T020022-01 -D
Pointing Requirements in Advanced LIGO
Part I

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1 Pointing Specifications

General Remarks:

The pointing of the input field can be described as a additional sidebands separated by the pointing (noise) frequency f in a TEM_{10} mode on top of the carrier and RF-sidebands. These fields will be tranformed into TEM_{00} light inside the interferometer if one or more of the optical components are misaligned. This light will pass the output mode cleaner and will mimic a gravitational wave signal.

To calculate the requirements on the pointing, we need to calculate the transfer function of TEM_{10} light into TEM_{00} at the dark port for a misaligned interferometer.

$$a_{00}^{DP}(f) = T_{1->0}(f)a_{10}^{in}$$

DC-Locking

In a DC-locking scheme the RF-sidebands will not pass the output mode cleaner and will not contribute to the noise. Only the carrier and the noise sidebands around it will contribute. I wrote an Octave (Matlab) routine [Ali] that calculates the power fluctuations in the dark port field by beating the noise sidebands with the carrier a_c at the dark port:

$$P_{pointing}(f) = a_c\left(T_p(f)a_{10}(f) + T_p(-f)a_{10}(-f)\right)$$

The most problematic misalignment for DC-locking is a differential tilt of the ITM mirrors [Ali]. The results presented here are based on the assumption that the ITM are tilted by 10^{-10} rad with opposite signs. As long as this tilt angle does not increase the carrier power in the TEM_{00} mode (needs to be compared to the carrier power and phase for optimum detection [req]), the subsequent power fluctuations caused by pointing scale with the tilt angle. This seems to be the case for tilt angles up to 10^{-8} rad. Above 10^{-8} rad, the dark port power and the phase of the carrier will depend on the tilt angle and the requirements do not longer scale.

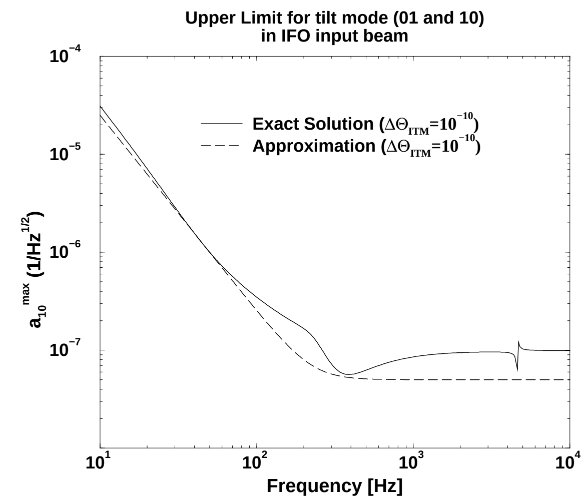

Figure 1: The upper limit for the amplitude of the TEM_{10}-mode for a slightly misaligned Advanced LIGO. A differential tilt of 10^{-10}rad rms between the two ITM mirrors is assumed. This includes already a safety factor of 10 and assumes similar pointing and tilt in the horizontal and vertical direction. The solid line is the solution of the detailed analysis. The dashed line is the approximation from eq. 1 for future use.

The requirements for power fluctuations caused by technical noise sources are derived in [req]. This leads to the final requirements in the pointing (see Fig. 1). The dashed line in Fig. 1 is the following approximation of the exact solution:

$$a_{10}^{max}(f) = \sqrt{\left(\frac{2.5\cdot 10^{-3}}{f^2}\right)^2 + (5\cdot 10^{-8})^2}\,\frac{[10^{-10}\mathrm{rad}]}{\Delta\Theta_{ITM}}\,\frac{1}{\sqrt{\mathrm{Hz}}} \tag{1}$$

RF-Sensing

In an RF-locking scheme the signal has essentially two different contributions. The first contribution is the beat between the RF-sideband and the pointing sidebands around the carrier. This contribution will scale with a differential ITM misalignment. The second contribution is the beat between the leaking carrier and the pointing sidebands around the RF-sidebands. This contribution will scale with a common ITM misalignment as the Michelson interferometer is bright for the RF-sidebands. The output mode cleaner will now filter all higher order modes but will transmit every field in the fundamental spatial mode. Which of the two contributions will contribute more to the signal depends on the different transfer functions of the pointing sidebands into light in the fundamental mode at the dark port and on the amplitude of the carrier with respect to the amplitude of the RF-sidebands. These fields are the local oscillators for the noise fields. For the parameters used here (see [req]) the RF-sidebands are about 3 orders of magnitude larger than the carrier in the dark port. If this ratio changes, the following requirements need to be derived again.

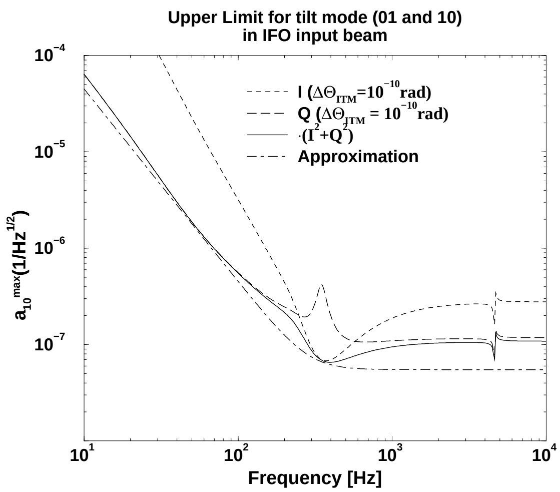

Figure 2: The upper limit for the amplitude of the TEM_{10}-mode for a slightly misaligned Advanced LIGO. A differential tilt of 10^{-10}rad rms between the two ITM mirrors is assumed. This includes already a safety factor of 10 and assumes similar pointing and tilt in the horizontal and vertical direction. The solid line is the solution of the detailed analysis. The dot dashed line is the approximation from eq. 2 for future use. Also shown are the single inphase and quadrature components of the demodulated signal. It shows that an optimum demodulation phase could reduce the requirements in specific frequency regions. But this has to be compared to the optimum signal demodulation phase and it is most likely not a major change.

Under assumptions described above we can approximate the pointing requirements for the input field (see also Fig. 2) for differentially tilted ITMs:

$$a_{10}^{max}(f) = \sqrt{\left(\frac{4.5\cdot 10^{-3}}{f^2}\right)^2 + (5.5\cdot 10^{-8})^2}\,\frac{[10^{-10}\mathrm{rad}]}{\Delta\Theta_{ITM}}\,\frac{1}{\sqrt{\mathrm{Hz}}} \tag{2}$$

The requirements for a common tilt of 10^{-10}rad in the ITMs are shown in Fig. 3. They can be approximated by

$$a_{10}^{max}(f) = \sqrt{\left(\frac{3.5\cdot 10^{-3}}{f^2}\right)^2 + (5\cdot 10^{-7})^2}\,\frac{[10^{-10}\mathrm{rad}]}{C\Theta_{ITM}}\,\frac{1}{\sqrt{\mathrm{Hz}}} \tag{3}$$

2 IO Requirements

The above requirements on the pointing in the input beam turn into a few requirements for the input optics.

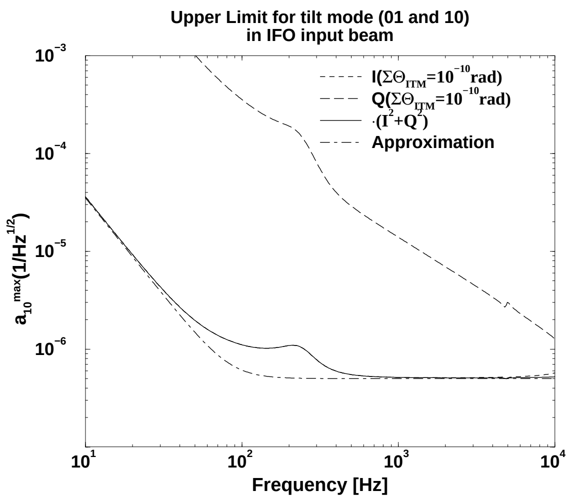

Figure 3: The upper limit for the amplitude of the TEM_{10}-mode for a slightly misaligned Advanced LIGO. A common tilt of 10^{-10}rad of the two ITM mirrors is assumed. This includes already a safety factor of 10 and assumes similar pointing and tilt in the horizontal and vertical direction. The solid line is the solution of the detailed analysis. The dot dashed line is the approximation from eq. 3. Also shown are the single inphase and quadrature components of the demodulated signal. It shows that an optimum demodulation phase could reduce the requirements for this case dramatically. But as differential tilt seems to be the driving noise source, it is probably not possible to utilize this.

Mode Cleaner angular stability

The angular stability of the mode cleaner mirrors has to be sufficient. The transfer of TEM_{00} mode cleaner input light into TEM_{10} mode cleaner transmitted light in a mode cleaner with jittering mirrors can be calculated using the modal picture and matrices that describe the tilt. This is comparable to the calculation I performed for the pointing requirements in a misaligned interferometer. But in contrast to the main interferometer, it is not necessary to perform a frequency dependent analysis for the mode cleaner. The line width of the mode cleaner is about 4.5 kHz and the jittering is only of interest in frequency regions far below that corner frequency. A simple DC-analysis is sufficient. The transfer function for 00-mode light into 10-mode light in a misaligned MC is:

$$t^{MC}_{00\to 10} = \frac{t_1 t_2 L_1}{1 - L_1 M_1 L_3 M_3 L_2 M_2}$$

where

$$M_i = \begin{pmatrix} \sqrt{1-4\Theta_i^2} & -2i\Theta_i \\ -2i\Theta_i & \sqrt{1-4\Theta_i^2} \end{pmatrix}$$

is the tilt matrix for mode cleaner mirror $i = 1,2,3$ in reflection. The propagators L_i incorporate the Gouy phase shift into the calculation. The off-diagonal element $t^{MC}_{00\to 10}(1,2)$ gives the coupling:

$$t^{MC}_{00\to 10} \approx 5\cdot 10^{-8} \frac{[5\cdot 10^{-13} rad]}{\Theta_1} \frac{[5\cdot 10^{-13} rad]}{\Theta_2} \frac{[5\cdot 10^{-13} rad]}{\Theta_3}$$

Taking into account the frequency dependence of the pointing requirements leads to the requirements for the MC-mirror jitter shown in Fig. 4 and approximated by:

$$\Theta_i^{max}(f) = \sqrt{\left(\frac{2.5\cdot 10^{-10}}{f^2}\right)^2 + (5\cdot 10^{-13})^2} \frac{[10^{-10} rad]}{\Delta\Theta_{ITM}} \frac{1}{\sqrt{Hz}}$$

Mode Cleaner Alignment

A misaligned mode cleaner couples pointing of the input beam into frequency noise in the transmitted 00-mode. The calculation goes as follows: The pointing can be described as noise sidebands at noise frequency f in the TEM_{10} mode on top of the fundamental carrier mode. The TEM_{10} field is transformed into TEM_{00} light behind the slightly misaligned mode cleaner. This field alters the phase of the fundamental TEM_{00} light. It creates frequency noise.

The pointing in the mode cleaner input beam has to be sufficiently small that the amplitude of the TEM_{10} mode in the filtered beam behind the mode cleaner is below the requirements given in Fig. 1 . The mode cleaner suppresses the amplitude of the tilt mode roughly by a factor 1000. This leads to the following limitations for the tilt mode in the MC input beam:

$$a_{10}^{max}(f) = \sqrt{\left(\frac{2.5\cdot 10^{-2}}{f^2}\right)^2 + (5\cdot 10^{-7})^2} \frac{[10^{-8} rad]}{\Delta\Theta_{ITM}} \frac{1}{\sqrt{Hz}} \tag{4}$$

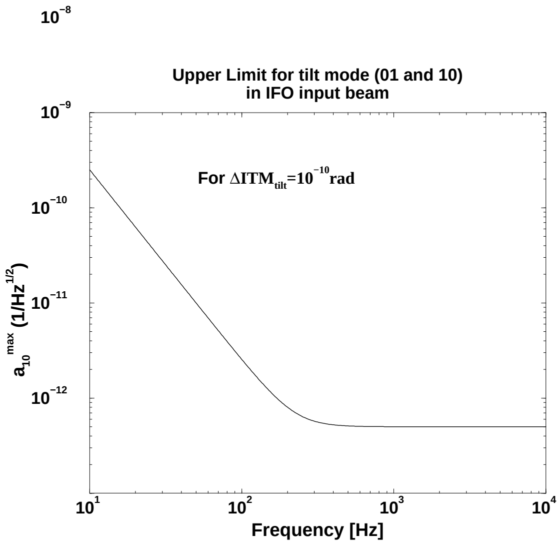

Figure 4: The MC-jitter has to be below $2.5 \cdot 10^{-10} rad/\sqrt{Hz}$ at 10 Hz and below $5 \cdot 10^{-13} rad/\sqrt{Hz}$ above 300 Hz if the differential rms-tilt between the ITMs can be limited to $10^{-10} rad$.

It still depends on our ability to align the ITMs in the main interferometer.

The requirements for the mode cleaner frequency noise can be found in [Fri]. It can be approximated by:

$$\delta\nu_{IFO}^{max}(f) < \sqrt{\left(\frac{3 \cdot 10^{-2}}{f}\right)^2 + (3 \cdot 10^{-5})^2} \frac{Hz}{\sqrt{Hz}} \tag{5}$$

This can be expressed in terms of PM-sidebands:

$$E_{in} = E_o e^{i\omega_0 t} e^{i\frac{\delta\nu(f)}{2\pi f}\cos 2\pi f t} = E_0 e^{i\omega_0 t}\left[1 + i\frac{\delta\nu(f)}{4\pi f}\left(e^{i2\pi f t} + e^{-i2\pi f t}\right)\right]$$

$$E_{in} \equiv E_0 e^{i\omega_0 t}\left[1 + i a_{00}(f)\left(e^{i2\pi f t} + e^{-i2\pi f t}\right)\right]$$

The relative amplitude of the PM-sidebands has to be lower than

$$a_{00}^{max}(f) = \frac{\delta\nu^{max}(f)}{4\pi f} < \sqrt{\left(\frac{2.4 \cdot 10^{-3}}{f^2}\right)^2 + \left(\frac{2.4 \cdot 10^{-6}}{f}\right)^2} \frac{1}{\sqrt{Hz}} \tag{6}$$

One approach to define the requirements on the misalignment in the mode cleaner is to say that it should not change the requirements on the pointing and the frequency noise. This means that the transfer function of the mode cleaner for $a_{10}(f) \rightarrow a_{00}(f)$ has to be lower than

$$t_{10\rightarrow 00}^{MC}(f) < \frac{a_{00}^{max}(f)}{a_{10}^{max}(f)} \qquad \text{at all GW} - \text{frequencies}$$

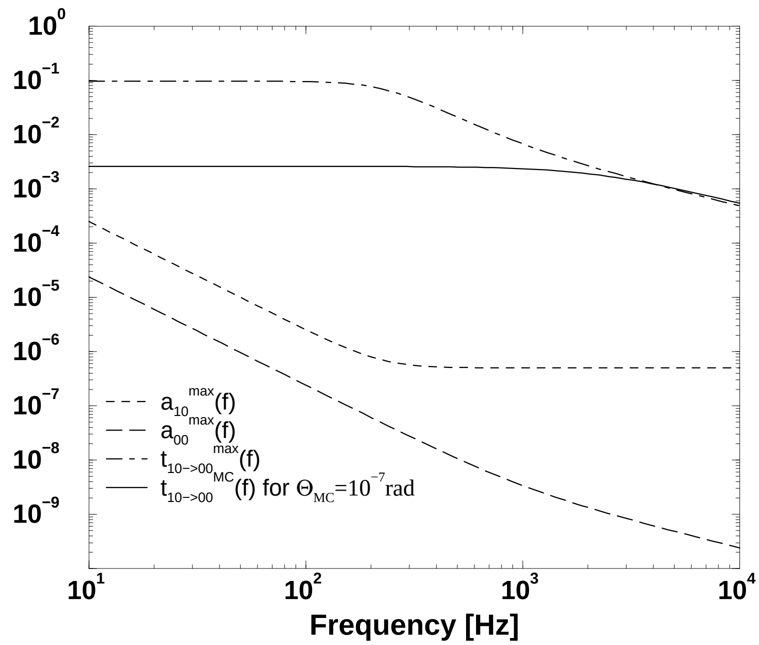

Figure 5: This graph shows the requirements on the PM-sidebands $a_{00}^{max}(f)$ as set in 6, the requirements on the tilt mode $a_{10}^{max}(f)$ for a main interferometer with misaligned ITM-mirrors ($\Delta\Theta_{ITM} = 10^{-8}$rad). They also show the maximum allowed transfer $t_{10\to 00}^{max}(f)$ from the tilt mode into the 00-mode and the transferfunction $t_{10\to 00}^{MC}(f)$ for a mode cleaner where all the MC-mirrors are tilted by 10^{-7}rad.

The frequency dependence of $a_{00}^{max}(f)$ and $a_{10}^{max}(f)$ leads to the fact that the maximal allowed transfer function becomes a function of the upper GW-frequency. This is shown in Fig. 5. If the pointing just matches its requirements set by the misalignment in the core optics, the mode cleaner mirrors have to be aligned within 10^{-7}rad. The additional frequency noise generated by the pointing of the laser field would then match the requirements at all frequencies above about 3 kHz and would be below the required level at frequencies below 3 kHz.

We have to trade off between the requirements for mode cleaner alignment and requirements on the amplitude of the tilt mode. In the low frequency region below 200 Hz we only need an alignment of $3 \cdot 10^{-5}$rad for MC-mirrors. Such an alignment accuracy could be good enough if the amplitude of the pointing sidebands $a_{10}(f)$ would continue to fall off with $1/f^2$ up to 1000 Hz and $1/f$ above that similar to the requirements for $a_{00}(f)$.

References

[Ali] *Pointing and Alignment in Advanced LIGO* (LIGO-G010154-00-Z). Presentation at March 2001 LSC-Meeting in Baton Rouge, Guido Mueller, A publication is in preparation.

[req] *Signal Sensing in Advanced LIGO, Part I* (LIGO-T020023-00-D), Guido Mueller.

[Fri] Advanced LIGO Systems Design, LSC ed. Peter Fritschel LIGO-T010075-00-D (27 June 2001) pg. 18.