Vibrational Modes of the BSC Seismic Isolation System

(Initial LIGO SEI)

The modal calculations and visualization in this *Mathematica* notebook are based upon the Hytec mass and stiffness matrices documented in:
E. Ponslet, "HAM and BSC Dynamic Models", Hytec Inc., 4/22/98, LIGO-T990128-00 (http://www.ligo.caltech.edu/docs/T/T990128-00.pdf)
These mode shapes are for the passive isolation stack and a 12 dof representation of the elastic support structure.
Notes:
(1) The latest version of this document (T990128) was issued later than the model files that are used (and the only ones available at this date) for this analysis. The latest document refers to additional information in the model files which are not present in the available files. The model files used were issued in conjunction with a draft release of T990128 on 4/22/1998. According to Hytec, the additional data in the model files referred to in the later version of T990128-00 do not exist.
(2) The eigenvector components for the optics table (the modal participation factors) are listed for all of the modes. In addition, for all modes that have a significant participation by the optics table, there are modal displacement plots and modal animations. One can read the notebook and display the modes dynamically with either Mathematica or the free MathReader application available from Wolfram research at:

http://www.wolfram.com/products/mathreader/

To animate the modes, select the bracket at the right edge of the window, with an arrow at the bottom, and then select the menu item "Cell" --> "Animate Selected Graphics" (or just type Ctrl+Y). The cell bracket with an arrow at the bottom represents a closed set of cells; Double-clicking will open and display the sequence of graphics used to create the animation of the mode.
(3) If you have Mathematica and attempt to recalculate the mode shapes, you will need the data file BSC_SEI_MK.txt, which is available in the same directory:

■ Version History

■ version 1

1/30/2002 Calcualtion & visualization of the natural modes based on the Hytec mass and stiffness matrices. [Version 1 was done only for the BSC SEI system, but to keep the versions in synch, I will use the same version numbering for the HAM analysis.] Reads the mass and stiffness matrices from the file MK_BSC_SEI.txt. This file is created from the original Hytec Matlab model file (described in T980128) with the Matlab script HytecModel_2_Mathenatica.m

■ version 2

3/26/2002

a) Corrected the Euler angle convention/sequence used for visualizing the mode shapes.

b) Added mode number and modal mass in the optics table modal participation factor table (and generated the table for all modes).

c) Attempted to get a better diagonalization of the modal mass matrix by trying other transformations from the generalized to the standard eigenproblem, but was not sucessful; There modal mass matrix has some significant, complex off-diagonal terms. Perhaps there is a problem with the *Mathematica* routine?

d) Generated a list of all modes with any optics table modal participation factor greater than a threshold; Used this list to plot mode shapes relevant to motion of the optics table.

■ Initialization

■ Test rotation & translation directions

■ Read Global Stiffness and Mass Matrices

■ Eigenproblem:

```
In[25]:= nDOFs = Round[km]

Out[25]= 90
```

Check that M is positive definite (symmetric & m[[i,i]] > 0):

```
In[26]:= Min[Abs[Table[M[[i, i]], {i, 1, nDOFs}]]] > 0

Out[26]= True

In[27]:= Md = Flatten[M - Transpose[M]];
         Md.Table[1, {Dimensions[Md][[1]]}] == 0

Out[28]= True
```

■ transformation to the standard eigenproblem by Inverse[M].K

N.B.: The resulting modal matrix is not quite diagonal and has some complex entries. Attempts to factor M and K to transform from the generalized eigenproblem to the standard form eigenproblem (see subsections in the companion *Mathematica* notebook for the HAM SEI, T020045) were not sucessful. The eigenvalues (vibration frequencies) and the mode shapes appear to be reasonable.

```
In[29]:= Clear[vals, vecs];
         {vals, vecs} = Eigensystem[Inverse[M].Kr];

In[31]:= test = Table[Max[Abs[Inverse[M].Kr.vecs[[i]] - vals[[i]] vecs[[i]]]], {i, 1, nDOFs}];
         Chop[test, 10^-5]

Out[32]= {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
          0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
          0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

In[33]:= Max[Im[vals]]
         Min[Im[vals]]

Out[33]= 5.113×10^-11

Out[34]= -5.113×10^-11

In[35]:= freqs = Reverse[Sqrt[Re[vals]] / (2 Pi)]

Out[35]= {1.35737, 1.359, 2.2585, 2.35176, 2.3599, 2.83947, 4.98739, 4.98739, 4.98739, 4.98739, 4.98739,
          5.60543, 5.62885, 5.6444, 6.30876, 6.38233, 6.39484, 6.39484, 6.39484, 6.43517, 6.5482, 6.57707, 8.3473,
          8.3473, 8.3473, 8.3473, 8.3473, 9.71462, 9.75254, 9.93123, 10.2245, 10.2461, 10.5912, 10.6962, 11.0571,
          11.3285, 11.3554, 11.3554, 11.3554, 11.3554, 11.3554, 11.4108, 12.5569, 12.5665, 12.5665, 12.5665,
          12.8593, 12.8838, 12.8969, 12.9329, 12.9329, 12.9329, 12.9329, 12.9329, 12.9483, 13.5482, 13.6453,
          13.8256, 15.2552, 15.2552, 15.2552, 15.2552, 15.2552, 15.2616, 15.2784, 15.2788, 15.5485, 16.2246,
          16.2631, 16.2638, 16.2638, 16.2638, 16.2638, 16.2638, 17.3568, 17.3635, 17.3635, 17.3635, 17.4942,
          19.4628, 25.3238, 42.4117, 66.7825, 70.3363, 85.6947, 110.244, 134.035, 183.954, 214.359, 227.187}

In[36]:= Mm = vecs.M.Transpose[vecs];

In[37]:= TableForm[Chop[Mm, 10^-9]]

In[38]:= Flatten[Mm].Table[1, {Dimensions[Flatten[Mm]][[1]]}]

Out[38]= 18239.9 + 1.77482×10^-15 i
```

```
In[39]:= Tr[Re[Mm]]

Out[39]= 18342.2

In[40]:= Tr[M]

Out[40]= 39253.8

In[41]:= modalMass = Table[Re[Mm][[i, i]], {i, 1, nDOFs}]

Out[41]= {1970.32, 4547.94, 6250.83, 506.885, 501.998, 368.868, 95.7541, 433.649, 263.636, 374.931, 385.695, 11.8421, 6.65443,
  6.65443, 6.65443, 6.65938, 4.11479, 4.11479, 4.11479, 0.84532, 0.84532, 4.17798, 4.15699, 6.79033, 9.49275, 9.4231,
  6.65772, 8.93139, 1.80693, 1.80693, 8.93139, 8.93139, 10.897, 9.26241, 15.75, 163.669, 7.14731, 7.14731, 2.35532,
  2.35532, 7.14731, 167.61, 80.3904, 76.5345, 5.91187, 5.91187, 5.91187, 5.93091, 5.6017, 4.96498, 4.96498, 2.9304,
  2.9304, 4.96498, 5.43367, 3.98062, 6.31431, 10.7366, 4.4483, 166.344, 154.842, 16.2308, 16.8412, 3.53389, 3.53389,
  3.53389, 0.536357, 0.536357, 7.08681, 8.50242, 8.92894, 5.36306, 5.36306, 5.36306, 239.552, 6.31983, 94.6412,
  81.617, 165.711, 41.3431, 41.3431, 41.3431, 41.3431, 41.3431, 268.39, 27.5798, 28.1413, 24.4975, 168.734, 168.489}

In[42]:= vecs = Reverse[vecs];

In[43]:= opticstableVecs = Transpose[Take[Transpose[vecs], -6]];

In[44]:= threshold = 0.01;
         opticstableModes = 1;
         Do[
           If[
            Max[Abs[opticstableVecs[[i]]]] > threshold,
            opticstableModes = Flatten[{opticstableModes, i}],
           ], {i, 2, nDOFs}];
         opticstableModes

Out[47]= {1, 2, 3, 4, 5, 6, 13, 14, 15, 16, 20, 21, 22, 28, 29, 31, 32, 33, 34, 36, 42, 47, 48, 55, 56, 57, 58, 65, 66, 67, 68, 69}

In[48]:= nTableLength = nDOFs;
         tableHead = {"freq (Hz)", "x", "y", "z", "Rx", "Ry", "Rz"};
         tableLabel = Table[i, {i, nTableLength}];
         opticsTableModeShapes = Transpose[Partition[
             Join[Take[freqs, nTableLength], Flatten[Transpose[Take[opticstableVecs, nTableLength]]]], nTableLength]];
         Print[TableForm[Chop[opticsTableModeShapes, 10^-5], TableHeadings -> {tableLabel, tableHead}]]
```

	freq (Hz)	x	y	z	Rx	Ry	Rz
1	1.35737	0	0.435461	0	0.317466	0	0
2	1.359	0.435826	0	0	0	-0.318945	0
3	2.2585	0	0	0	0	0	-0.239951
4	2.35176	0	-0.0129692	0	0.17485	0	0

5	2.3599	0.013688	0	0	0	0.172993	0
6	2.83947	0	0	−0.517961	0	0	0
7	4.98739	0	0	0	0	0	0
8	4.98739	0	0	0	0	0	0
9	4.98739	0	0	0	0	0	0
10	4.98739	0	0	0	0	0	0
11	4.98739	0	0	0	0	0	0
12	5.60543	0	0	0	0	0	0
13	5.62885	0	−0.0838879	0	−0.0277983	0	0
14	5.6444	−0.0877056	0	0	0	0.0389316	0
15	6.30876	0	0	0	0	0	−0.0496092
16	6.38233	0	0	0.315252	0	0	0
17	6.39484	0	0	0	0	0	0
18	6.39484	0	0	0	0	0	0
19	6.39484	0	0	0	0	0	0
20	6.43517	0	−0.0177749	0	0.0599427	0	0
21	6.5482	0.0192896	0	0	0	0.0591652	0
22	6.57707	0	0	0	0	0	0.0818928
23	8.3473	0	0	0	0	0	0
24	8.3473	0	0	0	0	0	0
25	8.3473	0	0	0	0	0	0
26	8.3473	0	0	0	0	0	0
27	8.3473	0	0	0	0	0	0
28	9.71462	0	0.0358599	0	−0.0394634	0	0
29	9.75254	−0.0322856	0	0	0	−0.0308697	0
30	9.93123	0	0	0	0	0	0
31	10.2245	0	0	−0.14512	0	0	0
32	10.2461	0	0	0	0	0	0.105377
33	10.5912	0	0.0298519	0	−0.0846827	0	0
34	10.6962	0.0239568	0	0	0	0.0661138	0
35	11.0571	0	0.00118593	0	−0.00542228	0	0
36	11.3285	0.00863706	0	0	0	0.0179367	0
37	11.3554	0	0	0	0	0	0
38	11.3554	0	0	0	0	0	0
39	11.3554	0	0	0	0	0	0
40	11.3554	0	0	0	0	0	0
41	11.3554	0	0	0	0	0	0
42	11.4108	0	0	0	0	0	−0.0346004
43	12.5569	0	0	0	0	0	−0.00125141
44	12.5665	0	0	0	0	0	0
45	12.5665	0	0	0	0	0	0
46	12.5665	0	0	0	0	0	0
47	12.8593	0	0.0252688	0	−0.0158714	0	0
48	12.8838	−0.0140297	0	0	0	0.0106278	0
49	12.8969	0	0	0	0	0	0
50	12.9329	0	0	0	0	0	0
51	12.9329	0	0	0	0	0	0
52	12.9329	0	0	0	0	0	0
53	12.9329	0	0	0	0	0	0
54	12.9329	0	0	0	0	0	0

55	12.9483	0	0	0.0606264	0	0	0
56	13.5482	0	0.0426839	0	−0.0979921	0	0
57	13.6453	0	0	0	0	0	0.115057
58	13.8256	0.0360611	0	0	0	0.0811272	0
59	15.2552	0	0	0	0	0	0
60	15.2552	0	0	0	0	0	0
61	15.2552	0	0	0	0	0	0
62	15.2552	0	0	0	0	0	0
63	15.2552	0	0	0	0	0	0
64	15.2616	0	−0.00216221	0	0.003891	0	0
65	15.2784	0.00639526	0	0	0	0.0128948	0
66	15.2788	0	0	0	0	0	0.021338
67	15.5485	0	−0.0131286	0	0.0282008	0	0
68	16.2246	−0.00677177	0	0	0	−0.0145348	0
69	16.2631	0	0	0	0	0	0.0204369
70	16.2638	0	0	0	0	0	0
71	16.2638	0	0	0	0	0	0
72	16.2638	0	0	0	0	0	0
73	16.2638	0	0	0	0	0	0
74	16.2638	0	0	0	0	0	0
75	17.3568	0	0	0	0	0	0.00326728
76	17.3635	0	0	0	0	0	0
77	17.3635	0	0	0	0	0	0
78	17.3635	0	0	0	0	0	0
79	17.4942	0	0.00285008	0	−0.00608468	0	0
80	19.4628	0	0	0	0	0	0.00249573
81	25.3238	0.0000876824	0	0	0	0.00018542	0
82	42.4117	0	0	0	0	0	0
83	66.7825	0	0	0	0	0	0
84	70.3363	0	0	0	0	0	0
85	85.6947	0	0	0	0	0	0
86	110.244	0	0	0	0	0	0
87	134.035	0	0	0	0	0	0
88	183.954	0	0	0	0	0	0
89	214.359	0	0	0	0	0	0
90	227.187	0	0	0	0	0	0

```
In[53]:= Min[Table[Max[Abs[vecs[[i]]]], {i, 1, nDOFs}]]

Out[53]= 0.239951
```

■ Stack Geometry

■ Modal Plots

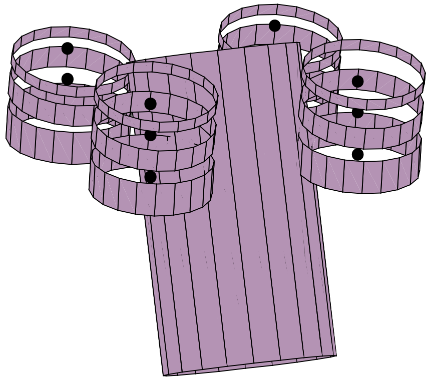
```
In[85]:= massdof = Drop[massdof, 1];

In[86]:= nElements = Dimensions[massdof][[1]]

Out[86]= 15

In[87]:= nPlots = Dimensions[opticstableModes][[1]];

In[88]:= transdof = Flatten[Transpose[Drop[Transpose[massdof], -3]]];

In[89]:= rotatedof = Flatten[Transpose[Drop[Transpose[massdof], 3]]];

In[90]:= range = 40;
         factorT = 4000;
         factorR = 10;
```

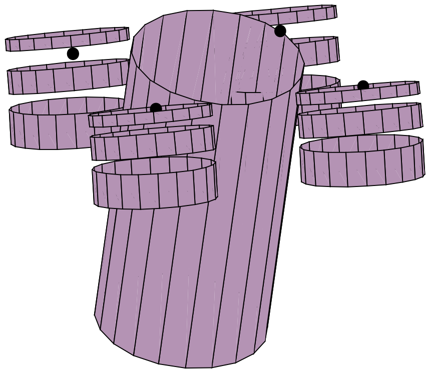
■ Animated Mode Plots

Static Mode Plots

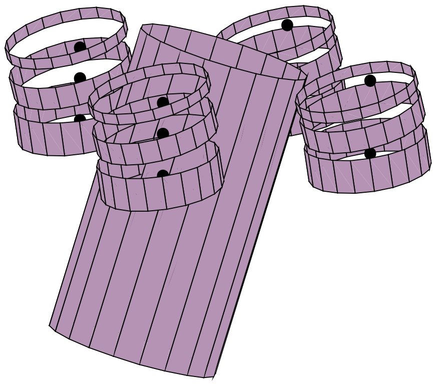
In[93]:=
```
modePlot[k_, factorT_, factorR_] := Block[{j},
transvec = Partition[Table[vecs[[k, transdof[[j]]]], {j, 1, Length[transdof]}], 3];
rotatevec = Partition[Table[vecs[[k, rotatedof[[j]]]], {j, 1, Length[rotatedof]}], 3];
   jsin = Table[Sin[j Pi], {j, 0.5, 1.5, 1}];
   Table[
    Show[
     Table[
      TranslateShape[
       RotateShape[
        RotateShape[
         Graphics3D[Cylinder[legMassRadius, massCylinderHalfHeights[[i - 2]], 20]],
         Pi / 2, -jsin[[j]] factorR rotatevec[[i, 2]], -Pi / 2 - jsin[[j]] factorR rotatevec[[i, 3]]],
        0, -jsin[[j]] factorR rotatevec[[i, 1]], 0],
       massPositions[[i - 2]] + jsin[[j]] factorT transvec[[i]]],
      {i, 3, nElements - 1}],
     TranslateShape[
      RotateShape[
       RotateShape[
        Graphics3D[Cylinder[downtubeRadius, downtubeMassHeight / 2, 20]],
        Pi / 2, -jsin[[j]] factorR rotatevec[[nElements, 2]], -Pi / 2 - jsin[[j]] factorR rotatevec[[nElements, 3]]],
       0, -jsin[[j]] factorR rotatevec[[nElements, 1]], 0],
      massPositions[[nElements - 2]] + jsin[[j]] factorT transvec[[nElements]]],
     Graphics3D[{PointSize[0.02], pts}],
     BoxRatios → {1, 1, 1}, PlotRange → {{-range, range}, {-range, range}, {-1.2 range, 0.8 range}},
     ViewPoint → {40000, 100000, 30000}, Boxed → False],
    {j, 1, 2}];
Print["Mode # ", k, "     ", freqs[[k]], " Hz"];
  ];
```

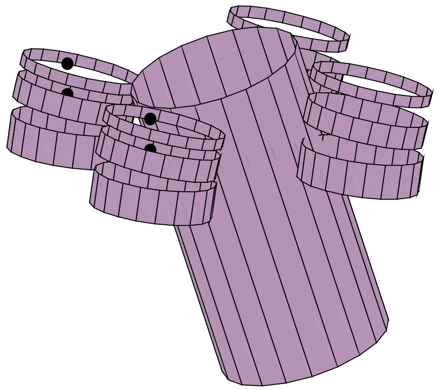
In[95]:=
```
Do[modePlot[opticstableModes[[k]], 20, 1], {k, 1, nPlots}];
```



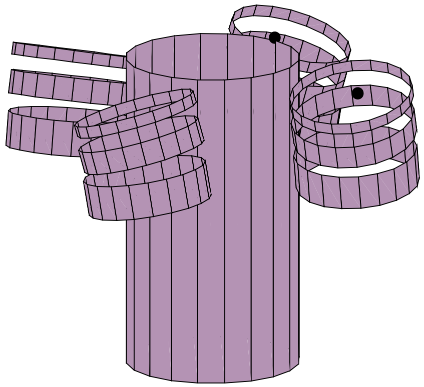


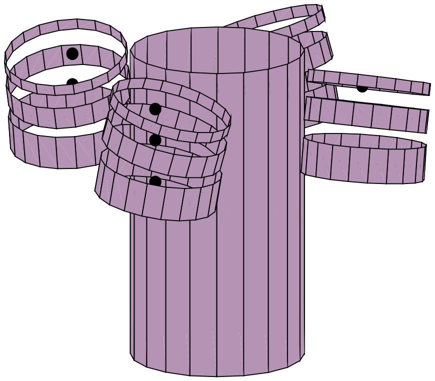
Mode # 1   1.35737 Hz



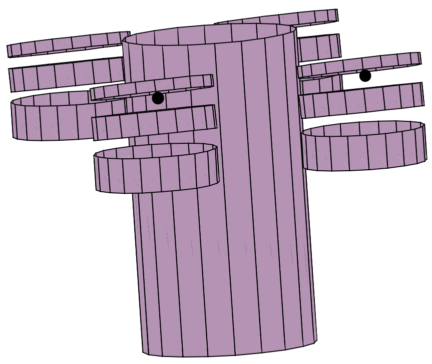


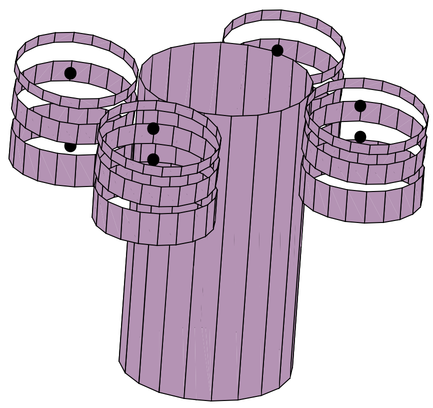
Mode # 2   1.359 Hz



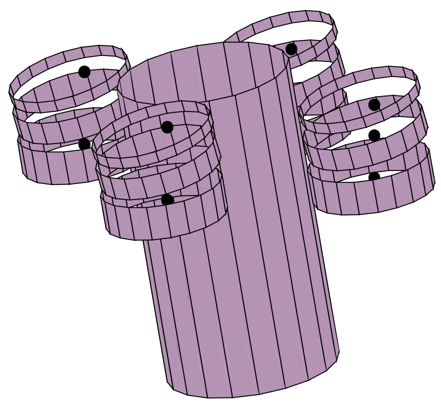


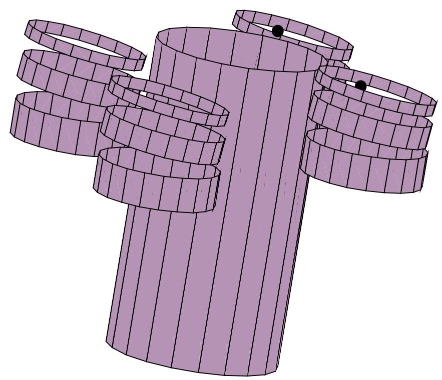
Mode # 3 2.2585 Hz



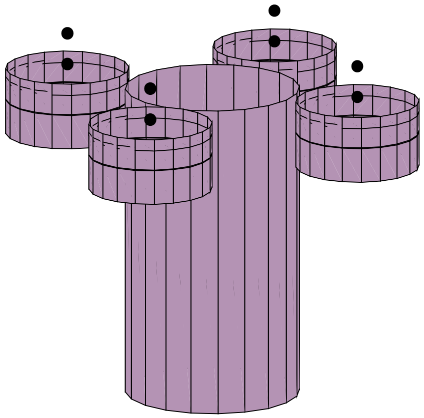


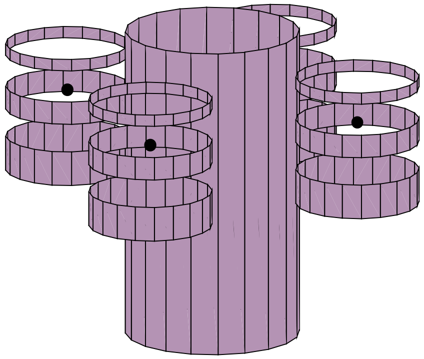
Mode # 4 2.35176 Hz



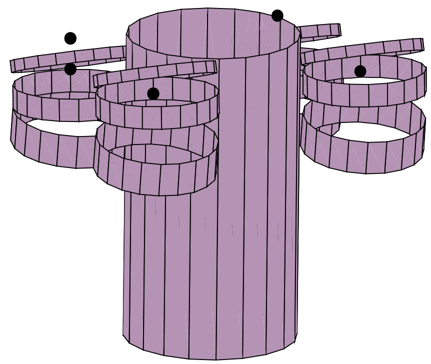


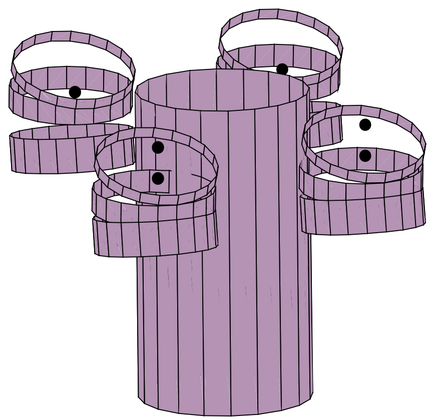
Mode # 5 2.3599 Hz



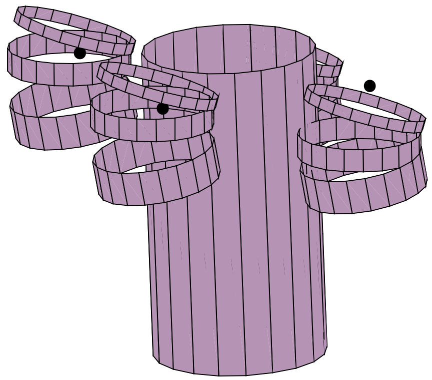


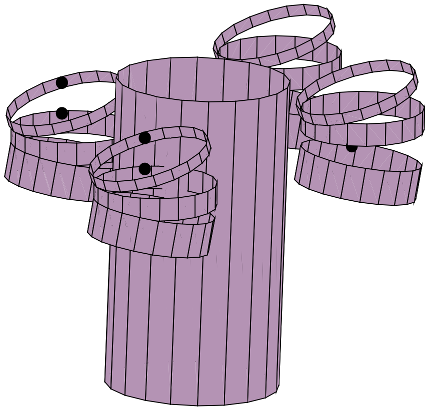
Mode # 6   2.83947 Hz



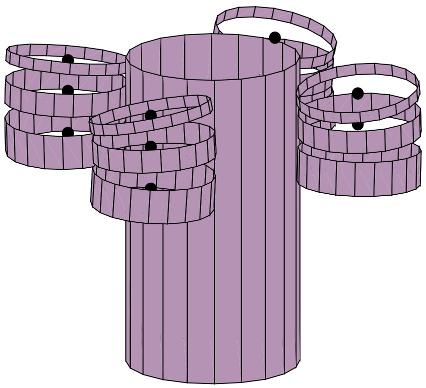


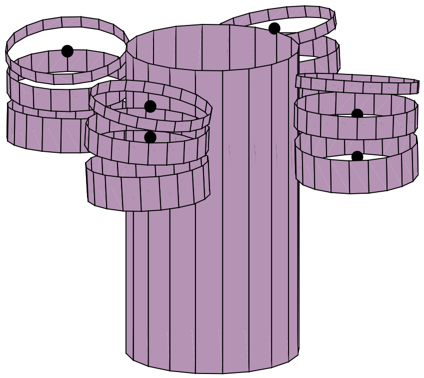
Mode # 13   5.62885 Hz



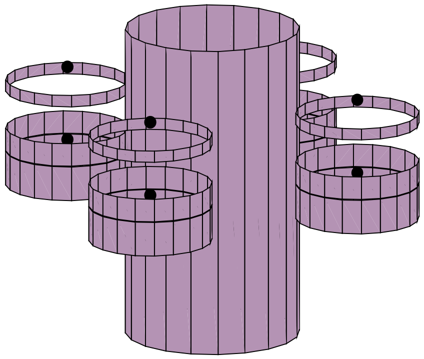


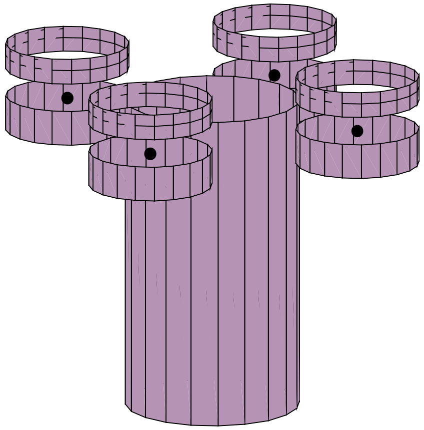
Mode # 14 5.6444 Hz



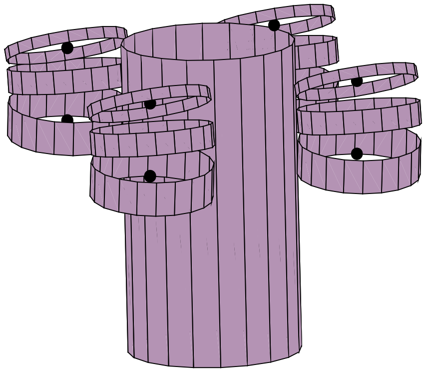


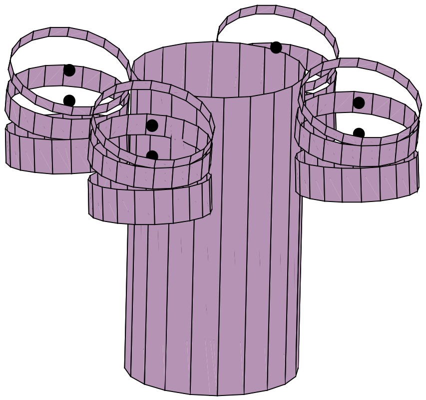
Mode # 15 6.30876 Hz



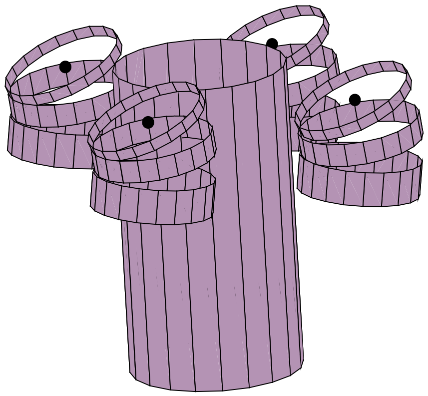


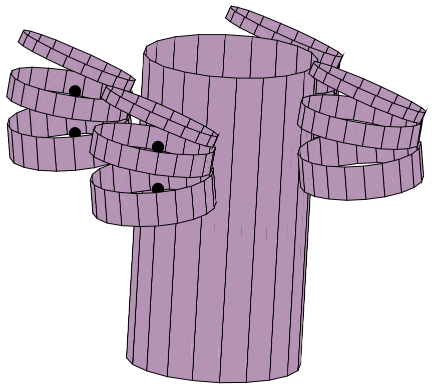
Mode # 16 6.38233 Hz



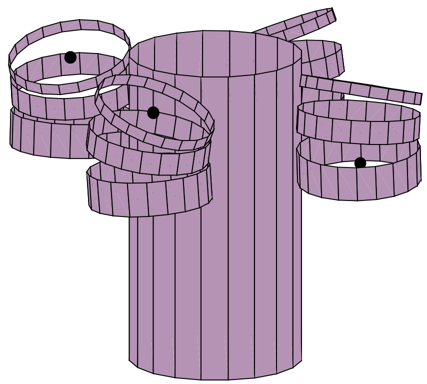


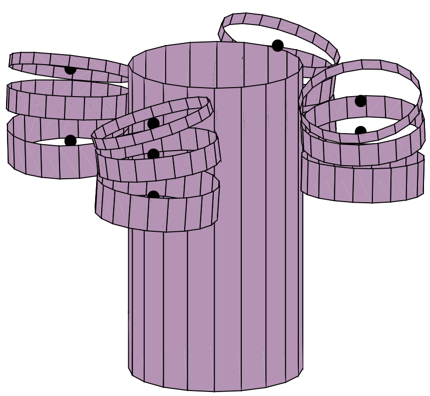
Mode # 20   6.43517 Hz



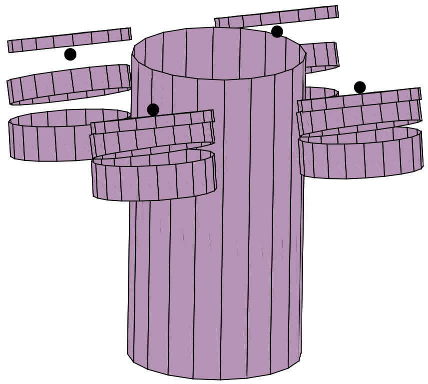


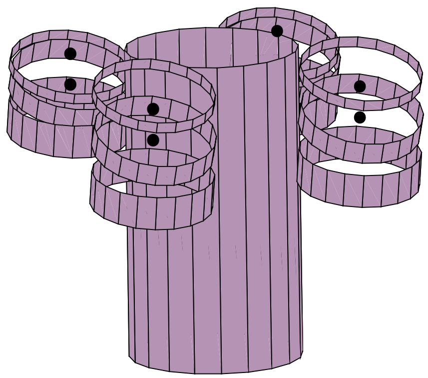
Mode # 21 6.5482 Hz



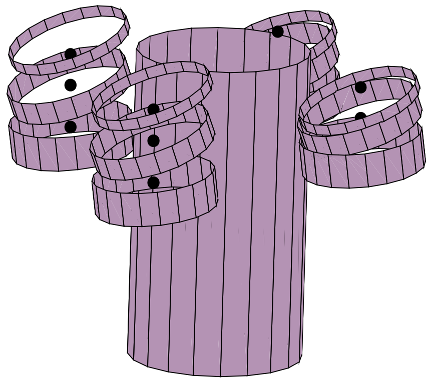


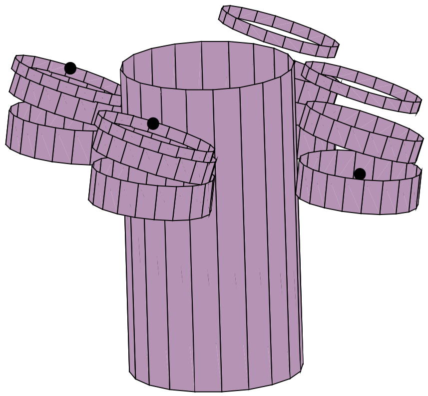
Mode # 22 6.57707 Hz



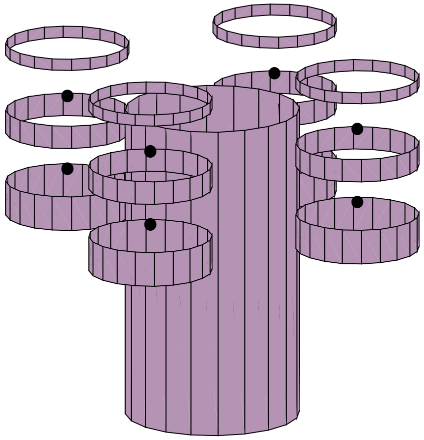


Mode # 28   9.71462 Hz

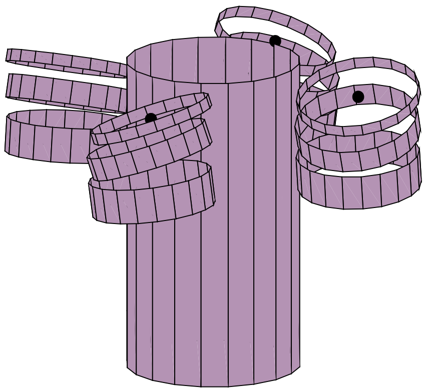


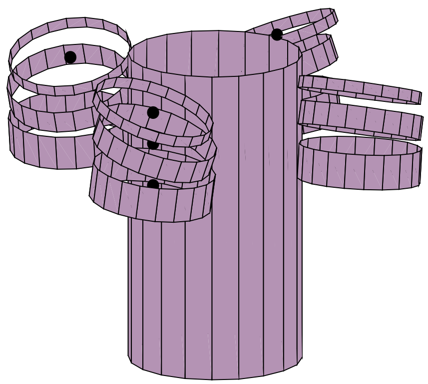


Mode # 29 9.75254 Hz

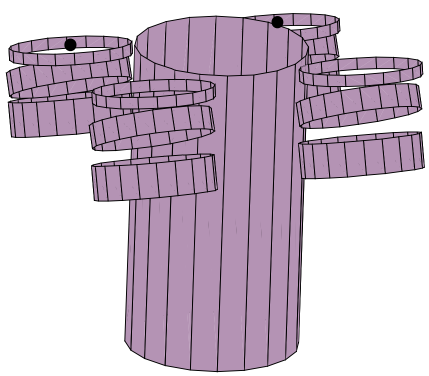


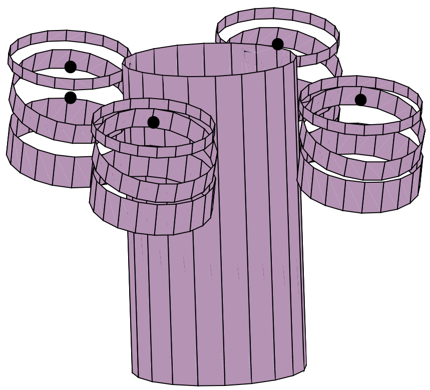
Mode # 31   10.2245 Hz



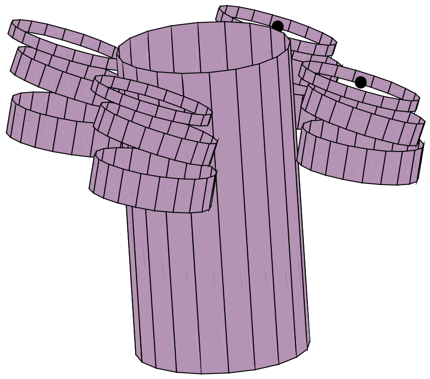


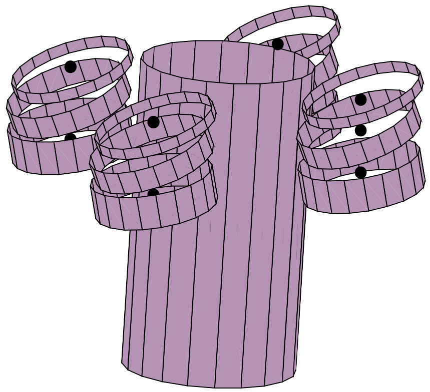
Mode # 32   10.2461 Hz



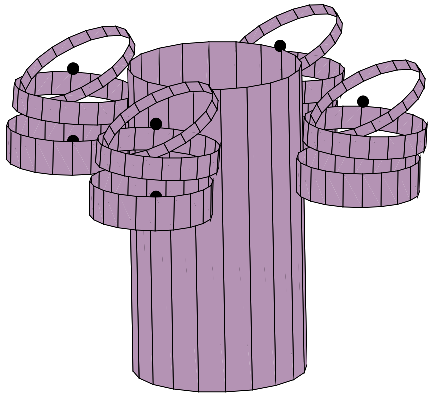


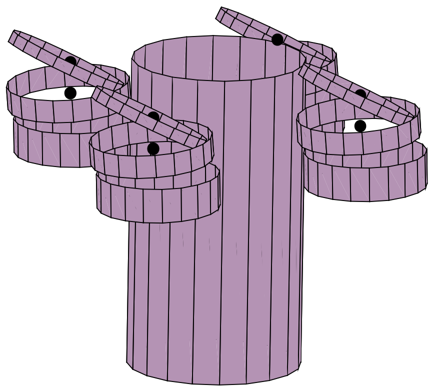
Mode # 33 10.5912 Hz



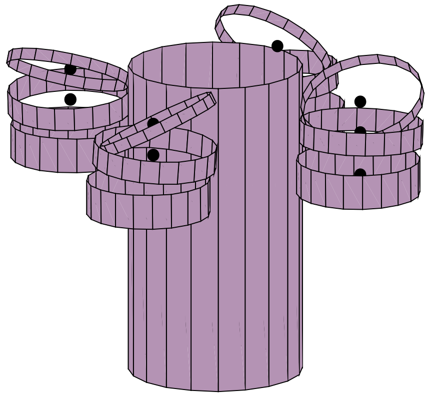


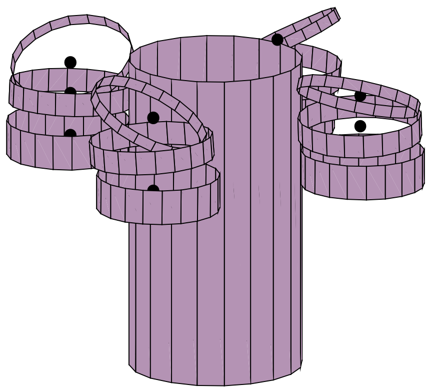
Mode # 34   10.6962 Hz



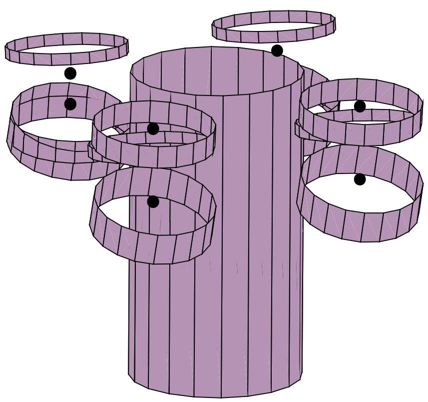


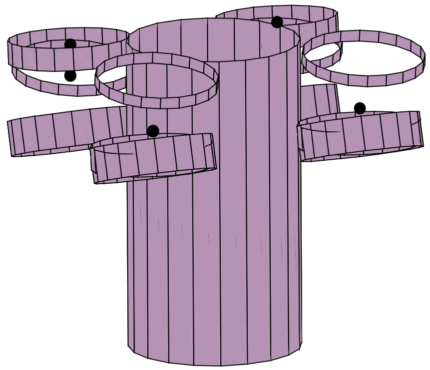
Mode # 36   11.3285 Hz



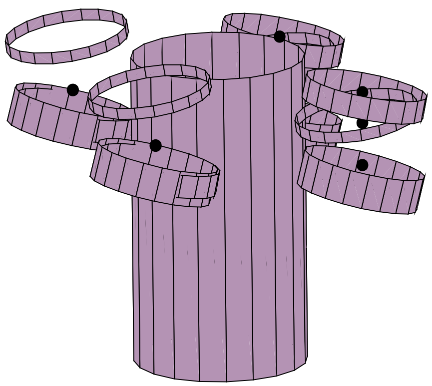


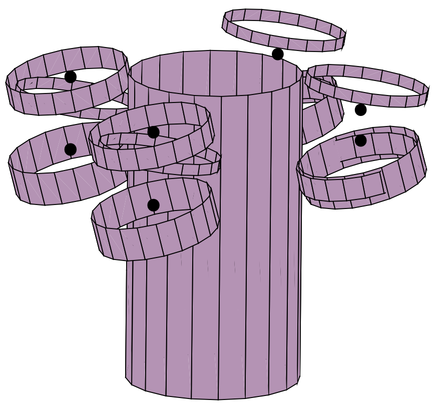
Mode # 42   11.4108 Hz



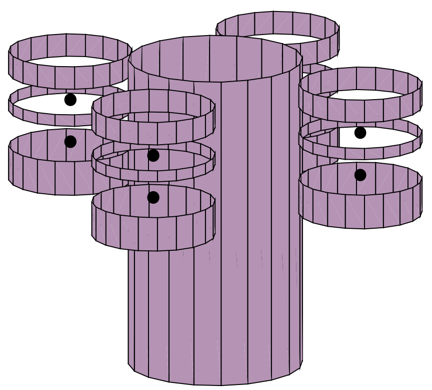


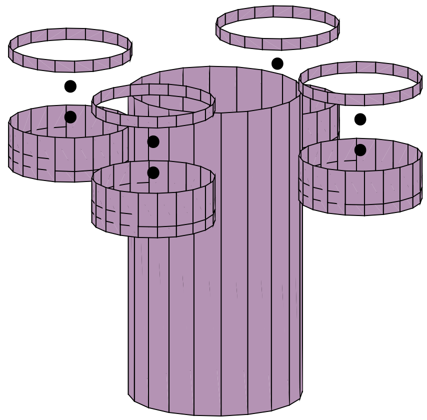
Mode # 47 12.8593 Hz



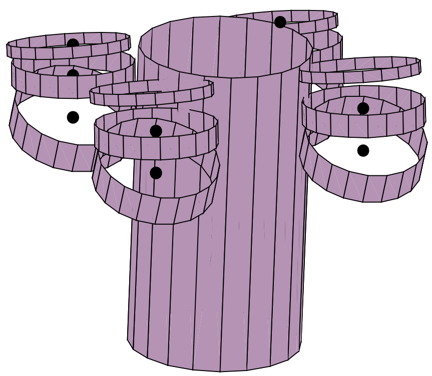


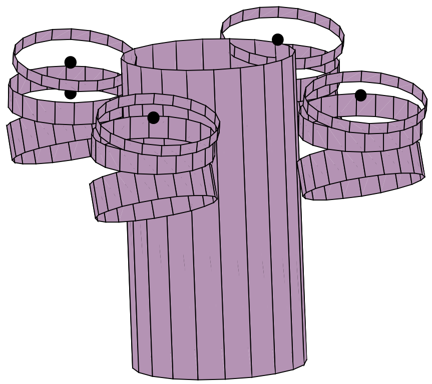
Mode # 48   12.8838 Hz



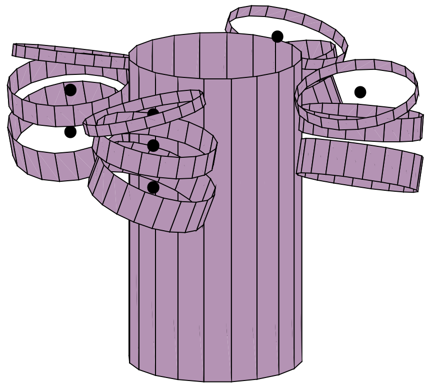


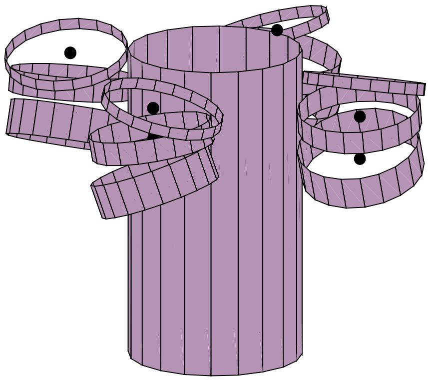
Mode # 55   12.9483 Hz



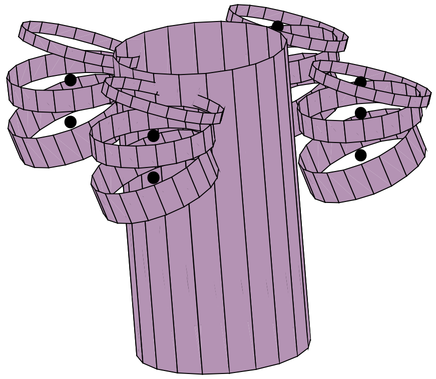


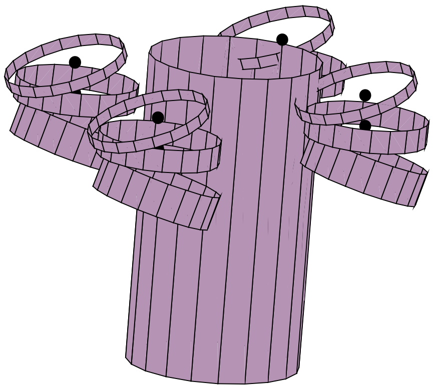
Mode # 56   13.5482 Hz



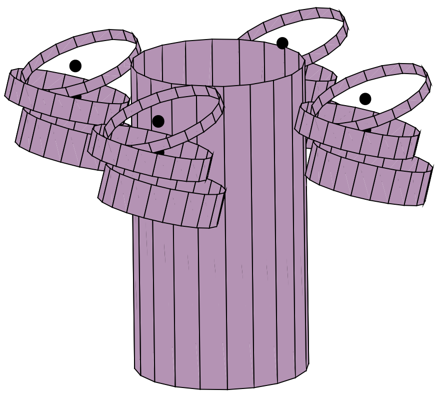


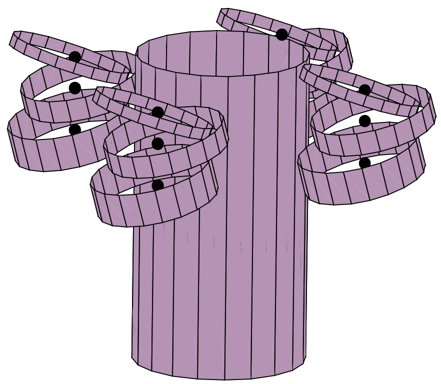
Mode # 57 13.6453 Hz



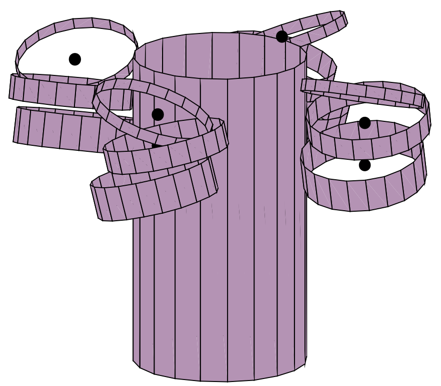


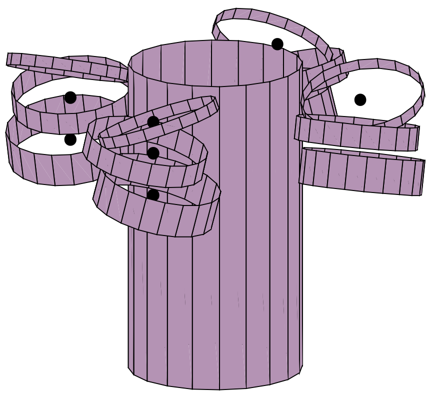
Mode # 58   13.8256 Hz



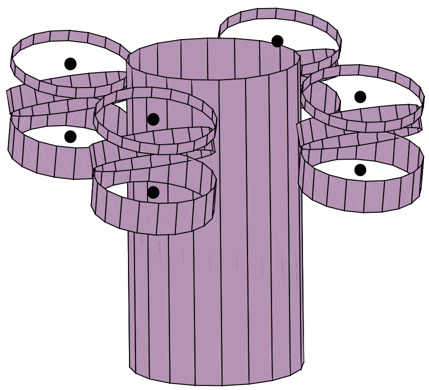


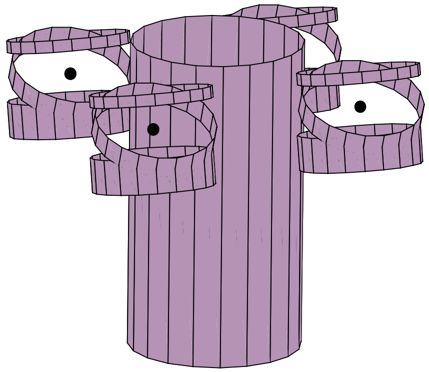
Mode # 65 15.2784 Hz



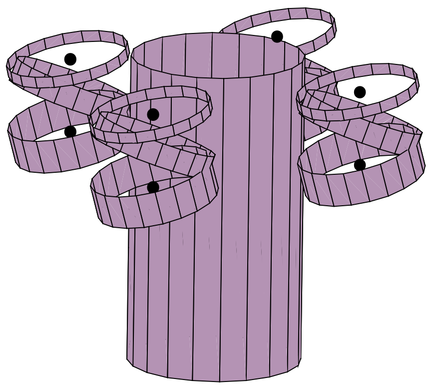


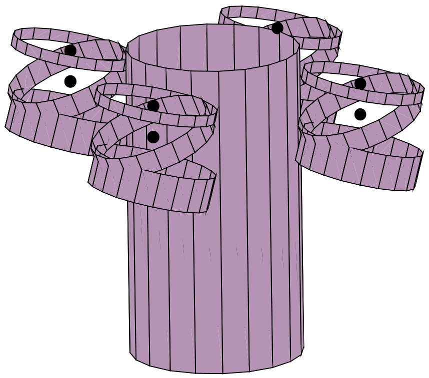
Mode # 66   15.2788 Hz



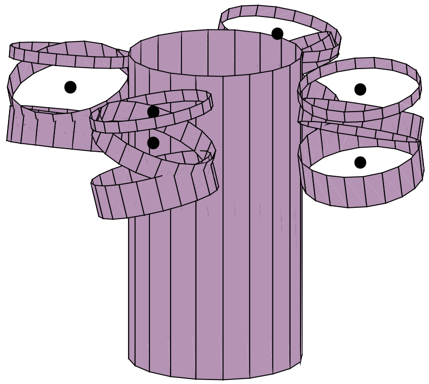


Mode # 67 15.5485 Hz





Mode # 68 16.2246 Hz



Mode # 69 16.2631 Hz