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Mode Matching in Advanced LIGO
Part I

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The real spatial eigenmode will be distorted by several thermal lensing problems. Although these are difficult to predict or quantify at this moment, the planned spatial eigenmode is beset by thermal noise problems and practical problems.

Sapphire

From a thermal noise point of view, we would like to have a beam size that is as large as possible to average over the distortions caused by thermoelastic damping. There are certainly practical limitations such as the size of the mirror. The target for sapphire [1] is now to have spatial eigenmodes with 6cm Gaussian beam radius on each test mass:

$$w_{ETM} = 6\,cm = w_{ITM} = w$$

This symmetry requires a symmetric resonator with

$$g_{ETM} = g_{ITM} = g = \sqrt{1 - \frac{L^2\lambda^2}{\pi^2 w^4}}$$

with $L = 4000\,m$, $\lambda = 1.064\,\mu m$, we get the following parameters for the interferometer eigenmode:

Standard Solutions

The standard solution calls for one PR-mirror inline with one of the arm cavities as shown in Fig. 1:

Solution 1:

Arm Cavities:

$\mathrm{g-factor:}\quad g_{ETM} = g_{ITM} = g = 0.9264$
$\mathrm{waist\ size\ (at\ 2000\,m):}\quad w_0 = 5.89\,cm$
$\mathrm{Radii\ of\ curvatures:}\quad R_{ETM} = R_{ITM} = 54416\,m$

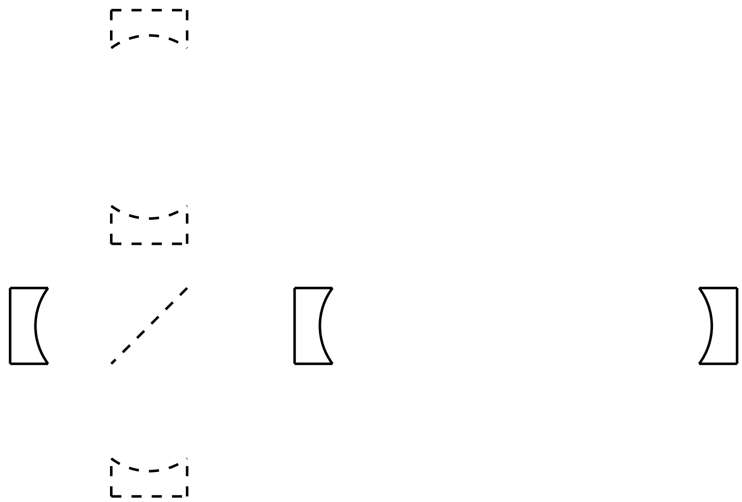

Figure 1: The standard adv. LIGO configuration. The recycling cavities are linear cavities inline with one of the arm cavities. For this discussion we can reduce the system to a linear 3 mirror cavity (solid lines) and ignore the second arm cavity and the second recycling cavity (dashed lines) and the beam splitter.

Rayleigh range : $z_R = 10239\,m$
Half trip Gouy phase : $\phi^{HT}_{Gouy} = acos(g) = .3858\,rad$
Transversal mode spacing $\Delta\nu_{TEM} = 4.6\,kHz$

Recycling Cavities

Beam size on PR: $w_{PR} = 6.0\,cm$
Radius of curvature of PR: $R_{PR} = 31.06\,km$
Radius of Phasefront at ITM (inside PR-cavity): $R_{ITM2} = -31.12\,km$
Rayleigh range: $z_R = 9522.1\,m$
g-factor: $g_{itm2} = 1.0003$ $g_{PR} = 0.99973$
Half trip Gouy phase: $\phi^{HT}_{Gouy} = acos(\sqrt{g_{itm2}g_{PR}}) = 0.000784\,rad$
Transversal mode spacing: $\Delta\nu_{TEM} = 4.486\,kHz$
The length of the power recycling cavity is 8.34 m; the length of the signal recycling cavity is 8.327 m. The difference is so small that the modes are (nearly) identical.

Solution 2:

Arm Cavities:

g-factor: $g_{ETM} = g_{ITM} = -0.9264$ $g = g_{ITM} \cdot g_{ETM} = 0.8582$
waist size $(\text{at } 2000\,m)$: $w_0 = 1.15\,cm$
Radii of curvatures : $R_{ETM} = R_{ITM} = 2076.4\,m$
Rayleigh range : $z_R = 390.9\,m$

Half trip Gouy phase : $\phi^{HT}_{Gouy} = acos(\sqrt{g}) = .3858\,rad$
Transversal mode spacing : $\Delta\nu_{TEM} = 4.6\,kHz$

Recycling Cavities:

Beam size on PR: $w_{PR} = 6.04\,cm$
Radius of curvature of PR: $R_{PR} = 1194.7\,m$
Radius of Phasefront at ITM (inside PR-cavity): $R_{ITM2} = -1186.4\,m$
Rayleigh range: $z_R = 130.7\,m$
g-factor: $g_{itm2} = 1.007$ $g_{PR} = 0.99302$
Half trip Gouy phase: $\phi^{HT}_{Gouy} = acos(\sqrt{g_{itm2}g_{PR}}) = 0.000779\,rad$
Transversal mode spacing: $\Delta\nu_{TEM} = 4.456\,kHz$
The length of the power recycling cavity is 8.34 m, the length of the signal recycling cavity is 8.327 m. The difference is so small that the modes are (nearly) identical.

Remarks:

The finesse of the recycling cavity is in the order of 50 (R=0.94, assume about impedance matched). The linewidth is therefore in the range of 370 kHz. The higher order modes of the carrier are in this case close to antiresonant because the arm cavities are only resonant for the fundamental mode. However, no such filtering is going to happen for the RF-sidebands in this design.

Fused Silica

Fused Silica has different problems with thermal distortions and no best choice has yet been determined [1].

Mode matching between MC and IFO

A two-mirror telescope will match the spatial mode of the mode cleaner output to that of the interferometer. A steering mirror on HAM1 directs this output first to a mirror (MMT1) located on HAM2, then to a second mirror (MMT2) again on HAM1, after which it encounters the power-recycling mirror and enters the IFO. The mode-matching is optimized for the cold interferometer - without thermal lenses. Small deviations in the mirror radii can be compensated, and adequate modematching maintained, by changes to the telescope length of approximately half the error in the radius. One possible set of solutions for the 55k and the 2k-design are:

	Solution 1 ($R_{ITM} = 55k$)	Solution 2 ($R_{ITM} = 2.076k$)
R_{MMT1}	-1.550 m	-1.500 m
R_{MMT2}	27.505 m	27.019 m
$L_{MC\rightarrow MMT1}$	15.4 m	15.4 m
$L_{MMT1\rightarrow MMT2}$	13.0 m	13.0 m
$L_{MMT2\rightarrow PR}$	18.0 m	18.0 m

The mode matching between the MC and the IFO needs to be studied in the context of thermal lensing and how thermal lensing changes the spatial eigenmode in the IFO.

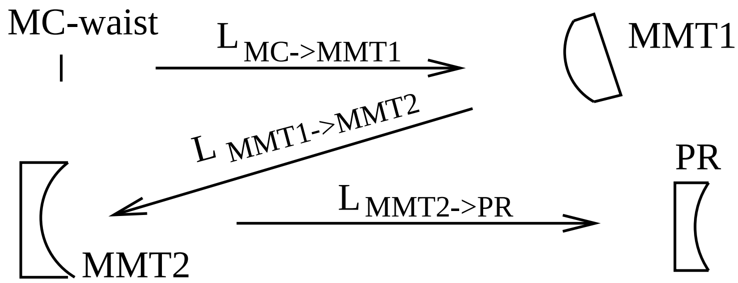

Figure 2: One possible solution for the mode matching. MMT1 blows up the size of the incoming beam that it matches the size of the IFO eigenmode at MMT2. MMT2 changes then the radius of curvature of the incoming field that it matches the radius of curvature of the IFO eigenmode.

This is still a subject of ongoing research and its consequences are not fully understood. There are also the two solutions for the interferometer as discussed above and it is not yet clear which of the two solutions is actually the better one. The mode matching we present here will make certain assumptions about the size of the thermal lenses and needs to be revised as new results in this field of research emerge.

This does not include any thermal lensing inside the IFO. Nevertheless, the thermal lensing and the subsequent changes of the spatial eigenmode can be estimated under the assumption that it is still a Gaussian fundamental mode. The main change will be an additional lens in the ITM substrates in the order of 15km. The main problem with thermal lensing is caused by the asymmetry originating in the beam splitter and the difference between the spatial modes in the recycling cavity and in the arm cavities. That is ignored so far.

Thermal Lensing

Thermal lensing in Advanced LIGO has its most pronounced effect in the sapphire input test masses (ITM's) of the Fabry-Perot arm cavities. We propose a possible solution to this problem. If radiative heaters are permitted to change the focal lengths of input optics and the power-recycling mirror, acceptable mode-matching may be maintained throughout the initial heating and operating (detection) phases.

'Melody' calculations predict that the anticipated power within the arm cavities will produce a 15 km thermal lens within the substrate of the ITM and a 60 km change to its radius of curvature such that the new R_{ITM} is given by:

$$R_{ITM}^{cold} = \frac{1}{\left| \frac{1}{R_{ITM}^{hot}} + \frac{1}{60000} \right|}$$

The following calculations apply to the solution with intra-cavity waist size $w_0 = 1.15 cm$ and hot ITM, ETM radii $R_{ITM}^{hot} = 2076$m. This gives a R_{ITM}^{cold}= 2007 m ($w_{ITM}^{cold} = 6.83$ cm). Complimentary changes must take place in the power-recycling mirror in

	initial (cold)	step1	intermediate	step 2	final (hot)
R_{PR}	1296	heating	1155	heater off	1296
R_{ITM}	2006		2006	th. lensing	2076
R_{MMT1}	-1.480	heating	-1.534	heater off	-1.480
R_{MMT2}	27.020		27.020		27.020

Table 1: The subsequent changes that are necessary to stay mode matched during lock aquisition and heat up phase in Advanced LIGO.

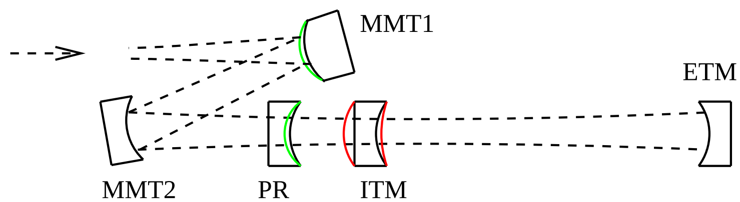

Figure 3: The ITM seen from within the arm cavity will be flatter when the cavity is hot. Also the substrate of the ITM mirror will form a thermal lens that changes the spatial eigenmode of the power recycling cavity as well as the mode matching into the arm cavity. The power recycling mirror and MMT1 can be changed with radiative heaters to match the eigenmode of the IFO when its cold. The radiative heaters can be turned off for the hot IFO.

order to maintain adequate mode-matching. The corresponding values are $R_{PR}^{hot} = 1296$ m (with a 6 cm spot size) and $R_{PR}^{cold} = 1155$ m (with w_{PR}^{cold}=6.88 cm).

We propose the following 2-step process for heating the cavities to the operating point. First, the F-P cavity is in its cold configuration, with the laser light poorly coupled into the cavities. Next, radiative heaters increase the curvature of the power recycling mirror to the point where coupling is sufficient that thermal lensing in the ITM commences. The desired R_{ITM} of 2076 m be-reached, the heating stops and radiative cooling takes over until the original curvature of the power-recycling mirror, optimized for the hot configuration, is restored. Note that thermal lensing effects in the PR- mirror are negligible with respect to those of the ITM. The question remains whether the input optics, specifically the mode-matching telescope, are sufficiently adaptive to follow this process throughout. We found that a 4% change in the radius of curvature of one telescope mirror could take us from the cold to the hot solution. Table 1 summarizes these changes.

An illustration is shown in Figure 3.

Other possible designs

Two mirror design

One of the problems is that the recycling cavities in both solutions are close to a flat-flat configuration and that they are barely stable. One way to avoid this is to refocus

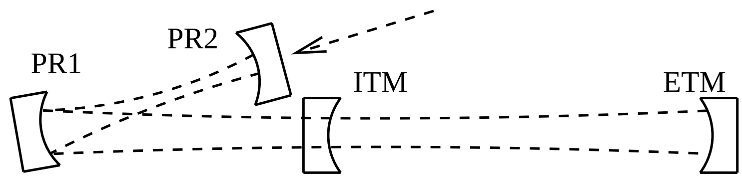

Figure 4: The recycling cavity could be folded. The input would be through PR2. PR1 would focus the eigenmode between PR1 and 2. Depending on the distance between PR1 and 2 and the focal length of PR1, the cavity can be made very stable, but will have a small spot size on PR2.

the beam inside the recycling cavity with either a lens or a curved mirror. The curved mirror design is shown in Fig.4.

PR1 would have a radius of curvature in the ten meter range. It will produce a focus around the position of PR2.

Stability

The stability of cavities with more than two mirrors can be calculated using the roundtrip ABCD-matrix. In this case the matrix would be:

$$M_{RT} = M_{ITM} \cdot M_{L1} \cdot M_{PR1} \cdot M_{L2} \cdot M_{PR2} \cdot M_{L2} \cdot M_{PR1} \cdot M_{L1} \equiv \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

M_{L1} and M_{L2} describe the propagation between the different mirrors. The stability can be calculated from the m-value:

$$m \equiv \frac{A+D}{2}$$

This corresponds to the common g-values as follows:

$$g = g_1 g_2 = \frac{m+1}{2} \qquad 0 \le g \le 1 \Leftrightarrow -1 \le m \le 1$$

The round trip phase difference between consecutive Hermite-Gaussian modes is

$$\Phi = \arccos m = 2 \arccos \sqrt{g}$$

The cavity can be very stable (g-factors everywhere between 0 and 1). The disadvantage is that the beam size and the radius of curvature on PR2 will be very small. This would be a trade-off between acceptable beam size on PR2 and stability of the cavity. In any case, this change in the recycling cavity could be done with both types of arm cavities. We focus on the second type of arm cavity with R_{TM} = 2.1km, but a similar solution exists for the first type (R_{TM} = 55km)

Different Solutions:

Distance $ITM \leftrightarrow PR1$: $8.34m$

Distance $PR1 \leftrightarrow PR2$: $16.65m$

Radius of Curvature $PR1$	33m	32.1m	30m
Radius of Curvature $PR2$	-0.089m	0.383m	1.46m
Beam Size $PR2$	0.31mm	1.4mm	5.8mm
Stability m	0.815	0.991	0.9995
Stability g_1g_2	0.9075	0.9955	0.99975

Looking at these solutions, it doesn't seem to help. Either the beam size and radius of curvature on PR2 are both to small or the gain in stability is very limited. Only an increase in the length of the recycling cavity could really help. The reason can be understood if we look at the following formula for the beam size:

$$w(z) = w_0\sqrt{1+\left(\frac{z}{z_R}\right)^2} \Rightarrow w_0^2 = \frac{w^2}{2}\left[1 \pm \sqrt{1-\frac{4z^2\lambda^2}{\pi^2 w^4}}\right]$$

The waist size of this mode is determined by the size of the mode on the first PR-mirror ($\approx 6\,cm$) and the distance z between the waist and this mirror:

$$w_0^2 \approx \frac{w^2}{2}\left[1 \pm \sqrt{1-\left(\frac{z}{5314.7\,m}\right)^2}\right]$$

To achieve a stable recycling cavity we have to place the second power recycling mirror within the Rayleigh range of the beam waist of the recycling cavity eigenmode. The power recycling length is set to be about $8.4\,\mathrm{m} + n \cdot 16.65$ m ($n = 0, 1$). This requires that z is in the 10 m range and we can approximate:

$$w_{0a} \approx w \qquad w_{0b} \approx w\frac{z}{10^4}$$

The first solution is the typical flat-flat solution similar to the traditional one mirror solution. The second solution is the one for which a few cases where presented above. The waist size is about 3 orders of magnitude smaller than the original beam.

Three mirror design

The problem with the two mirror design is the huge Rayleigh range of the IFO mode compared to the recycling cavity length. This problem can be solved using three mirrors as shown in Fig. 5. For simplicity reasons PR3 is assumed to be flat. A mode with a Rayleigh range of 20m has a waist of:

$$w_0 = \sqrt{\frac{z_R\lambda}{\pi}} = 2.6\,mm$$

The length of the power recycling cavity in this design is $24.99\,m$. For the stability of the cavity it would be necessary to have a fair distance between PR2 and PR3. On the other hand, we can't decrease the distance between PR1 and PR2 and between PR1 and ITM as much as we want, because it would increase the astigmatism caused by the non perpendicular angle of incidence. As a starting point, let's assume that all mirrors are separated by the same amount.

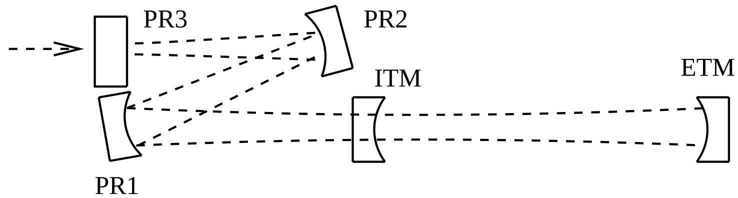

Figure 5: A three mirror folded recycling cavity can stabilize the recycling cavity. The basic idea is to place PR3 into a focus of a mode with a Rayleigh range in the 20m range. The distances between the three PR-mirrors and the ITM are all in the 8m range. PR2 and the inline ITM could share one chamber, while PR3 and PR1 could share another chamber. Footprints and tuning ranges need to be discussed.

Then the following set of parameters would work and should fit at least in the vacuum system:

R_{PR1}	15.813 m
R_{PR2}	0.73698 m
R_{PR3}	∞
L_{PR}	24.99 m
w_{PR1}	6.04 cm
w_{PR2}	2.82 mm
w_{PR3}	2.6 mm
m	0.725
g	0.8625

A cavity with a g-factor of 0.8625 will have a transversal mode spacing of 2.2 MHz. It would be a nice spatial filter for the higher order modes of the sidebands, provides a lot additional flexibility for mode matching between the subsystems, and is stable.

The beam sizes on all mirrors seem to be in an acceptable range. This design is also very flexible. If we assume that we can mode match into any fundamental mode inside the recycling cavity, the problem would be to mode match the recycling cavity mode into the arm cavity mode. This mode matching will depend mainly on the distance between $PR1$ and $PR2$ and their radii of curvatures. The distance can be adjusted similar to the adjustments we made in the mode matching telescope in LIGO I. The subsequent length change in the recycling cavity can be compensated with the distances between $PR2$ and $PR3$ or between $PR1$ and the ITMs.

The open questions are:

1. How much does the eigenmode change when the recycling cavity is thermally loaded

2. Is the astigmatism caused by the non perpendicular angle of incidence small enough (compared to the BS induced astigmatism for example).

3. ?

1.) To answer the first question, we can estimate the thermal deformation (sagitta change) using the following formula:

$$\delta s = \frac{dn/dT}{4\pi\kappa} P_a = \frac{8.7 \cdot 10^{-6}/K}{4\pi 1.38\,W/mK} 0.6 \cdot 10^{-6} \cdot P_{PR}$$

The PR-mirrors will be fused silica mirrors, the absorption of $0.6\,ppm$ is the coating absorption found in melody. The power in the recycling cavity is about 3.3 kW max. That creates a $\delta s \approx 1nm$ or a radius of curvature of 3.4 km on the flat PR1 mirror ($w = 2.6\,mm$). Doesn't really matter.

2.) The astigmatism can be calculated from the angle of incidence. The effective radius of curvature in the tangential plane is $R\cos\Theta$, and $R/\cos\Theta$in the sagittal plane. The angle Θ is determined by the diameter of the mirrors and the size of the suspensions. I assume that we need about $40\,cm$ lateral offset between the beam hitting $PR2$ and ITM.

$$\Theta = \frac{1}{2}\frac{0.4}{8} = 0.025\,rad \qquad \cos\Theta = 0.9997$$

I doubt that this is a problem.

3.) ?

Melody

The two solutions presented above need to be compared. For this we need to optimize the radius of curvatures and the input mode. For the first attempt, we will use melody results to calculate the expected changes in the radii of curvatures.

The parameters used in melody as set in the parameter files are: 4000m arm length, T_{ITM} =0.5%, T_{PR} = 6%, etc. The test mass substrates are non-compensated sapphire, the recycling mirrors standard fused silica, and the beam splitter advanced fused Silica. Max Input Power: 120W. As mentioned, Solution1 is flat-flat, Solution 2 the near-concentric.

Results:

Power Levels	Solution 1	Solution 2
Power (PR-Cavity)	2.53kW	2.5kW
Power (Dark Port)	4.1mW	3.9mW
Power (inline Cav.)	935.8 kW	927.9kW
Power (outline Cav.)	936.5 kW	928.5kW
Thermal Focal lengths	Solution 1	Solution 2
ITM_1 (PR-side)	15.92 km	16.11 km
ITM_2 (PR-side)	16.03 km	16.23 km
BS (inline, x-plane)	942 km	963 km
BS (inline, y-plane)	471 km	482 km
BS (outline, x-plane)	3094 km	3161 km
BS (outline, y-plane)	1547 km	1581 km
ITM_1 (Cavity-side)	59.26 km	59.92 km
ITM_2 (Cavity-side)	59.56 km	60.23 km

The values are very similar for both designs. First observation is that we have more power in the cavities in the first solution. But the difference is very small and it might be caused by the close to flat arm cavities which are now resonant for all low order modes. This includes the Bullseye mode, or in other words, it's hard not to stay mode matched into this cavity. The price is a very unstable arm cavity with effective radii of curvatures above about 400 km. In this case, I would argue that the (expected) increased sensitivity to pointing and beam breathing does out weight the small advantage of the 1% increase in arm cavity power. On the other hand, the arm cavities in the second solution stay stable. The effective radii of curvatures have changed from 2.1km to 2.18km. Nothing scary, nothing dramatic. The power recycling cavity is in both cases not affected by the change. The thermal lensing in the BS and the ITM (on the PR-side) is worst in the y-plane in the inline arm. But the 471 km has to be compared to the radii of curvatures in the other optics. This can be close to negligible when compared to the 55 km for the test masses and the 31 km for the PR-mirror in the first solution. It is even less a problem if we compare it to the 2.1 km and 1.1 km in the second solution. Again, the second solution seems to have advantages.

A similar run up to 85W with fused silica for the beam splitter yields:

Power Levels	Solution 1	Solution 2
Power (PR-Cavity)	1.98kW	1.96 kW
Power (Dark Port)	179 mW	169 mW
Power (inline Cav.)	715.6 kW	709.5 kW
Power (outline Cav.)	772.6 kW	766.7 kW
Thermal Focal lengths	Solution 1	Solution 2
ITM_1 (PR-side)	20.86 km	21.03 km
ITM_2 (PR-side)	19.78 km	20.03 km
BS (inline, x-plane)	130 km	132 km
BS (inline, y-plane)	65 km	66 km
BS (outline, x-plane)	3716 km	3825 km
BS (outline, y-plane)	1858 km	1912 km
ITM_1 (Cavity-side)	77.62 km	78.18 km
ITM_2 (Cavity-side)	72.90 km	73.78 km

There is also no difference between the two solutions when we use standard fused silica for the beam splitter. The difference between advanced FS and standard FS is the absorption coefficient: SFS: 5ppm/cm, AFS: 0.3ppm/cm. In both solutions, the asymmetric thermal lensing generates a huge amount of light at the dark port. We need to look into different radii of curvatures in the two arms, but we first need to understand first how the code deals with unmatched radii of curvatures in a cold interferometer.

A similar run up to 120W with advanced fused silica for the beam splitter and compensated sapphire yields:

Power Levels	Solution 1	Solution 2
Power (PR-Cavity)	3.29 kW	3.28 kW
Power (Dark Port)	0.079 mW	.075 mW
Power (inline Cav.)	1345 kW	1340 kW
Power (outline Cav.)	1321 kW	1315 kW
Thermal Focal lengths	Solution 1	Solution 2
ITM_1 (PR-side)	117 km	117 km
ITM_2 (PR-side)	119 km	120 km
BS (inline, x-plane)	713 km	724 km
BS (inline, y-plane)	356 km	362 km
BS (outline, x-plane)	2348 km	2385 km
BS (outline, y-plane)	1174 km	1192 km
ITM_1 (Cavity-side)	424 km	427 km
ITM_2 (Cavity-side)	432 km	435 km

This represents something close to the optimum solution. A solution that reaches these power levels with un-compensated sapphire and/or standard fused silica would be fine. The influence of the beam splitter could be reduced if we add a compensation plate into the outline arm. This could at least reduce the asymmetry between the two arms which seems to drive the dark port power.

References

[1] Advanced LIGO Systems Design, LSC ed. Peter Fritschel, LIGO-T010075-00-D (27. June 2001)