LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
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Cantilever Spring and Flexure Sizing for the ETF

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The payloads which must be dealt with in the LIGO II HAM system could have significant variations in moments of inertia and ratios of static and dynamic mass. The seismic isolation system must be able to deal with these payload variations and the ETF prototype would be an ideal system in which to investigate the resulting parameter space.

The purpose of this document is an attempt to provide a very approximate definition of the LIGO II HAM payloads and provide some cantilever spring and flexure sizing for the ETF prototype (following on from Brian's spring sizing document "ETF_Spring_Size_V2.PDF'). The 4km interferometer at Livingston (LHAM1-LHAM5) and the 2km and 4km at Hanford (WHAM1-WHAM5 and WHAM6-WHAM12 respectively) have similar optical layouts and we will therefore only consider the Livingston 4km layout. The "main" optical components are as follows:

LHAM1:	Mode Cleaner 1 Mode Cleaner 3 Mode Matching Telescope 1 Mode Matching Telescope 3 Steering Mirror 1
LHAM2:	Mode Cleaner 2 Mode Matching Telescope 2
LHAM3:	Recycling Mirror Pick-Off Telescope 1
LHAM4:	Pick-Off Telescope 2 Pick-Off Telescope 2 Pick-Off Telescope 2
LHAM5:	

For LIGO II, lets assume that the mode cleaner suspensions, steering mirror and recycling mirror will be replaced with GEO triple pendulums and all other optical components will remain unchanged (except for the inclusion of a signal recycling mirror). I have not made any assumptions about the mode matching telescopes in LIGO II.

From E990303-02 ("LIGO II Seismic Isolation Design Requirements Document") the masses of the triple suspensions and pick-off telescopes are assumed to be 100kg and ≈60kg respectively. I further assume that each triple suspension includes 10kg of static mass in the form of a cradle. I have no good idea about the mode matching telescopes, so let us assume they have a static mass of 20kg and a dynamic mass of 20kg?

The mass budget for the HAM chambers can then be determined:

	m_{static} (kg)	$m_{dynamic}$ (kg)
LHAM1:	70	340
LHAM2:	30	120

LHAM3:	70	100
LHAM4:	180	0
LHAM5:	0	0

Each isolated stage must include some trim mass in order to account for variations in the cantilever spring rate/errors in the fabrication. 20% trim mass should be a conservative estimate. Therefore lets assume that each stage weighs 350kg (300kg +≈20% trim mass). In addition to this there should be ballast/counterbalance mass on the inner stage to allow for variations in payload. For starters, assume that there is sufficient ballast/counterbalance mass to attain a total payload of 800kg (as stated in the "Seismic Isolation Design Requirements Review"). The static and dynamic mass for the worst case HAM systems (LHAM1 and LHAM4/LHAM5) is therefore:

		m_{static} (kg)	$m_{dynamic}$ (kg)
LHAM1:	outer stage	350	1150
	inner stage	810	340
LHAM4/5:	outer stage	350	1150
	inner stage	1150	0

Cantilever Spring Design

The cantilever spring design is somewhat constrained in the sense that there is not arbitrary freedom to choose their length and width. I think that keeping the spring length ≤10" (0.254m) and the spring width ≤6" (0.152m) would be preferable for HPD.

Using ETF_Deisgner3.m (the spring sizing programme written by Brian and Dan) we could design the following springs for LHAM1:

12.5 Hz outer stage:	0.270 x 0.128m x 12.4mm	(length x width x thickness)
5.5Hz inner stage:	0.250m x 0.110m x 11.3mm	(length x width x thickness)

For LHAM4/5 the inner stage frequency would drop to 4.6Hz (due to the larger static mass) whilst the outer stage frequency would remain around 12.5Hz.

Another scenario would be to allow the total mass which must be supported by the springs to be reduced (to attempt to make the spring design more compact). Consider that the total mass of the inner stage is allowed to be 800kg (i.e. the combined mass of the payload and stage). The resulting springs for LHAM1 would be:

11 Hz outer stage:	0.24m x 0.113m x 10.9mm	(length x width x thickness)
6Hz inner stage:	0.23m x 0.104m x 9.3mm	(length x width x thickness)

For LHAM 4/5 the inner stage frequency would drop to 4.5Hz whilst the outer stage frequency would remain around 11Hz. Reducing the payload allows the spring design to be made more compact (shorter and thinner). However, the variation of the frequencies for the different payloads becomes larger as the ratio of static to dynamic mass between LHAM1 and LHAM4/5 increases.

Flexure Design

We can make a first cut at a flexure design by assuming that the first and second stage flexures must suspend 500kg and 383kg respectively. In order to support the load the radius of the flexures must be several mm, and thus the flexures will benefit from being as long as possible (from a bending stress point of view). If we start by assuming that we will keep a length of around 6" (close to the MIT prototype) then the radius can then be chosen in order to keep the combined bending stress (for a 1mm displacement) and dead load stress at about 20% of yield.

outer stage flexures:
- 5.2mm diameter
- dead load stress=$2.3 \times 10^8 N/m^2$
- bending stress=$1.6 \times 10^8 N/m^2$
- elastic stiffness≈1.9xgravitational stiffness
- zero moment point=36mm

inner stage flexures:
- 4.4mm diameter
- dead load stress=$2.5 \times 10^8 N/m^2$
- bending stress=$1.5 \times 10^8 N/m^2$
- elastic stiffness≈1.6xgravitational stiffness
- zero moment point=30mm

Of course, relaxing the requirements on the payload allows the flexures to be thinned, but the flexure design is probably not as constrained as the cantilever springs.