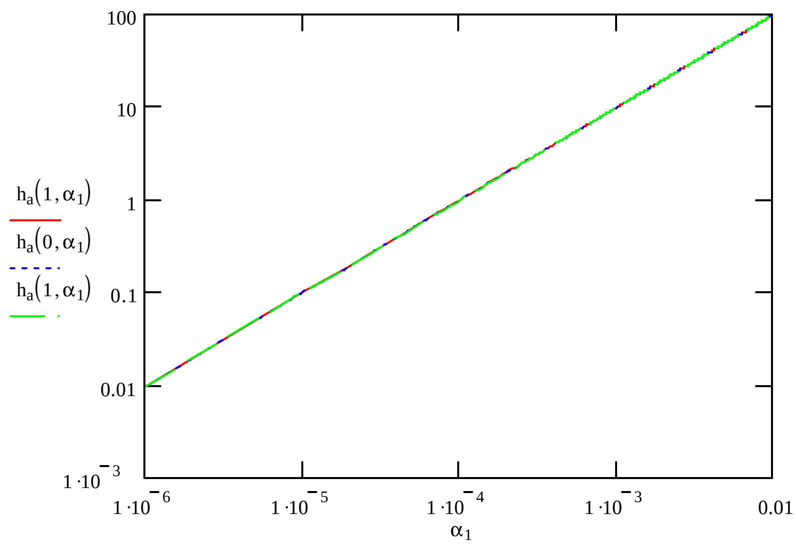

Figure 2: Angle sensor, QPD spot height versus beam angle

2.2.3 Cross-coupling Sensitivity of Angle Sensor to Position Input

The response of the angle sensor to a position input is caused by the error in positioning L4, as shown in Figure 3. In order to limit the cross-coupling of the angle sensor to < 0.0002 rad, which corresponds to the divergence angle of the IMC beam waist, for a position input of 1 mm, the error in the exact position of L4 must be < 0.05 mm.

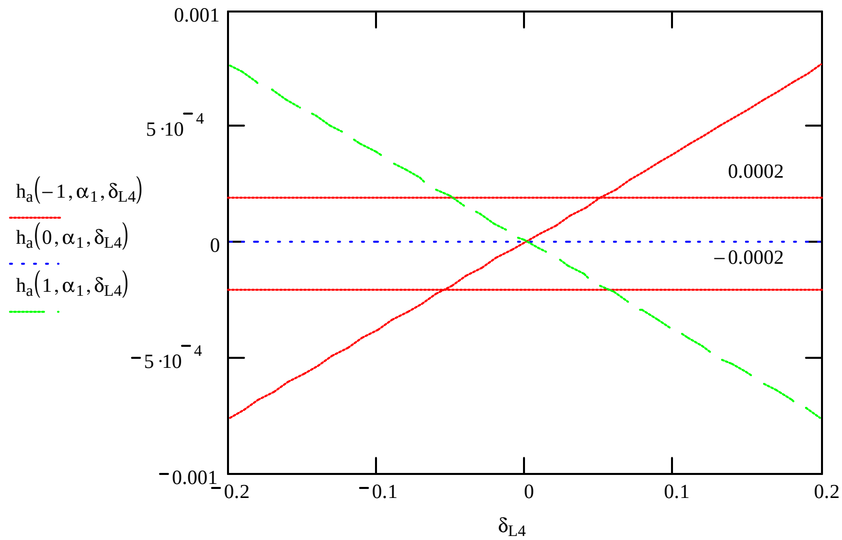

Figure 3: Position response of angle sensor versus position error of L4

2.3 Position Sensor

The principle of the position sensor is based on the formation of an image of the beam waist of the IMC at the QPD, so that the displacement of the spot at the QPD is proportional to the displacement of the beam at the IMC and is independent of the angular deviation of the beam from the optical axis.

The position sensing system consists of the following elements: 1) a beam splitter to sample a portion of the input beam incident on the input mode cleaner (IMC), 2) an imaging lens, L1, to form an image of the IMC beam waist at a virtual detection plane, and 3) an imaging lens, L5, that forms a magnified image of the spot at the virtual detection plane on the position quad photo diode (QPD) to match the size of the QPD.

The following parameters were used in the final design:

focal length of lens5, mm	$f_5 := 29.1$
focal length of lens6, mm	$f_6 := -29.1$
magnification of pos sensing signal	$m_{pos} := \frac{2}{0.194}$
distance from object plane to L5, mm	$l_{ol5}(m_{pos}) := \frac{(m_{pos}+1)\cdot f_5}{m_{pos}}$
distance from L5 to pos QPD, mm	$l_{l5qpd}(m_{pos}) := m_{pos}\cdot l_{ol5}(m_{pos})$

2.3.1 Calculation of Spot Size at the Position Sensor QPD

system matrix at image of L1

$$M_p(\delta1, f_1) := M_{s3}(\delta1) \cdot ML1(f_1) \cdot M_{imc1}$$

$$M_p(\delta1, f_1) = \begin{pmatrix} -0.119 & 2.484 \times 10^{-13} \\ -4.365 \times 10^{-3} & -8.393 \end{pmatrix}$$

Spot size at image of L1

$$A := M_p(\delta1, f_1)_{0,0} \qquad B := M_p(\delta1, f_1)_{0,1}$$

$$C := M_p(\delta1, f_1)_{1,0} \qquad D := M_p(\delta1, f_1)_{1,1}$$

$$w_{f1} := \left[\frac{\lambda}{\pi} \cdot \frac{-(B)^2 - (A)^2 \cdot \left(\pi \cdot \frac{w_{01}{}^2}{\lambda}\right)^2}{\pi \cdot \frac{w_{01}{}^2}{\lambda} \cdot (B \cdot C - A \cdot D)}\right]^{0.5}$$

$$w_{f1} = 0.194$$

system matrix at pos QPD

$$M_{pos}(m_{pos}) := M_{l5qpd}(m_{pos}) \cdot ML5(f_5) \cdot M_{l5}(m_{pos}) \cdot M_p(\delta1, f_1)$$

$$M_{pos}(m_{pos}) = \begin{pmatrix} 1.228 & -2.547 \times 10^{-12} \\ 4.518 \times 10^{-3} & 0.814 \end{pmatrix}$$

Spot size at pos QPD

$$A(m_{pos}) := M_{pos}(m_{pos})_{0,0} \qquad B(m_{pos}) := M_{pos}(m_{pos})_{0,1}$$

$$C(m_{pos}) := M_{pos}(m_{pos})_{1,0} \qquad D(m_{pos}) := M_{pos}(m_{pos})_{1,1}$$

$$w_p(m_{pos}) := \left[\frac{\lambda}{\pi} \cdot \frac{-\left(B(m_{pos})\right)^2 - \left(A(m_{pos})\right)^2 \cdot \left(\pi \cdot \frac{w_{01}{}^2}{\lambda}\right)^2}{\pi \cdot \frac{w_{01}{}^2}{\lambda} \cdot \left(B(m_{pos}) \cdot C(m_{pos}) - A(m_{pos}) \cdot D(m_{pos})\right)}\right]^{0.5}$$

spot size at angle QPD, mm $\quad w_p(m_{pos}) = 2$

2.3.2 Height Versus Beam Angle and Displacement at Position Sensor QPD

The beam height at the QPD is a function of the beam angle and displacement at the position of the IMC; it can be calculated using the ABCD matrix.

$$\begin{pmatrix} h_p \\ \alpha_p \end{pmatrix} := \begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} h_1 \\ \alpha_1 \end{pmatrix}$$

$$h_p(h_1, \alpha_1) := M_{pos}(m_{pos})_{0,0} \cdot h_1 + M_{pos}(m_{pos})_{0,1} \cdot \alpha_1$$

The linear response of spot height versus beam position is shown in Figure 4. Note that three graphs with different beam angles of a_1 = -0.01rad, a_1 = 0 rad, a_1 = 0.01 rad are plotted on the graph and the curves are exactly on top of each other, indicating that the position sensor is not sensitive to beam angle. The position sensor is capable of measuring a beam displacement of less than 0.01mm, which is limited by the minimum measurable displacement of approximately 0.01mm on the QPD.

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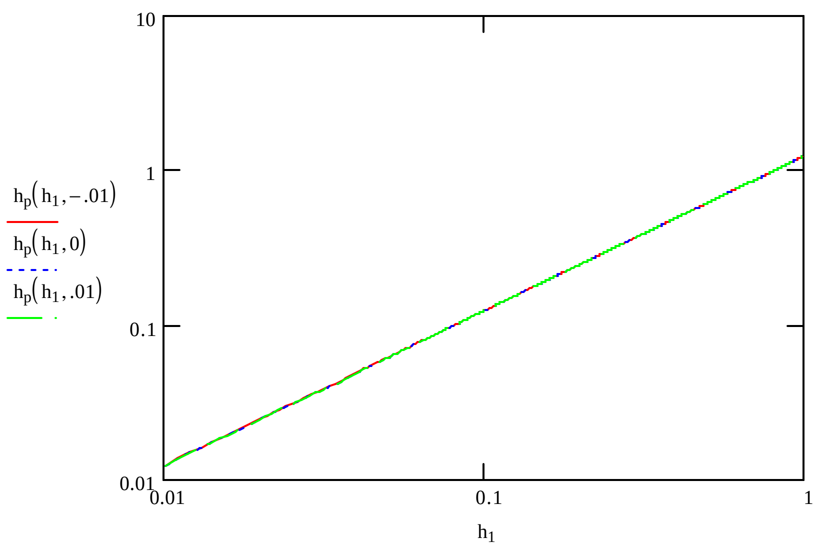


Figure 4: Position sensor, QPD spot height versus beam position

2.3.3 Cross-coupling Sensitivity of Position Sensor to Angle Input

The response of the position sensor to an angle input is caused by the error in positioning L5, as shown in Figure 5. In order to limit the cross-coupling of the position sensor to < 0.16 mm, which corresponds to 1/10 the beam waist size of the IMC beam waist, for an angle input of 0.001 rad, the error in the exact position of L5 must be < 2 mm.

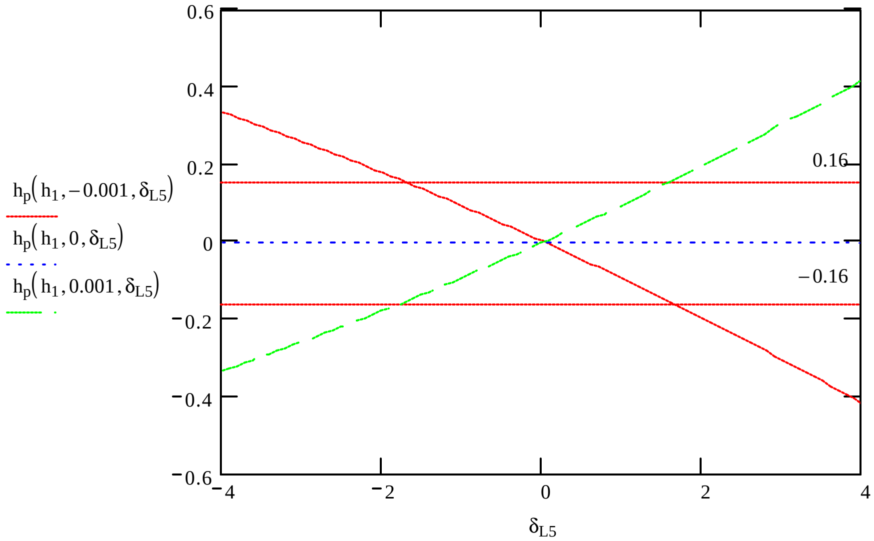


Figure 5: Angle response of position sensor versus position error of L5

3 Alignment and Calibration Procedures

The angle sensor and position sensor can be aligned and calibrated by introducing a known angle deviation and a known displacement of the IO beam.

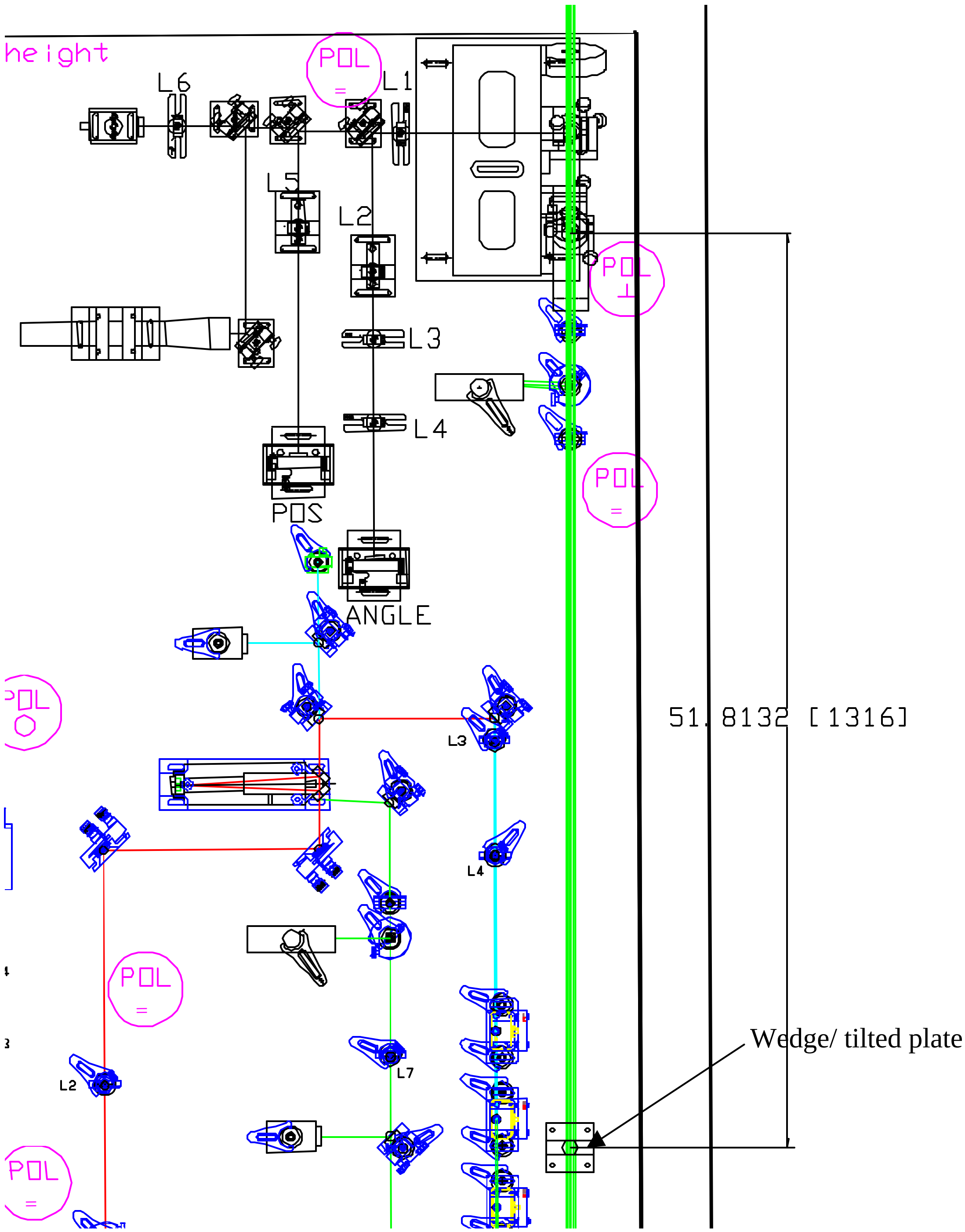


Figure 6: IO Sensor Alignment

3.1 Angle Deviation Caused by a Wedged Plate

The wedge angle of a wedged optical plate, with a wedge of T_d < 5/60 degrees, can be measured with an autocollimator. The angular deviation of the IO beam due to the small wedge angle is given by

$$T_d = T_w(n-1),$$

where n is the refractive index of the plate.

3.2 Displacement Deviation Due to a Tilted Plate

By tilting the wedged optical plate, the IO beam will be displaced laterally by the amount

$$?x = t \sin (T_o) - t \tan(T_i) \cos(T_o),$$

Where t is the thickness of the plate, T_o is the tilt angle of the plate, and T_i is the internal angle given by

$$T_i = \mathrm{asin} (\sin (T_o)/n)$$

3.3 Angle Sensor Alignment and Calibration

Place the wedged optical plate in the IO beam path at a location that is the same distance away from L1 as the distance to the IMC beam waist, as indicated in Figure 6.

3.3.1 Alignment of Angle Sensor L4

Tilt the plate +20 and –20 degrees and record the readings of the angle sensor QPD output. Average the two readings to obtain the null offset reading of the angle sensor; this null offset angle reading is caused by the wedge of the optical plate. Adjust the position of L4 until the tilt of the plate does not cause the angle sensor output to change.

3.3.2 Calibration of Angle Sensor

Set the tilt to zero. Rotate the wedged plate one full revolution about the axis of the beam and record the readings of the angle sensor. The maximum angle readings obtained in the horizontal and vertical directions should be symmetric about zero and are equal to the angular deviation caused by the known wedge.

3.4 Position Sensor

Place the wedged optical plate in the IO beam path at a location that is the same distance away from L1 as the distance to the IMC beam waist as indicated in Figure 6.

3.4.1 Alignment of Position Sensor L5

Set the tilt to zero. Rotate the wedged plate one full revolution about the axis of the beam and record the readings of the position sensor. Adjust the position of L5 until the rotation of the plate does not cause the position sensor output to change.

3.4.2 Calibration of Position Sensor

Tilt the plate +20 and –20 degrees and record the readings of the position sensor QPD output. The maximum position readings obtained in the horizontal direction should be symmetric about zero and are equal to the position deviations caused by the tilted plate.

4 Appendix A: Gaussian Beam Propagation

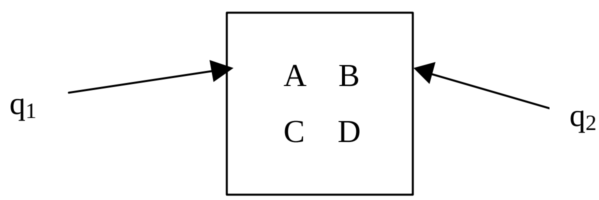
4.1 Gaussian Transformation by ABCD Matrix

The complex wave front curvature of a Gaussian beam is defined as follows[1]:

$$\frac{1}{q} := \frac{1}{R} + i\cdot\frac{\lambda}{\pi\cdot w^2}$$

R is the real radius of the spherical wavefront, and the imaginary term is the reciprocal of the Rayleigh range. The spot size w is the radius of the beam cross-section where the radial intensity has decreased to $1/e^2$ of the peak intensity.

The complex wavefront curvature, q, is transformed by a paraxial optical system, described by an ABCD matrix, in the following manner.



$$q_2 := \frac{A\cdot q_1 + B}{C\cdot q_1 + D}$$

If the input wavefront is at a beam waist, $1/R_1$ is zero and the complex curvature q_1 is purely imaginary.

$$q_1 := -i\cdot\frac{\pi\cdot w_{01}^2}{\lambda}$$

By writing the reciprocal of the transformed wavefront curvature q_2 in standard complex form,

$$\frac{1}{q_2} := \frac{A\cdot C\cdot q_1^2 + B\cdot D}{A^2\cdot q_1^2 + B^2} + \frac{(B\cdot C - A\cdot D)\cdot q_1}{A^2\cdot q_1^2 + B^2}$$

The real and imaginary terms can be related to the spherical radius and the spot size respectively as follows:

$$\frac{1}{R_2} := \frac{A\cdot C\cdot q_1^2 + B\cdot D}{A^2\cdot q_1^2 + B^2}$$

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[1] *Introduction to Optics*, F. L. Pedrotti and L. S. Pedrotti, 2nd Ed., 1993, Prentice Hall (New Jersey); A. E. Siegman in *Lasers*, 1986, University Science Books (California) defines the imaginary term with a minus sign; however, both definitions yield identical results.

$$i \cdot \frac{\lambda}{\pi \cdot w_2^{\,2}} := \frac{(B \cdot C - A \cdot D) \cdot q_1}{A^2 \cdot q_1^{\,2} + B^2}$$

Then, the transformed spot size w_2 and the radius of the spherical wavefront can be evaluated in terms of the input waist size w_{01}.

$$w_2 := \frac{\lambda}{\pi \cdot w_{01}} \cdot \left[\frac{\left(\frac{\pi \cdot w_{01}^{\,2}}{\lambda} \right)^2 \cdot A^2 + B^2}{A \cdot D - B \cdot C} \right]^{\frac{1}{2}}$$

$$R := \frac{A^2 \cdot \left(\frac{\pi \cdot w_{01}^{\,2}}{\lambda} \right)^2 + B^2}{A \cdot C \cdot \left(\frac{\pi \cdot w_{01}^{\,2}}{\lambda} \right)^2 + B \cdot D}$$

The ABCD system matrix for a complex optical system comprised of thick lenses, thin lenses, and mirrors is calculated by the ordered multiplication of each component matrix along the optical path.

$$M_{system} := M_n \cdot M_3 \cdot M_2 \cdot M_1$$

$$M_{system} := \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

Typical component matrices are the following:

Translation
$$M_T := \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

Thin lens
$$M_L := \begin{pmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{pmatrix}$$

Curved mirror
$$M_M := \begin{pmatrix} 1 & 0 \\ \frac{-2}{R} & 1 \end{pmatrix}$$

Thick lens surface 1
$$M_{TL1} := \begin{pmatrix} 1 & 0 \\ \frac{1-n}{n \cdot R1} & \frac{1}{n} \end{pmatrix}$$

Thick lens surface 2
$$M_{TL2} := \begin{pmatrix} 1 & 0 \\ \frac{n-1}{R2} & n \end{pmatrix}$$

4.2 Location and Size of the Beam Waist

The location of the beam waist from the last focusing element of an optical system can be calculated by explicitly writing the system matrix as the product of a sub-matrix that includes all the optical components including the last focusing element multiplied by the final translation matrix to the beam waist.

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} := \begin{pmatrix} 1 & l_0 \\ 0 & 1 \end{pmatrix} \cdot M_x$$

Carrying out the matrix multiplication, the ABCD elements can be identified.

$$A := M_{x_{0,0}} + l_0 \cdot M_{x_{1,0}} \qquad C := M_{x_{1,0}}$$

$$B := M_{x_{0,1}} + l_0 \cdot M_{x_{1,1}} \qquad A := M_{x_{1,1}}$$

At the beam waist, $1/R_2$ is zero, leading to the following condition that can be solved for l_0.

$$A \cdot C \cdot q_1^{\,2} + B \cdot D := 0$$

$$l_0 := \frac{-\left(\frac{\pi \cdot w_{01}^{\,2}}{\lambda}\right)^2 \cdot M_{x_{0,0}} \cdot M_{x_{1,0}} - M_{x_{0,1}} \cdot M_{x_{1,1}}}{\left(\frac{\pi \cdot w_{01}^{\,2}}{\lambda}\right)^2 \cdot \left(M_{x_{1,0}}\right)^2 + \left(M_{x_{1,1}}\right)^2}$$

The size of the beam waist can be calculated as before by using the full system matrix to the beam waist.

$$w_{02} := \frac{\lambda}{\pi \cdot w_{01}} \cdot \left[\frac{\left(\frac{\pi \cdot w_{01}^{\,2}}{\lambda}\right)^2 \cdot A^2 + B^2}{A \cdot D - B \cdot C}\right]^{\frac{1}{2}}$$