LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

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Thermal Noise Calculation with Inhomogeneous Loss using the Finite Element Method:

Application to Test Mass Optics with Coating Loss, Attachments and Composite Assemblies

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ROUGH DRAFT – INCOMPLETE

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Abstract

Several approaches using the Finite Element Method (FEM) to calculate of the low frequency thermal noise level for test masses are explored and applied to problems of relevance to LIGO.

1 Direct Calculation Methods

There are at least three methods by which a finite element based analysis may be used to calculate the thermal noise in a continuum with inhomogeneous loss:

1) Modal expansion: Similar to the modal expansion method used by Gillespie and Raab[1], but with the additional projection of the nodal damping matrix into modal space. This projection is performed in some finite element codes such as SDRC's IDEAS[2]. The resulting damping factors for each mode are then taken into account in the modal summation. This is tantamount to Yamamoto's[3] "advanced mode expansion" method.

2) Direct Integration: Many finite element codes can time integrate the governing equations including spatially varying structural (hysteric) or viscous damping, directly without performing a modal decomposition first. The response to a cycling varying Gaussian pressure applied to the front surface (mimicking the laser read-out), or the step response to the application of this force can then be used to determine the thermal noise.

3) Direct calculation of the generalized, low frequency (static) admittance: A generalization of Y. Levin's method using the finite element method, where the dissipation is the weighted sum of the strain energy and loss in each element. This approach is explored in this memorandum.

1.1 Homogeneous loss

Summarize Y. Levin's approach here

1.2 Extension for inhomogeneous loss

Summarize FEA analysis for the inhomogeneous case, especially method to incorporate coating loss -- TBW

2 Applications

2.1 Infinite half-space approximation (with homogeneous loss)

To be done

[1] A. Gillespie and F. Raab, Phys. Rev. D, 52, 577 (1995).

[2] Need reference for SDRC's IDEAS

[3] K. Yamamoto, "Study of the thermal noise caused by inhomogeneously distributed loss", Ph.D. thesis, Dept. of Physics, University of Tokyo, Dec 2000.

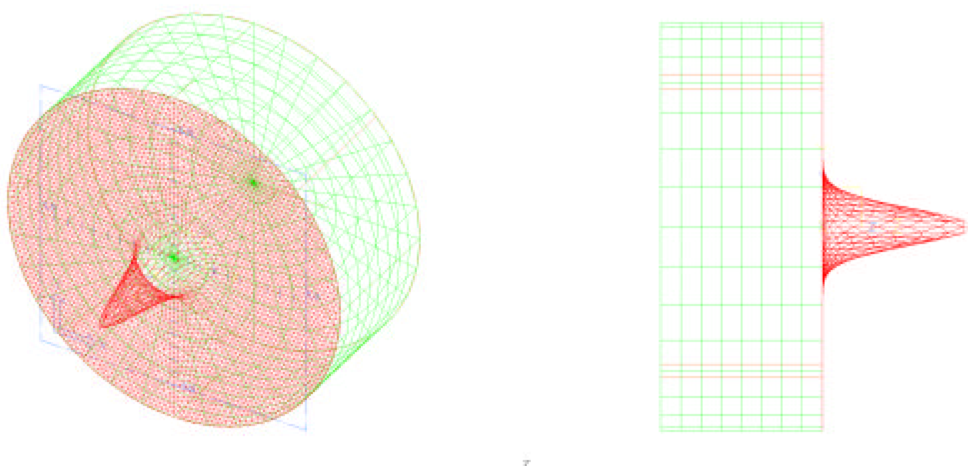
2.2 LIGO Test Mass: Right circular cylinder (with homogeneous loss)

For comparison with Gillespie and Rabb, Levin and Bondu et. al., the elastic strain energy for a Gaussian pressure load on an initial LIGO test mass (right circular cylinder with a radius of 0.125 m and a thickness of 0.100 m) has been calculated. The Gaussian pressure load results in a total force of 1 N, distributed as:

$$f(r)=\frac{1}{pr_o^2}e^{-r^2/r_o^2}$$

where ro = 1.56 cm is the Gaussian beam radius used by Gillespie and Raab. The material properties for the fused silica test mass are a modulus of elasticity, E = 71.8 GPa, Poisson's ratio, μ = 0.16, shear modulus, G = E/(2 (1+μ)) (isotropic material) and a structural damping loss factor of $\varphi = 10^{-7}$.. The applied pressure function and a typical mesh for the LIGO test mass is shown in the following figure.

Figure 1 Finite Element Mesh & Applied Gaussian Pressure Function: LIGO Test Mass



The FEA results are compared to the values reported in the literature in the following table. The deformation under the Gaussian pressure load and a contour of the elastic strain energy are given in the following figures. From the Table of results it appears that the best strategy is to refine the mesh in the region near the peak of the Gaussian pressure (say to a radius of $\approx \sqrt{2}r_o$ and a depth of $\approx 3r_o$) and to use parabolic tetrahedral elements (at least if using SDRC's IDEAS code). Note that the parabolic brick and wedge elements also converged to the same strain energy but at a much higher computational cost. If one does not need to model 3D aspects of the test mass (for example magnet and wire standoffs and wedge angles), then an axisymmetric model is computationally the best with either parabolic triangular or parabolic quad elements.

Figure 2 LIGO Test Mass, Deformation under a Gaussian Pressure Load

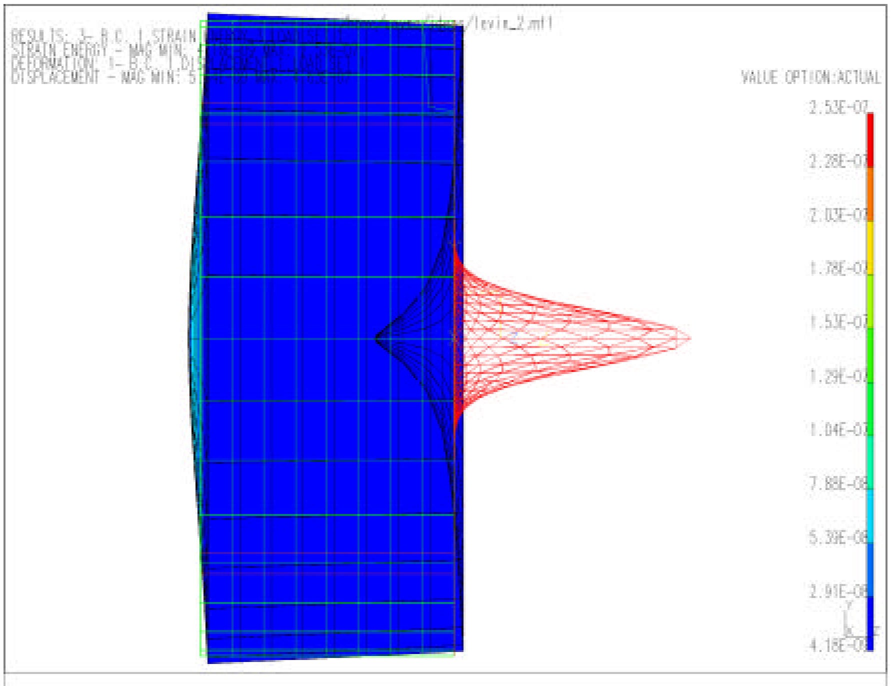


Figure 3 LIGO Test Mass, Contour of the Strain Energy Density: Face View

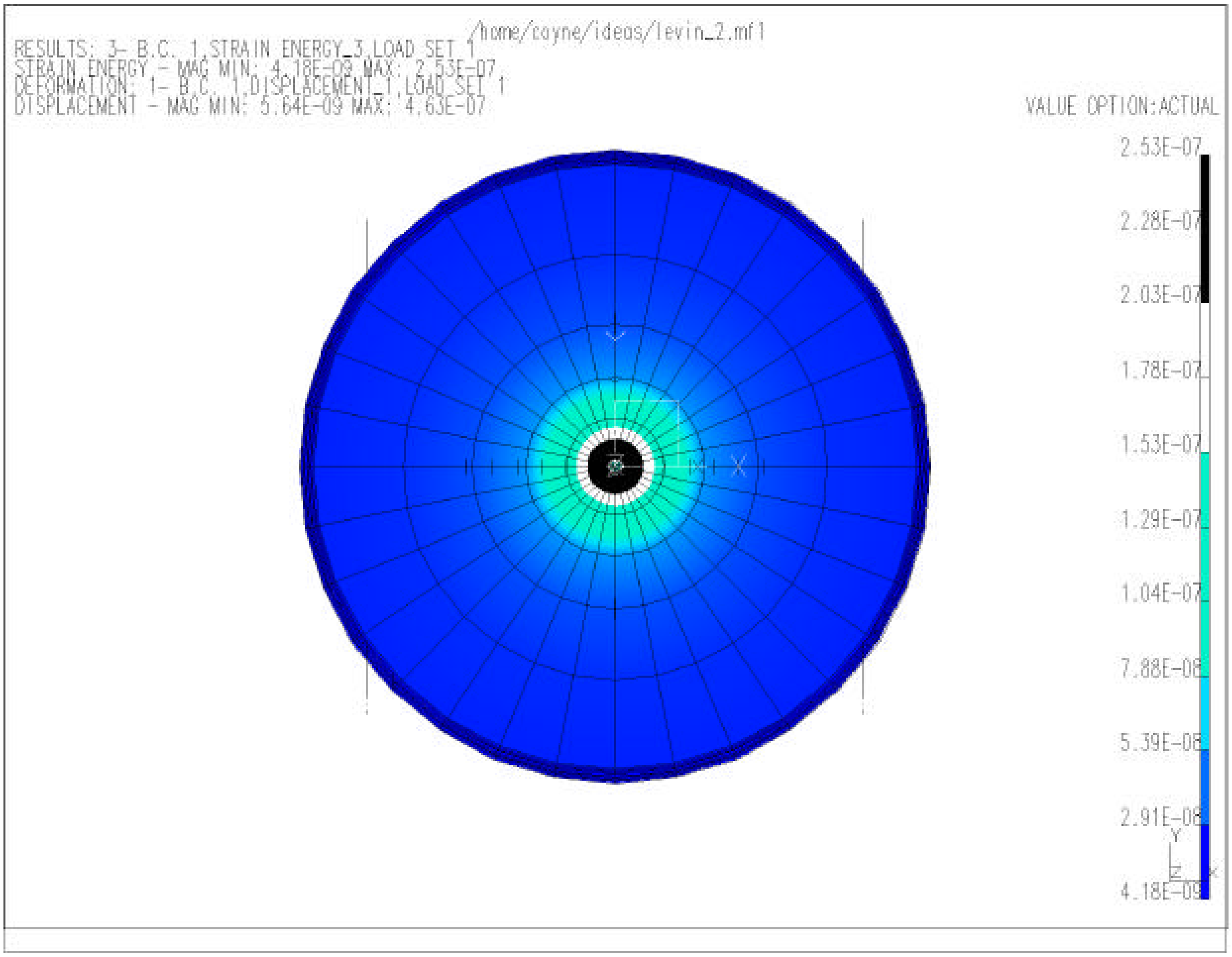


Figure 4 LIGO Test Mass, Contour of the Strain Energy Density: Cross-sectional View

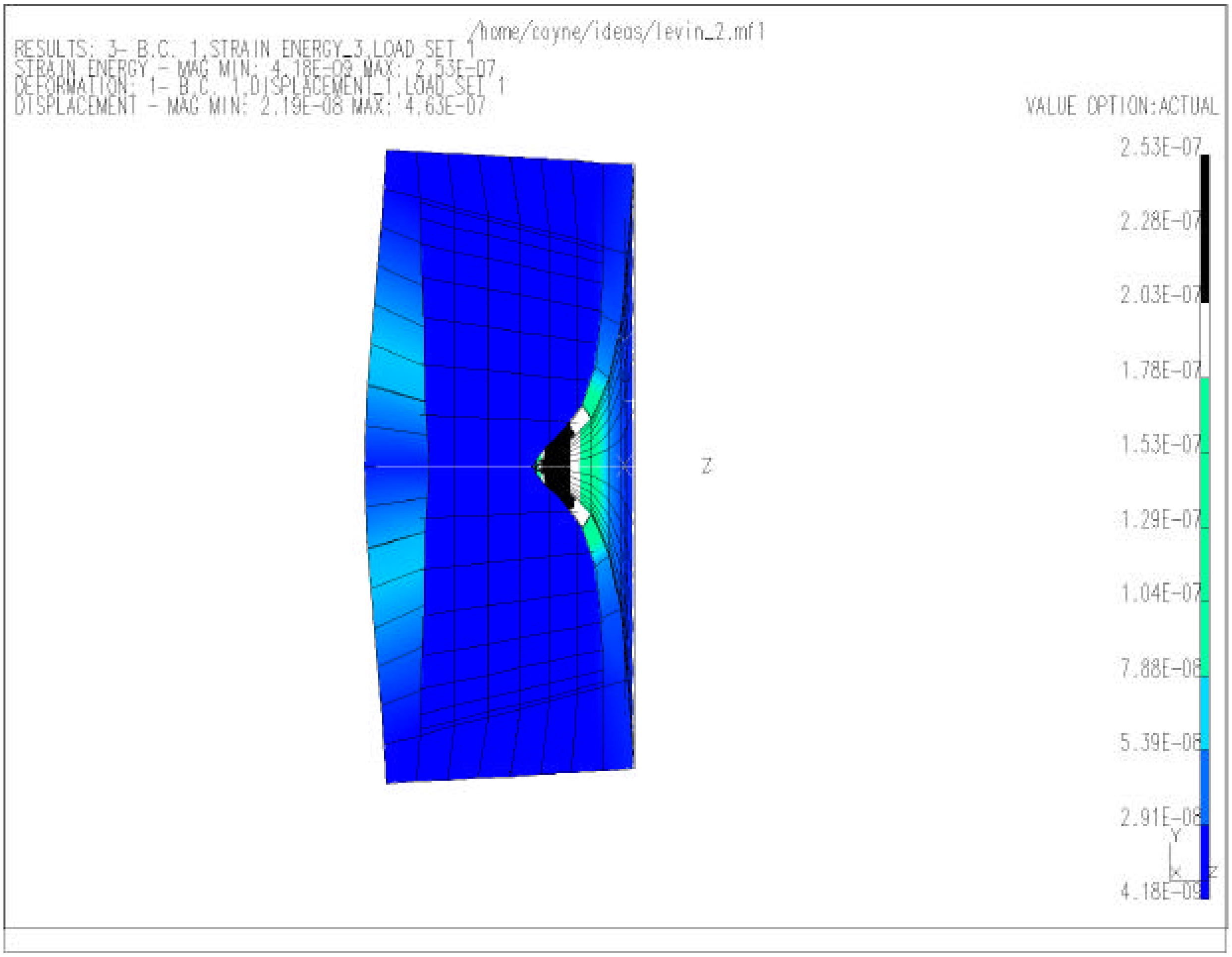


Figure 5 Axisymmetric model

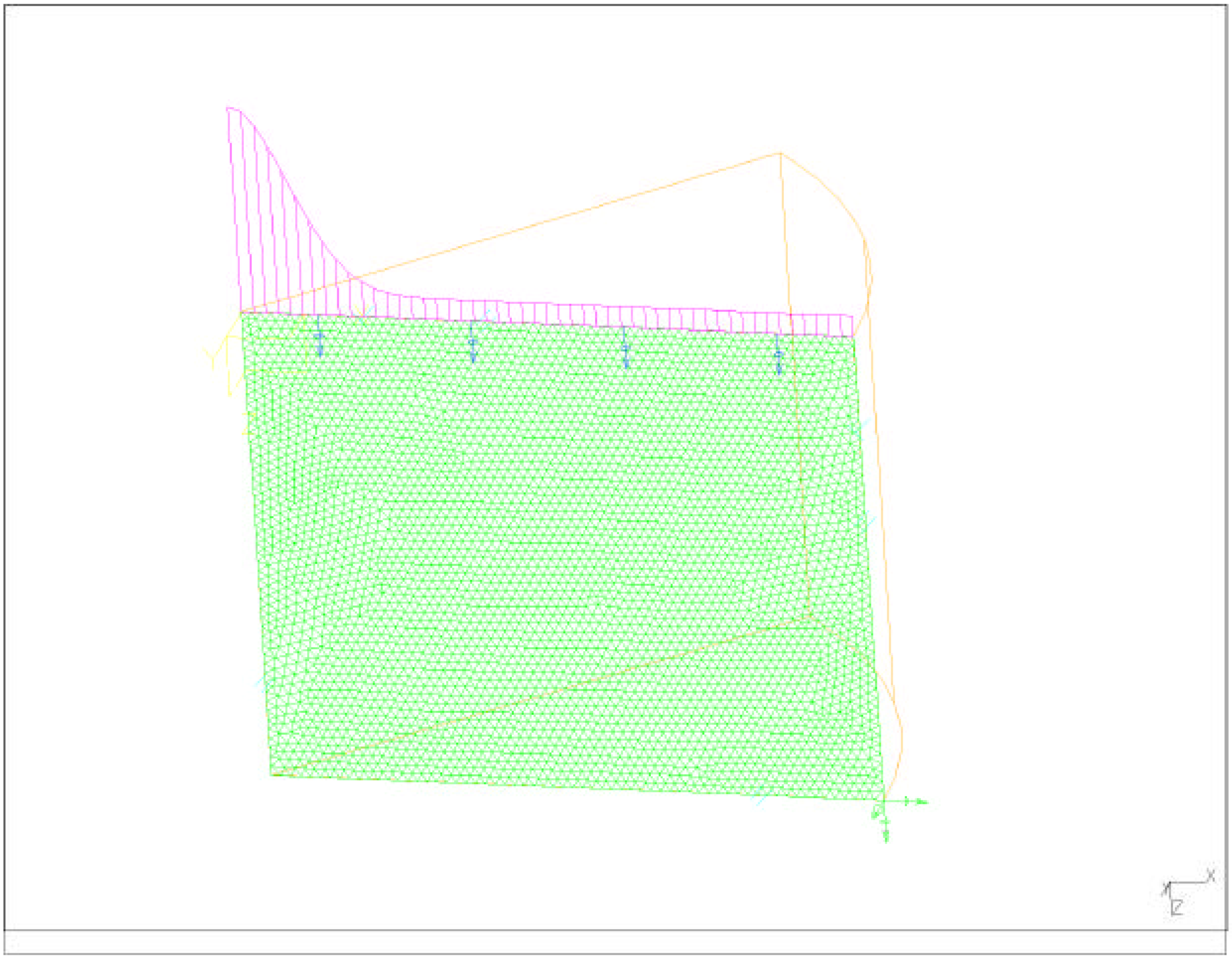
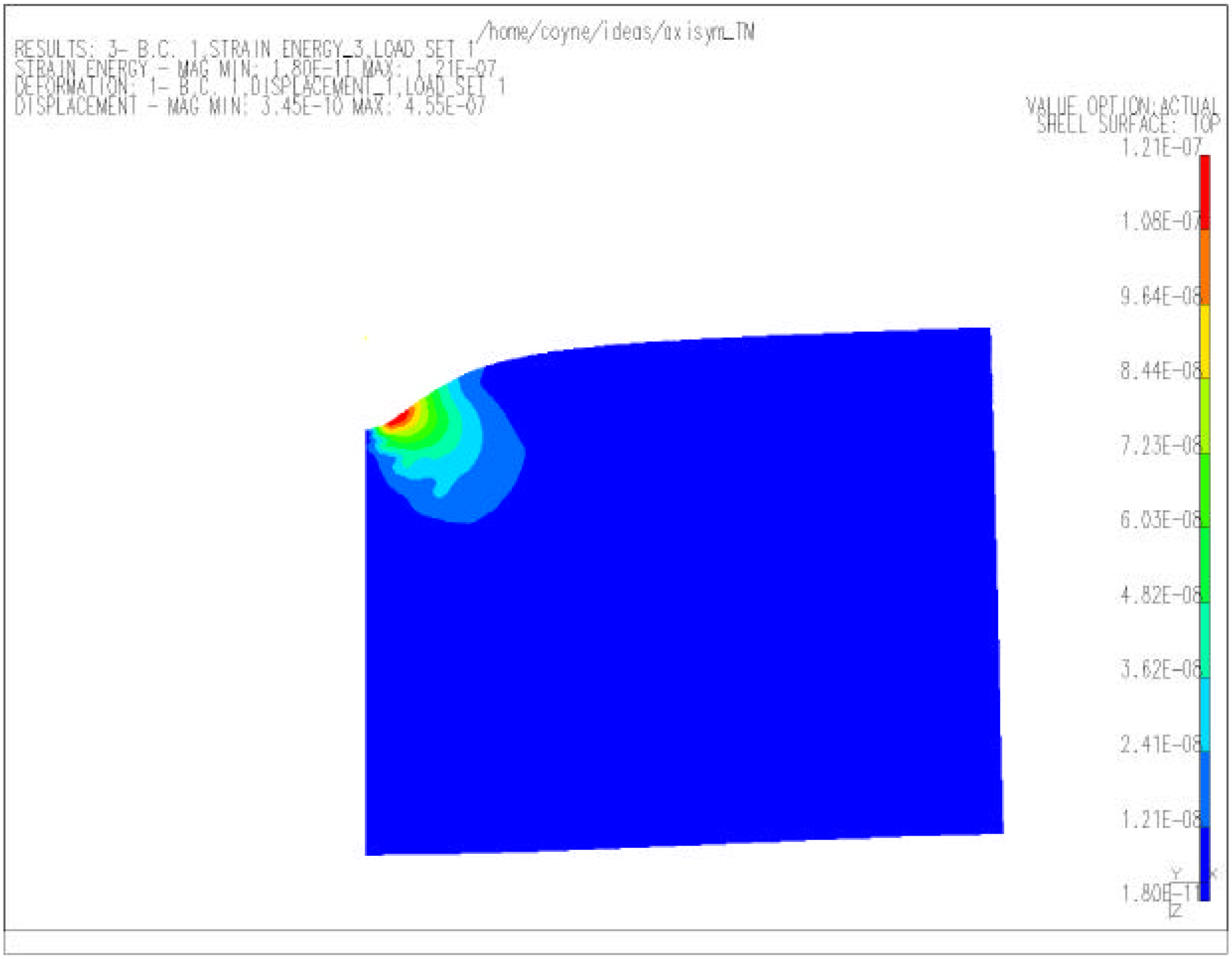


Figure 6 Axisymmetric strain energy contours & deformed model
(strain energy for 1000 N total force distributed as a Gaussian load with r_o = 1.56 cm)

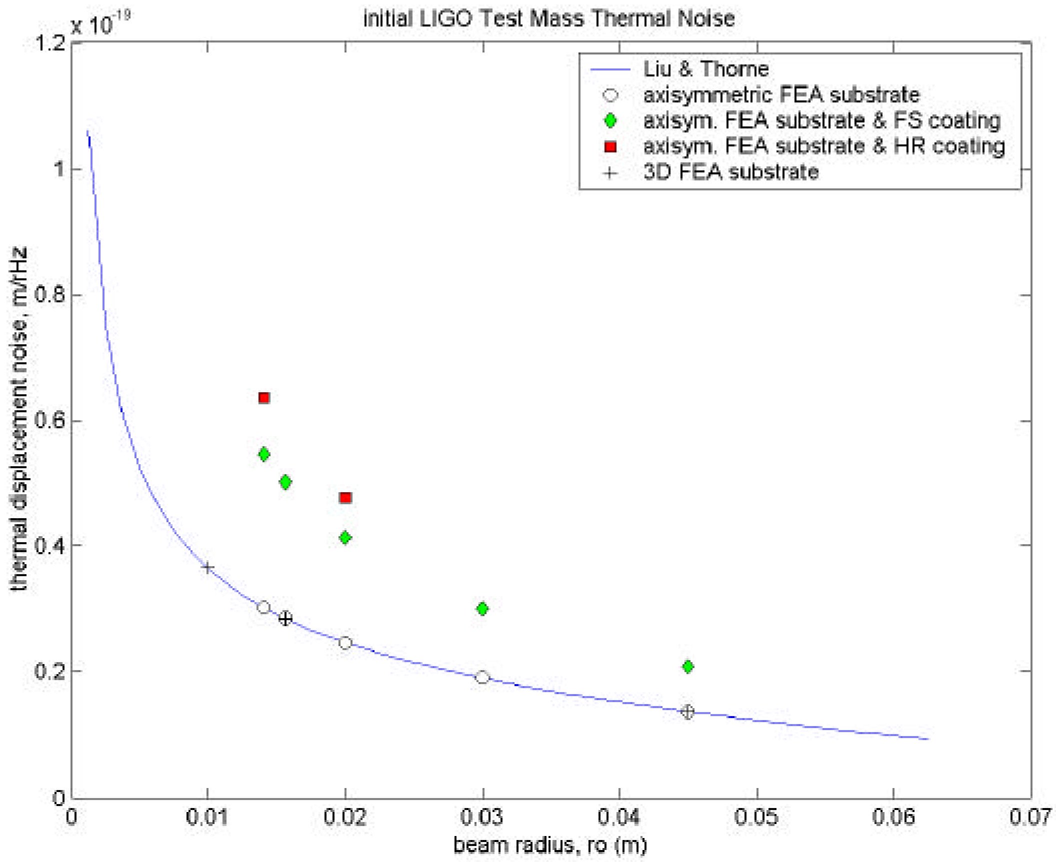


Case #	Mesh Type	Element Type(s)	No. of Elements	No. of Nodes	Time[4] (min)	Strain Energy $(10^{-10} J)$	Thermal Noise, $\lvert \hat{x} \rvert$ at 100 Hz $10^{-20}\left(m/\sqrt{Hz}\right)$
1	Infinite elastic half-space[5]	NA	NA	NA	NA	1.65	2.95
2	Infinite elastic half-space[6]	NA	NA	NA	NA	1.74	3.03
3	Analytical solution[7]	NA	NA	NA	NA	1.55	2.85
4	Normal Mode expansion[8]	NA	NA	NA	NA	NA	2.83
5a	Free mesh, uniform	Parabolic tetrahedral	2440	4107		1.441	
5b			3451	5630		1.465	
6a	Mapped mesh, with bias toward center of front face	Linear brick and wedge	2048			1.681	
6b			4000	4411		1.636	
7a	Solid free mesh, separately [illegible]	[illegible]	3805	5970	9	1.545	2.84
7b			6261	9693	17	1.548	2.85
7c			15616	23357	53	1.550	2.85
8a	Solid, mapped mesh, separately meshed central region	Linear bricks & wedges	13000	13546	22	1.770	
8b		Parabolic bricks & wedges	7392	30667	1252	1.559	2.86
9a	[illegible]	Parabolic triangular	1122	2335	0.5	1.520	
9b		Parabolic triangular	7148	14523	1.4	1.545	2.84
9c		Parabolic rectangular	500	1591	0.4	1.520	

[4] "Time" is clock time while running SDRC's IDEAS on LIGO's server named "sirius".
[5] Yu. Levin, Internal thermal noise in the LIGO test masses: A direct approach, Physical Review D., vol. 57, no. 2, 15 Jan 1998.
[6] F. Bondu, et. al., Physics Letters A, 246, (1998), pp. 227-236.
[7] Using the equations from Y. Liu and K. Thorne, "Thermoelastic noise and homogeneous thermal noise in finite sized gravitational-wave test masses", Physical Review D, vol. 62, 20 Nov 2000.
[8] A. Gillespie and F. Raab, Phys. Review D, 42, 2437 (1990).

2.3 LIGO Test Mass: Right circular cylinder with attachments and coating loss

To be written



2.4 Advanced LIGO Test Mass

To be written

2.5 Composite test mass

It was recently suggested by P. Fritschel[9] to consider, as an interim measure, using the existing initial LIGO, fused silica test masses with a cradle to adapt them to the dimensions of the advanced LIGO sapphire test mass and to provide more mass. One design, suggested by S. Rowan[10] is to

[9] P. Fritschel, D. Coyne, "Staged Approach to Advanced LIGO Implementation", G020230-00-M, 5/3/2002.

[10] S. Rowan, "Thermal noise estimate for a compound test mass for Advanced LIGO using a fused silica or heavy glass cradle to hold a LIGO I silica test mass", LIGO-G020242-00.

bond the test mass to the cradle with indium. The geometry, inertial properties, and first natural frequencies of several cradle designs were documented by D. Coyne[11]. In the following, an SF4 glass cradle bonded to a LIGO-1 fused silica test mass, with a uniformly thick and continuous layer of indium (between the barrel of the test mass and the inner diameter of the cradle) is modeled. The geometry is depicted in the following figure. The material properties used are given in the following table.

Material	density (g/cc)	Young's Modulus (GPa)	Poisson's Ratio (-)	Loss (-)
SF4	4.79	56	0.24	1.E-05
Indium	7.30	13	0.45	1.E-01
Fused Silica	2.20	73	0.17	1.E-07

The approach to modeling this composite test mass was to use a rather thick indium layer of 1 mm since it is plausible that one could mesh a layer of this thickness without an unrealistically large mesh size. Then thinner layers are approximated by adjusting the modulus of elasticity of the material of the bonding layer by the ratio of actual mess thickness (1 mm) to desired thickness. A complete 3D model (of axisymmetric form, without the flats on the side of the cradle) was created, as well as an axisymmetric model.

The 3D mesh, which is shown in figure below, was automatically ("freely") meshed with parabolic tetrahedral elements with a discretization of the fused silica test mass coarser in the central region than the mesh of case 7a of the table above for fused silica masses. The indium layer had only 1element through its thickness, which is really inadequate to capture gradients in the layer. Nonetheless, the total strain energy for the 3D model, as well as the total strain in the indium layer, is close to that of a more finely meshed axisymmetric model (as indicated in the table below).

Assuming the mechanical loss values listed in the table above, the thermal noise is as follows:

In layer thickness (mm)	**Contribution, U j** (10^{-14} J)			**Thermal Noise,** $|\hat{x}|$ at 100 Hz $10^{-20}\left(m/\sqrt{Hz}\right)$
	FS	**In**	**SF4**	
1.0	0.0017	9.7	0.011	226
0.1	0.0017	1.5	0.011	89

[11] D. Coyne, "Composite Test Mass", LIGO-G020241-00, 5/17/2002.

Table 1 Composite Mass/Indium Layer/Cradle Analyses

#	In layer thickness (mm)	Element Type(s)	No. of Elements	No. of Nodes	Time[12] (min)	Strain Energy ($10^{-10} J$)			
						FS	In	SF4	Total
1	1.0	3D automatically meshed with parabolic tetrahedral elements	29,133 total 14,219 FS test mass[13] 10,372 SF4 cradle 4,542 Indium layer		536		0.0077		1.79
2	1.0	Axisymmetric, parabolic, triangular elements	5,877 total 4099 FS test mass 1,578 SF4 cradle 200 In layer	11942	1.1	1.70	0.0097	0.114	1.83
3	0.1	Axisymmetric, parabolic, triangular elements	(same as above)	(same)	1.3	1.70	0.0015	0.115	1.82

[12] "Time" is clock time while running SDRC's IDEAS on LIGO's server named "sirius".

[13] with 139 elements in the central $\sqrt{2}r_o$ radius and to a depth of ½ the test mass thickness as compared to 517 elements in the FS test mass mesh 7a in the table above

Figure 7 Composite Test Mass: SF4 Cradle and indium bonded LIGO-1 fused silica test mass

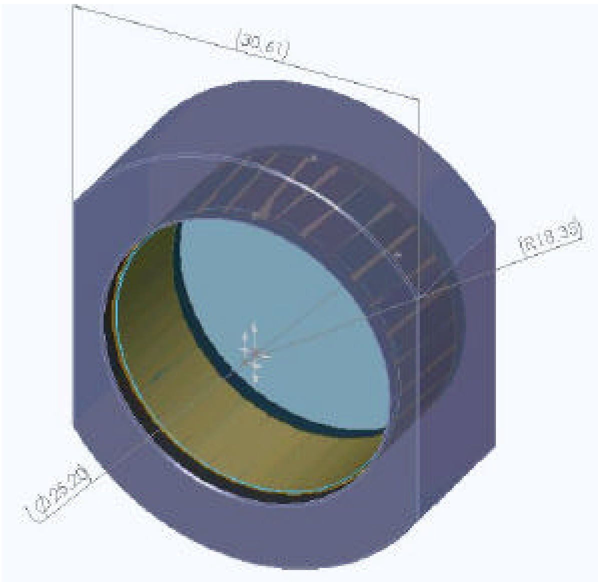


Figure 8 Composite Test Mass: FE mesh of the SF4 cradle, indium and fused silica test mass

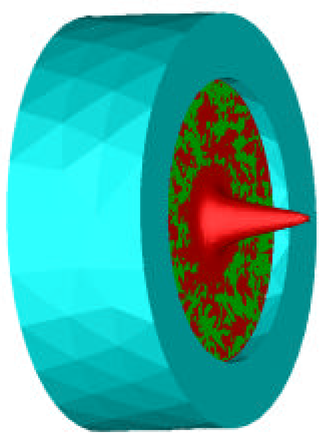


Figure 9 Composite Test Mass: Deformation under a Gaussian Pressure Load

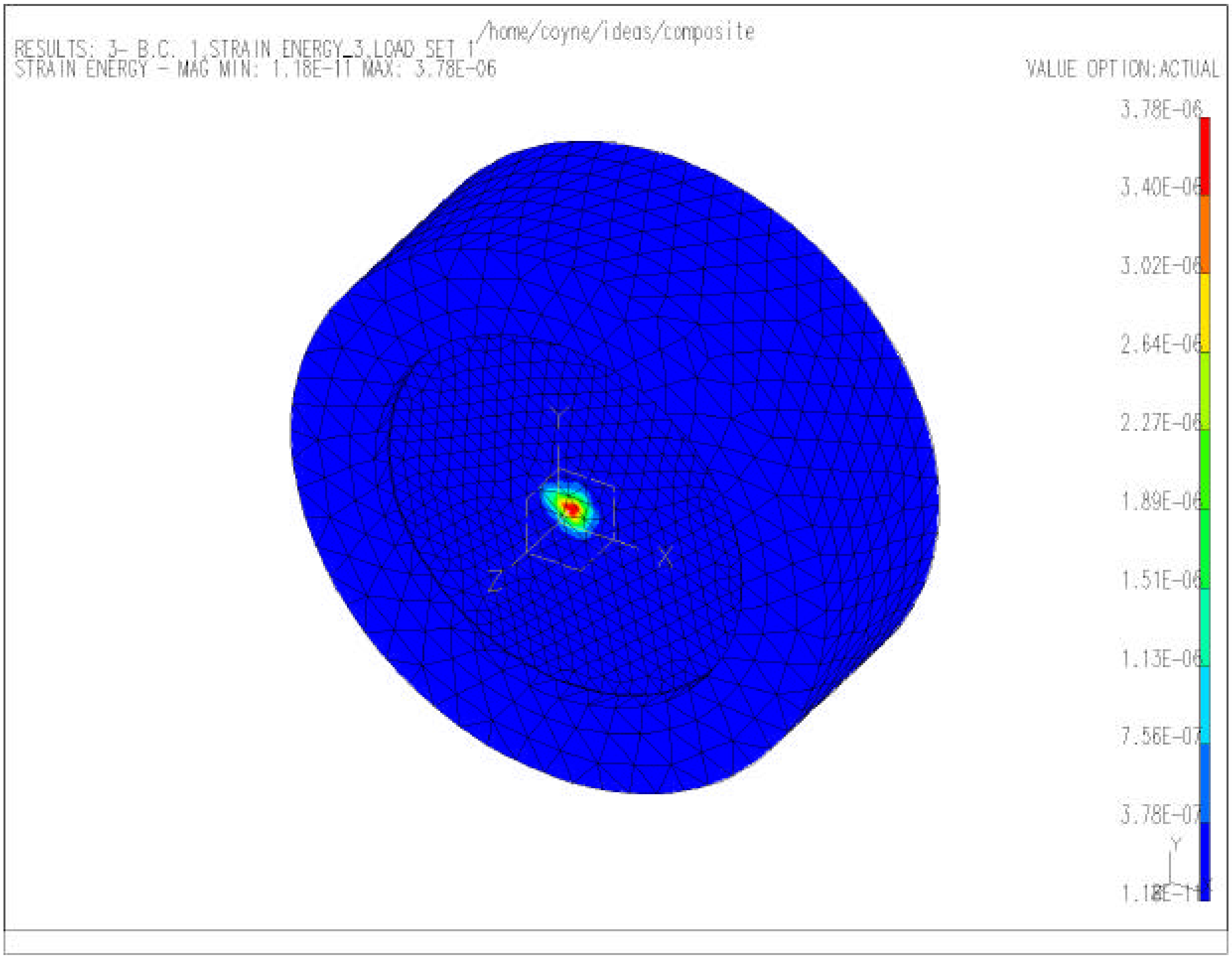


Figure 10 Axisymmetric Mesh for the Composite Test Mass

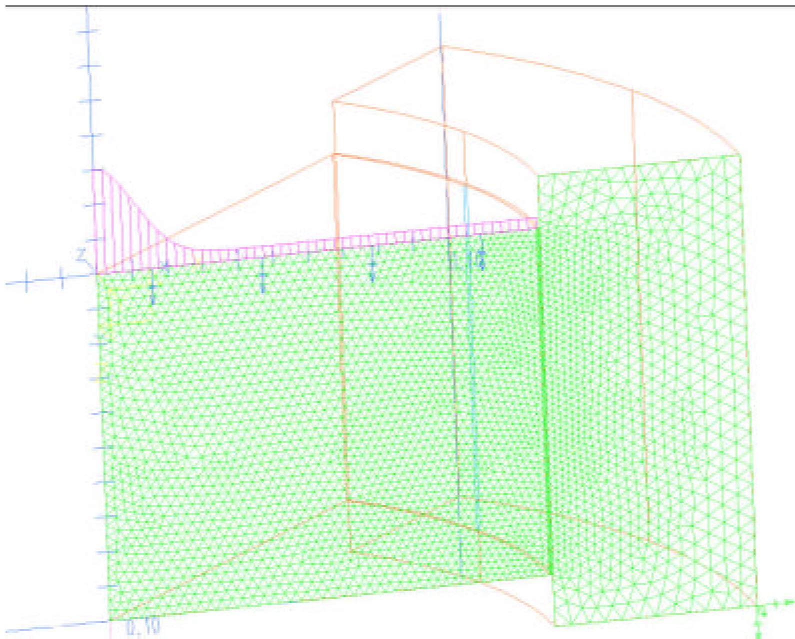


Figure 11 Strain Energy Contours for the Composite Test Mass with a 1 mm Indium Layer (contours from 0.1% to 100% of peak strain energy on a log scale)

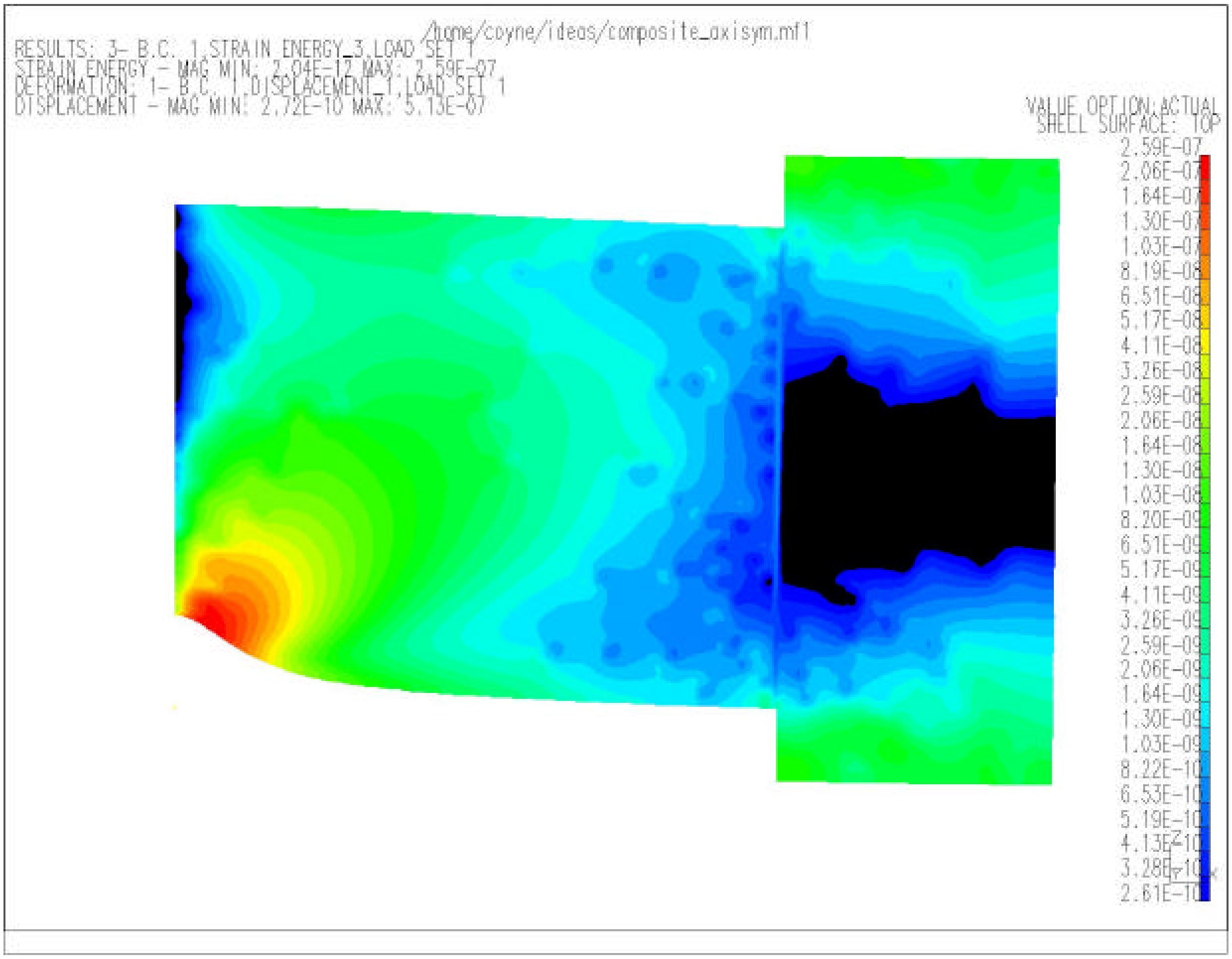
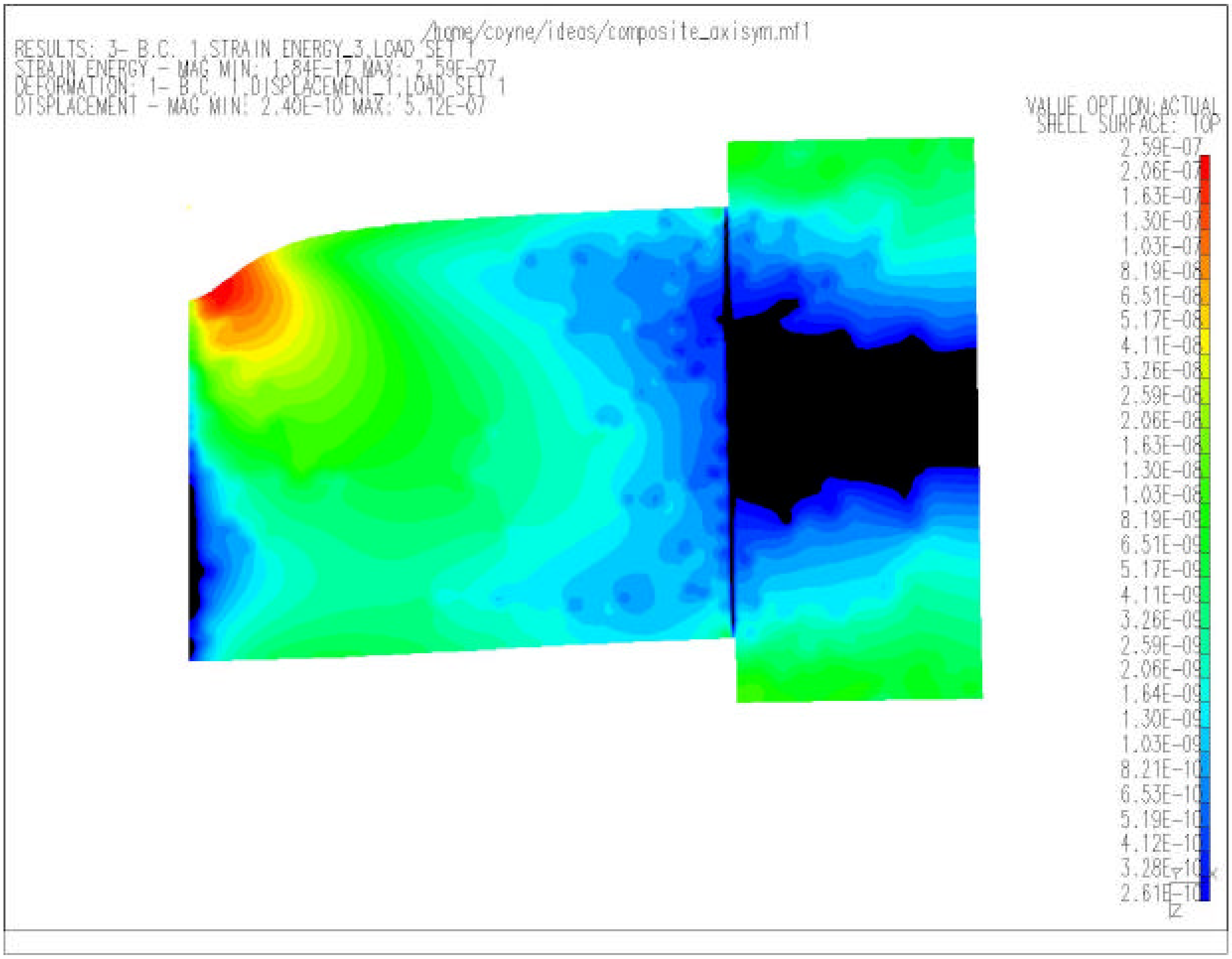


Figure 12 Strain Energy Contours for the Composite Test Mass with a 0.1 mm Indium Layer (contours from 0.1% to 100% of peak strain energy on a log scale)



3 Appendix

3.1 Implementation with IDEAS

Details on how to perform the calculation with SDRC's finite element code, IDEAS.

TBW

3.2 Implementation with Ansys

Maybe?