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The analysis of table-top quantum measurement with macroscopic masses

V.B.Braginsky, F.Ya.Khalili, P.S.Volikov
Dept. of Physics, Moscow State University,
Moscow 119899, Russia
e-mail: brag@mol.phys.msu.su,farid@mol.phys.msu.su

Abstract

The analysis of table-top quantum experiment with mechanical test mass is presented. The scheme of experiment is based on two principles: the difference between the free test mass and the oscillator sensitivity Standard Quantum Limits, and the use of mechanical rigidity produced by an optical pumping field in Fabry-Perot resonator to convert the free test mass into the mechanical oscillator having very low intrinsic noises. The analysis shows that proposed scheme allows to overpass the free test mass Standard Quantum Limit by the factor $\xi \simeq 0.1$.

1 Introduction

There is an evident steady progress in improvement of the sensitivity in many types of physical measurements. Particularly, in the previous century late 80-s several groups of experimentalists successfully demonstrated the resolution better than the Standard Quantum Limit (SQL) in optical domain using QND methods (e.g. see review article [1]). At the end of the 90-s even more impressive experiment was realized in the microwave domain. In this experiment single microwave quanta were counted without absorption [2]. At the same time experiments with resolution better than SQL using mechanical test objects are not yet realized.

There is at least one area in the experimental physics where the necessity to circumvent the SQL of sensitivity using mechanical test masses is crucially important. This is the terrestrial gravitational wave antennae creation. At the stage II of the LIGO project (years 2006-8) the antennae sensitivity is expected to be close to the SQL, and in the stage III the sensitivity will have to be better [3]. There were several articles with different schemes of measuring devices aimed to "beat" the SQL for mechanical objects (see, for example, articles [4, 5, 6, 7]. In the majority of these articles only concepts of new methods were presented and only one of them [7] has provided rather detailed analysis of the measurement scheme using mechanical object (mirror of the gravitational-wave antenna) based on QND principle. It was shown that proposed scheme can be implemented provided that very sophisticated cryogenic technique is used.

In this article we present the analysis key parts for a simple experiment scheme using relatively small test masses that is able to provide the sensitivity better than the free test mass SQL and can be realized in relatively modest laboratory conditions.

The first initial principle of the scheme is based on the difference between the sensitivity SQLs for the force F acting on the free mass m and mechanical oscillator having the same mass and eigenfrequency Ω_m:

$$F_{\rm SQL}^{\rm free\,mass} \simeq \sqrt{\frac{\hbar m \Omega_F^2}{\tau}}, \tag{1a}$$

$$F_{\rm SQL}^{\rm oscillator} \simeq \frac{\sqrt{\hbar m \Omega_F}}{\tau}, \tag{1b}$$

where Ω_F is the mean frequency of the force and τ is its duration. These equations are valid if $\tau \gtrsim 1/\Omega_F$ and $|\Omega_F - \Omega_m| \lesssim 1/\tau$.

Comparing equations (1) one may conclude that it is possible to "beat" the *free mass* SQL by the factor

$$\xi = \frac{F_{\rm SQL}^{\rm oscillator}}{F_{\rm SQL}^{\rm free\,mass}} \simeq \left(\frac{1}{\Omega_F \tau}\right)^{1/2}, \tag{2}$$

using test mass with sufficiently low noise rigidity $m\Omega_m^2$ attached to it. It can be shown (see Appendix A) that the exact form of this condition is

$$\xi = \left(\frac{\Delta\Omega}{\Omega_F}\right)^{1/2}, \tag{3}$$

where $\Delta\Omega \simeq 1/\tau$ is the bandwidth of the force.

The second initial principle of the scheme is based on the possibility to create mechanical rigidity provided by the dependence of light pressure on the detuned Fabry-Perot resonator mirrors position. We have already analyzed the possibility to create very low noise rigidity using the pumping frequency ω_p detuned much far from the resonator eigenfrequency ω_o [8]. It was supposed in article [8] that separate resonator must be used as a measuring device.

In this article we propose another simpler scheme where the same resonator serves as the meter and the rigidity source. In this case optimal detuning $\delta = \omega_p - \omega_o$ should be close to the resonator semi-bandwidth $\gamma = \omega_o/2Q_{\rm FP}$, where $Q_{\rm FP}$ is the quality factor of the resonator. We show that this scheme allows to reach the limiting value ξ [see formula (3)].

We have to note that the potential possibilities to use the mechanical rigidity of optical origin in different LIGO readout meters were already discussed in [9, 10]. These possibilities are indicating that SQL can be circumvented in narrow bandwidth. In the presented below analysis we were trying to give answers to most important practical issues which may appear in the implementation of such an experiment.

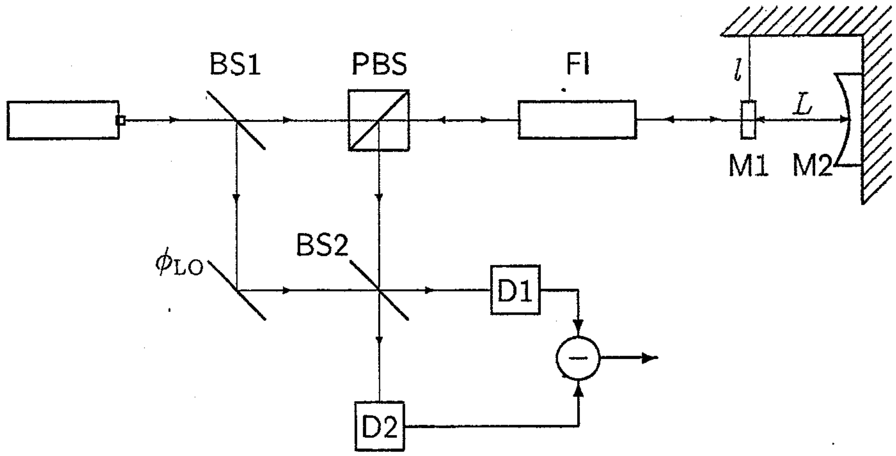

Figure 1: Sketch of the experimental scheme

2 The main elements of the design and the experiment scheme

2.1 The test mass suspension

The most important elements of the experimental setup are the test mass suspension, optical rigidity, and optical readout scheme. Simplified sketch of experimental scheme is presented on Fig.1.

The necessary condition for such kind of experiments is a sufficiently low dissipation in the suspension. The quality factor of the mechanical test oscillator has to exceed value

$$Q_m \gtrsim \frac{2\kappa T}{\hbar\Omega_F\xi^2} \simeq \frac{10^{10}}{\xi^2} \times \left(\frac{T}{300\text{K}}\right) \times \left(\frac{10^4\,\text{s}^{-1}}{\Omega_F}\right), \tag{4}$$

where κ is the Boltzmann constant, and T is the heatbath temperature. In fact, this inequality represents the condition that the fluctuating force originated from mechanical losses in the suspension (according to FDT) has to be ξ^{-1} times smaller than the force $F_{\rm SQL}^{\rm free\,mass}$.

The estimate (4) show that an ordinary mechanical spring can't be used here. It is necessary to use "artificial" rigidity with very low intrinsic noises, and optical ponderomotive rigidity does look promising. In this case the suspension can be similar to the Galileo pendulum with eigenfrequency $\Omega_{\rm pend} \ll \Omega_F$. The relaxation time of $\tau^*_{\rm pend} \simeq 2 \times 10^8$s has been already obtained for all fused silica suspension of the LIGO mirror model. This value of $\tau^*_{\rm pend}$ allows in principle to obtain the quality factor of $Q_m \simeq \Omega_m\tau^*_{\rm pend} \simeq 2\times 10^{12}$ (even if the viscous model of friction is valid) and thus allows to reach $\xi \simeq 0.1$. In the Appendix C more rigorous analysis of the suspension noises is presented, which shows that these noises do not prevent from obtaining the sensitivity of $\xi \lesssim 0.1$.

It is evident that the platform the suspending fiber has to be welded to must be a compact one: the mechanical eigenmodes of the platform have to be substantially higher than the chosen value of Ω_F. If the value $m \simeq 2 \times 10^{-2}$g (a few millimeters in dimension cylinder covered with high-reflectivity multilayer coating) then for $\Omega_F \simeq 10^4\text{s}^{-1}$ the meter has to register the oscillations of the mass with the amplitude of

$$\Delta x = \sqrt{\frac{\hbar}{m\Omega_F}} \simeq 2 \times 10^{-15}\text{cm}\,.$$

We omit here calculations which show that if the platform has sizes of a few centimeters and is manufactured from fused silica then quality factor of the eigenmodes that is higher than 10^5 will be sufficient to register this value of Δx.

It seems appropriate to use the first mirror of the Fabry-Perot resonator M1 as the test mass and the second mirror M2 must be attached rigidly to the platform (see Fig.1).

2.2 The optical rigidity

It is a relatively easy task to "convert" the mass $m \simeq 2\times 10^{-2}$g into a mechanical oscillator with eigenfrequency $\Omega_m \simeq 10^4\text{s}^{-1}$. If the mass is the Fabry-Perot resonator mirror (see Fig. 1) and the laser is tuned on the one of the resonator resonance curves slope then the rigidity will be equal to

$$m\Omega_m^2 = \frac{16\omega_o W \mathcal{F}^2}{\pi^2 c^2}\frac{\delta/\gamma}{\left[1+(\delta/\gamma)^2\right]^2} \simeq 2\times 10^6\text{dyn/cm} \times \left(\frac{W}{50\text{mW}}\right) \times \left(\frac{\mathcal{F}}{10^3}\right)^2 \times \frac{\delta/\gamma}{\left[1+(\delta/\gamma)^2\right]^2}\,. \tag{5}$$

where $\omega_o = 2 \times 10^{15}\text{s}^{-1}$ is the optical pumping frequency, $\mathcal{F}$ is the finesse of the optical resonator, W is the pumping power.

Fig.2 illustrates the dependence of the laser ponderomotive force on the distance between the mirrors. Dashed line corresponds to the pendulum rigidity $m\Omega_{\text{pend}}^2$. There is a relatively big number n of static equilibrium points (which correspond to the crossings of the right slopes of the resonant curves with the horizontal axis; these points are marked by Xs on the Fig.2):

$$n = \frac{4\omega_o W\mathcal{F}}{\pi^2 m c^3 \Omega_{\text{pend}}^2} \simeq 500 \times \left(\frac{W}{50\text{mW}}\right) \times \left(\frac{\mathcal{F}}{10^3}\right) \times \left(\frac{10\text{s}^{-1}}{\Omega_{\text{pend}}}\right)^2\,. \tag{6}$$

Thus by choosing one of these points experimentalists may change the rigidity. Another way for changing it is to apply a d.c. force onto the mirror (using for example the light pressure from another laser).

This rigidity has one disadvantage: it is associated with negative friction which corresponds to the characteristic time ("negative relaxation time") equal to

$$\tau_{\text{instab}} = \frac{\gamma}{2\Omega_m^2}\left[1+(\delta/\gamma)^2\right]. \tag{7}$$

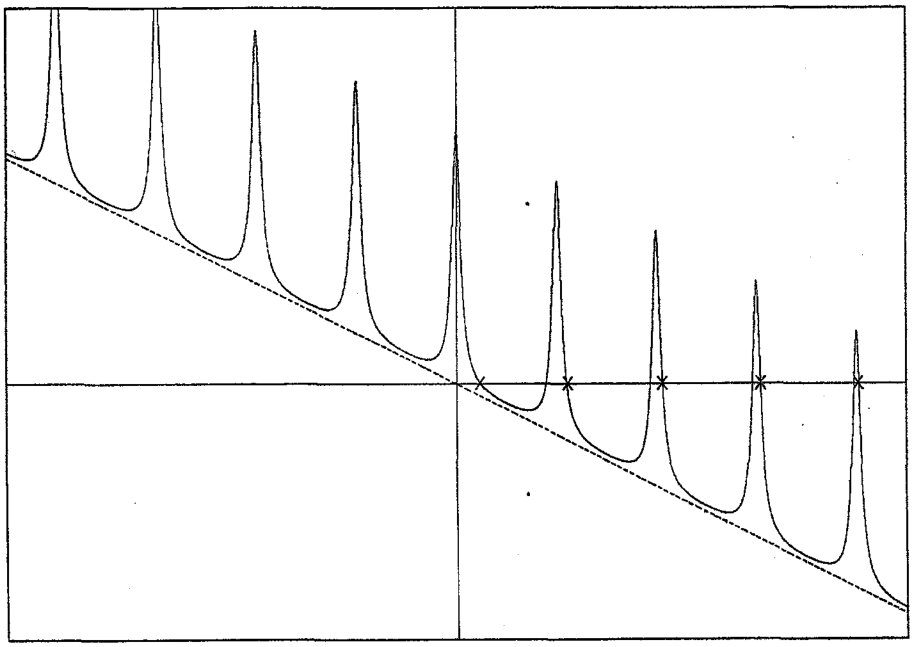

Figure 2: Dependence of the ponderomotive force on the distance between the mirrors

If the finesse of the resonator is $\mathcal{F} \simeq 10^3$ and its length is $L \lesssim 1$cm then $\gamma \gtrsim 5 \times 10^7 \mathrm{s}^{-1}$, and the value of τ_{instab} can be large enough to provide sufficient time for the measurement, $\Omega_m \tau_{\mathrm{instab}} \gtrsim 10^4$.

2.3 The optical readout scheme

The optical readout scheme is presented on the left part of the Fig.1 The pumping laser beam is splitted into two beams (the signal beam and the reference one). The signal beam enters the Fabry-Perot resonator and passes back carrying information about the displacement of the mirror M1 relative to M2 in its phase. The reflected beam is separated from the input one by polarization beam splitter PBS and Faraday isolator FI, and then is combined with the reference beam on the beamsplitter BS2. This beamsplitter together with photodetectors D1 and D2 form standard balanced homodyne detector. It is assumed that an arbitrary phase shift ϕ_{LO} can be added to the reference beam.

The difference between the photocurrents in such a scheme depends on the phase shift of the signal beam relative to the reference one,

$$I_1 - I_2 = \frac{2e\sqrt{W W_{\mathrm{ref}}}}{\hbar\omega_o} \cos\left(\phi_1 + \frac{2\gamma\omega_o}{\gamma^2+\delta^2}\frac{x}{L} - \phi_2 - \phi_{\mathrm{LO}}\right)$$

and thus provides information about x. Here W_{ref} is the reference beams power, ϕ_1, ϕ_2 are the initial phases of the signal and reference beams.

It can be shown (see Appendix B) that if the Fabry-Perot resonator bandwidth is defined by the nonzero transmittance of the mirror M1 only, if there are no losses in all optical elements which the signal beam passed through, if quantum efficiency of the photodetectors are equal to unity and quantum state of the pumping beam is pure coherent one then the value (3) can be achieved in this ideal case.

In more realistic case when the above conditions are not fulfilled the total achievable value of ξ^2 is a sum of two terms: the "ideal" value described by formula (3) and additional value $\xi^2_{\rm optics}$ which depends on the parameters of the optical scheme. General expression for $\xi^2_{\rm optics}$ is very cumbersome. In asymptotic case when $\Delta\Omega/\Omega_F \ll 1$, losses are small and sufficiently large value of the pumping power can be provided, the value of $\xi^2_{\rm optics}$ can be presented as

$$\xi^2_{\rm optics} \approx \frac{1}{\sqrt{P}}\,\frac{1+(\delta/\gamma)^2}{(\delta/\gamma)^{3/2}} + \frac{\mathcal{A}}{\delta/\gamma}\,, \qquad (8)$$

where P is a dimensionless parameter proportional to the pumping power:

$$P = \frac{64\omega_o W}{mc^2\Omega_F^2(1-\mathcal{R}_2)^2} \approx 1.5\times 10^4 \times \left(\frac{W}{50\mathrm{mW}}\right) \times \left(\frac{20\mathrm{mg}}{m}\right) \times \left(\frac{5\times 10^{-5}}{1-\mathcal{R}_2}\right)^2 \times \left(\frac{10^4\mathrm{s}^{-1}}{\Omega_F}\right)^2, \qquad (9)$$

$\mathcal{R}_2$ is the mirror M2 reflectivity,

$$\mathcal{A} = 1-\eta_{\rm PD}(1-\mathcal{A}_0)(1-\mathcal{A}_1) \approx 1-\eta_{\rm PD}+\mathcal{A}_0+\mathcal{A}_1 \qquad (10)$$

is total "external losses", $\eta_{\rm PD}$ is quantum efficiency of the photodetectors, $\mathcal{A}_0$ is total absorption factor of the optical elements between the Fabry-Perot resonator and the photodetectors, $\mathcal{A}_1$ is the mirror M1 absorption factor. Expression (8) is valid if $P \gg 1$ and $\mathcal{A} \ll 1$.

It is evident that the sensitivity depends on the detuning δ. Optimal value of δ depends on whether first or second term prevails in the expression (8). If $\mathcal{A} \ll 1/\sqrt{P}$ then the optimal value is $\delta = \sqrt{3}\gamma$ and

$$\xi^2_{\rm optics} = \frac{4}{\sqrt{3\sqrt{3}P}} \approx \frac{1.75}{\sqrt{P}}\,. \qquad (11)$$

In the opposite case the detuning must be large, $\delta/\gamma = (4\mathcal{A}^2P)^{1/3} \gg 1$ and in this case

$$\xi^2_{\rm optics} = \frac{3}{2}\left(\frac{\mathcal{A}}{P}\right)^{1/3}. \qquad (12)$$

These estimates show that even in the case of moderate conditions for the optical elements parameters losses in them don't prevent from obtaining the value of $\xi_{\rm optics} \simeq 0.1$ when the pumping power is sufficiently large, *e.g.* $W \gtrsim 50$mW.

If such a value of pumping power can't be provided then, nevertheless, the sensitivity slightly better than the SQL can be obtained, i.e. $\xi_{\rm optics} \simeq 0.3 \div 0.5$. Sensitivity for this

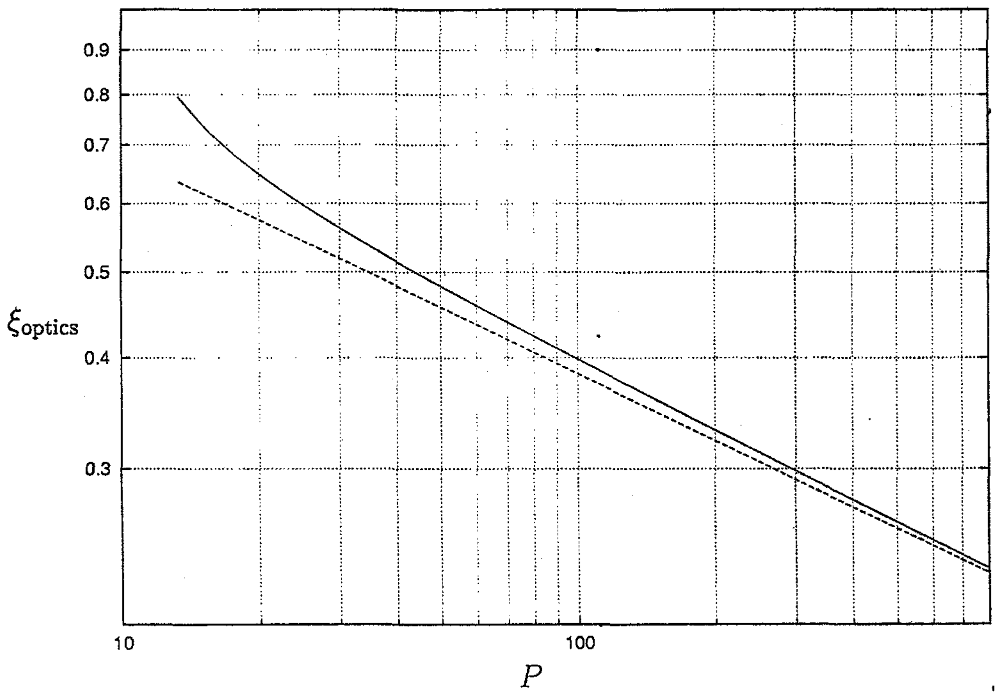

Figure 3: Sensitivity as a function of the pumping power

case is calculated numerically. The results for the case when $\Delta\Omega/\Omega = 0.01$ are presented on the Fig.3 (solid line). Dashed line is the asymptote (29).

3 Conclusion

The scheme of measurement presented above may be regarded only as the first step along the route of "divine quantum" measurements with macroscopic quantum objects. The main goal of the proposed experiment will be to show that various noises of nonquantum origin do not prevent to achieve sensitivity better than the SQL even at room temperatures. Evidently there are others, more sophisticated schemes of measurements which may provide better sensitivity. At present there are two evident "candidates". In the first one (which is the simplier one) it is possible to use variational measurement [6] by periodic modulation of the phase of the reference beam $\phi_{\rm LO}$. The second one is the stroboscopic measurement [5] which is in essence a Quantum Non-Demolition measurement of the probe oscillator quadrature amplitude. In the second case it will be necessary to use two pumping lasers: the first one has to be permanently on and it will provide the rigidity, and the second one has to be turned on periodically during the time interval much shorter than Ω_m^{-1}.

The authors of this article have no doubts that the sensitivity better than the SQL may be obtained at the present level of experimental "culture" in the measurement with mechanical object.

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A The Standard Quantum Limits for the free mass and the oscillator

Total net noise of linear scheme for detection of classical force acting on the free test mass m is equal to [12]

$$F_{\text{free mass}}(t) = m\frac{d^2}{dt^2}x_{\text{fluct}}(t) + F_{\text{fluct}}(t)\,. \tag{13}$$

where $x_{\text{fluct}}(t)$ is the additive noise of the meter and $F_{\text{fluct}}(t)$ is its back-action noise.

For an ordinary position meter which sensitivity is limited by the SQL, these two noises are non-correlated and have frequency-independent spectral densities S_x and S_F, correspondingly, which satisfy the uncertainty relation

$$S_x S_F \geq \frac{\hbar^2}{4}\,. \tag{14}$$

We will suppose that noises are as small as possible and exact equality takes place in this formula.

In this case spectral density of the total noise (13) is equal to

$$S_{\text{free mass}}(\Omega) = m^2\Omega^4 S_x + S_F\,. \tag{15}$$

For any given observation frequency $\Omega = \Omega_F$ this value can be minimized by adjusting the ratio of the spectral densities $S_F/S_x = m^2\Omega_F^4$, giving

$$S^{\text{SQL}}_{\text{free mass}}(\Omega_0) = \hbar m\Omega_0^2. \tag{16}$$

This is the spectral form of the SQL for a free test mass [7].

In the case of an oscillator total net noise is equal to

$$F_{\text{oscillator}}(t) = \left(m\frac{d^2}{dt^2} + \Omega_0^2\right) x_{\text{fluct}}(t) + F_{\text{fluct}}(t)\,, \tag{17}$$

and its spectral density is equal to

$$S_{\text{oscillator}}(\Omega) = m^2(\Omega_0^2 - \Omega^2)^2 S_x + S_F\,. \tag{18}$$

By adjusting the ratio S_F/S_x in order to provide minimum of the spectral density at the edges of the narrow vicinity of the eigenfrequency Ω_0, $\Omega = \Omega_0 \pm \Delta\Omega/2$, where $\Delta\Omega \ll \Omega_0$, we obtain:

$$S^{\rm SQL}_{\rm oscillator}(\Omega_0 \pm \Delta\Omega/2) = \hbar m \Omega_0 \Delta\Omega\,. \tag{19}$$

The ratio of the spectral densities (16) and (19) is equal to (3).

B The sensitivity limitation due to optical losses

Spectral density of the total net noise We will suppose that the Fabry-Perot resonator bandwidth is much larger than the observation frequency. It can be shown (we omit lengthy but quite straightforward calculations) that in the case of the Fabry-Perot position meter spectral densities S_x and S_F of the noises $x_{\rm fluct}$ and $F_{\rm fluct}$ introduced in the previous Appendix and their cross spectral density are equal to

$$S_x = \frac{\hbar}{2m\Lambda^2}\frac{1+(\delta/\gamma)^2}{\eta\sin^2\phi_{\rm LO}}\,, \qquad S_F = \frac{\hbar m\Lambda^2}{2}\frac{1}{1+(\delta/\gamma)^2}\,, \qquad S_{xF} = \frac{\hbar}{2}\cot\phi_{\rm LO}\,, \tag{20}$$

and the electromagnetic rigidity is equal to

$$\mathcal{K} = m\Omega_0^2 = \frac{m\Lambda^2}{2}\frac{\delta/\gamma}{1+(\delta/\gamma)^2} = \frac{4\omega_o\gamma_1(1-\mathcal{A}_1)W}{L^2\gamma^3}\,\frac{\delta/\gamma}{\left[1+(\delta/\gamma)^2\right]^2}\,, \tag{21}$$

where

$$\Lambda^2 = \frac{4\omega_o\mathcal{E}}{mL^2\gamma}\,, \qquad \eta = \frac{\gamma_1}{\gamma}(1-\mathcal{A})\,, \qquad \gamma_{1,2} = \frac{1-\mathcal{R}_{1,2}}{4L/c} \qquad (\gamma_1+\gamma_2=\gamma)\,, \tag{22}$$

$\mathcal{E}$ is pumping energy in the resonator, $\mathcal{R}_1, \mathcal{R}_2$ are the mirrors M1, M2 reflectivities, $\mathcal{A}$ is total "external losses" [see formula (10)].

Spectral density of the total noise of the meter in this case is equal to

$$S(\Omega) = \frac{\hbar m}{2}\left\{\frac{(\Omega^2-\Omega_F^2)^2\left[1+(\delta/\gamma)^2\right]}{\Lambda^2\eta\sin^2\phi_{\rm LO}} + \frac{1-\eta\cos^2\phi_{\rm LO}}{1+(\delta/\gamma)^2}\Lambda^2\right\}. \tag{23}$$

where

$$\Omega_F^2 = \Omega_0^2 - \frac{1}{2}\,\frac{\Lambda^2\eta\sin 2\phi_{LO}}{1+(\delta/\gamma)^2} = \frac{4\omega_o\gamma_1(1-\mathcal{A}_1)W}{mL^2\gamma^3}\,\frac{\delta/\gamma-\eta\sin 2\phi_{\rm LO}}{\left[1+(\delta/\gamma)^2\right]^2}\,. \tag{24}$$

Large pumping power Now our goal is to minimize the expression

$$\xi^2 \equiv \frac{S(\Omega_F \pm \Delta\Omega/2)}{\hbar m\omega_F^2} = \frac{(\Delta\Omega)^2\left[1+(\delta/\gamma)^2\right]}{2\Lambda^2\eta\sin^2\phi_{\rm LO}} + \frac{1-\eta\cos^2\phi_{\rm LO}}{\delta/\gamma - \eta\sin 2\phi_{\rm LO}}. \tag{25}$$

It is evident that in order to obtain $\xi \ll 1$ it is necessary to have $\Delta\Omega/\Omega_F \ll 1$, $1-\eta \ll 1$ and $|\phi_{\rm LO}| \ll 1$. Taking it into account one can show that the expression (25) is minimal if

$$\phi_{\rm LO} = \phi_{\rm LO}^{\rm opt} \approx -\sqrt{\frac{\Delta\Omega}{\Omega_F}\,\frac{\delta/\gamma}{2}}\,, \tag{26}$$

and the minimum is equal to

$$\xi^2 \approx \frac{\Delta\Omega}{\Omega_F} + \frac{\gamma_2/\gamma + \mathcal{A}}{\delta/\gamma}\left(1+\frac{2\phi_{\rm LO}^{\rm opt}}{\delta/\gamma}\right). \tag{27}$$

From this expression it is evident that the larger is the ratio δ/γ the smaller is ξ. On the other hand, the larger is this ratio the larger is the pumping power required to provide given $\Omega_F \approx \Omega_0$, see formula (21). If $\mathcal{A}_1 \ll 1$ and $\gamma_2 \ll \gamma_1$ then from (24) it follows that

$$\gamma \approx \sqrt{\frac{4\omega_o W}{mL^2\Omega_F^2}}\,\frac{\sqrt{\delta/\gamma}}{1+(\delta/\gamma)^2}\left(1-\frac{\phi_{\rm LO}^{\rm opt}}{\delta/\gamma}\right). \tag{28}$$

Substitution of this expression into formula (27) gives that

$$\xi^2 \approx \frac{\Delta\Omega}{\Omega_F} + \frac{1}{\sqrt{P}}\,\frac{1+(\delta/\gamma)^2}{(\delta/\gamma)^{3/2}}\left(1+\frac{3\phi_{\rm LO}^{\rm opt}}{\delta/\gamma}\right) + \frac{\mathcal{A}}{\delta/\gamma}\left(1+\frac{2\phi_{\rm LO}^{\rm opt}}{\delta/\gamma}\right), \tag{29}$$

Omitting here small terms proportional to $\phi_{\rm LO}^{\rm opt}$ we obtain formula (8).

Small pumping power If $P \simeq 1$ then it is possible to neglect the second term in the formula (8) because any good optical components can provide the value $\mathcal{A} \lesssim 0.1$. In this case it is necessary to minimize expression (25) with respect to $\phi_{\rm LO}$, γ_1 and δ with given values of the W and γ_2 and with additional condition (24). This minimization was performed numerically. Results are presented on the Fig.3.

C The suspension noises

We will base our consideration on the formula (11) in the article [12]. If the observation frequency Ω_F satisfies condition $\Omega_{\rm pend} \ll \Omega_F \ll \Omega_{\rm v}$, where $\Omega_{\rm v}$ is the eigenfrequency of the suspension fiber fundamental violin mode then this formula can be rewritten as:

$$S_x^{\rm susp} = \frac{4\kappa T}{I^2\Omega_F^6 l^2}\left\{\left[\frac{I}{m} - Rh\right]^2\zeta_{\rm top} + \left[\frac{I}{m} - (R+l)h\right]^2\zeta_{\rm bot}\right\}, \tag{30}$$

where l is the length of the suspension fiber, I is the test-mass moment of inertia for rotation about the center of masses, R is the radius of the mirror face, m is the mass of the test mirror, h is the displacement of the laser beam spot from the center of the mirror, $\zeta_{\rm top}, \zeta_{\rm bot}$ are values characterizing dissipation at the top and the bottom of the fiber. Following authors of the article [12] we suppose that

$$\zeta_{\rm top} = \zeta_{\rm bot} = \zeta = \frac{\Omega_F \phi \sqrt{YJmg}}{2}, \tag{31}$$

where Y is the Young modulus of the fiber material and $J = S^2/4\pi$ is the fiber geometrical momentum of inertia. If h is chosen optimally:

$$h = \frac{2R+1}{R^2+(R+l)^2} \cdot \frac{I}{M} \approx \frac{I}{Ml} \tag{32}$$

then (we suppose that $R \ll l$)

$$S_x^{\rm susp} = \frac{4\kappa T\zeta}{m^2\Omega_F^6[R^2+(R+l)^2]} \approx \frac{4\kappa T\zeta}{m^2\Omega_F^6 l^2} \tag{33}$$

This value of S_x corresponds to the spectral density of the fluctuating force acting on the test mass

$$S_{\rm susp} = m^2\Omega_F^4 S_x^{\rm susp} = \frac{4\kappa T\zeta}{\Omega_F^2 l^2} = \frac{2\kappa T\phi\sqrt{YJmg}}{\Omega_F l^2}. \tag{34}$$

So the value of ξ^2 limited by the suspension noise is equal to

$$\xi^2_{\rm susp} = \frac{S_{\rm susp}}{\hbar m \Omega_F^2} = \frac{2\kappa T\phi}{\hbar\Omega_F^3 l^2}\sqrt{\frac{YJg}{m}} = \frac{\kappa T g}{\pi^2\hbar v_o^2}\,\frac{\phi r \Omega_{\rm v}^2}{\mu^{3/2}\Omega_F^3}, \tag{35}$$

where $v_o = \sqrt{Y/\rho}$ is the speed of sound in the fiber material, $\mu = \dfrac{mg}{YS}$ is dimensionless stress factor of the fiber.

For the room temperature and fused silica it will be

$$\xi^2_{\rm susp} \approx 4\times 10^{-4} \times \left(\frac{\phi r}{10^{-8}\,{\rm dyn/cm}}\right) \times \left(\frac{10^4\,{\rm s}^{-1}}{\Omega_F}\right) \times \left(\frac{\Omega_{\rm v}}{\Omega_F}\right)^2 \times \left(\frac{10^{-3}}{\mu}\right)^{3/2} \tag{36}$$

Taking into account that values $\phi r \lesssim 10^{-8}$dyn/cm has been already obtained experimentally [13, 14] it is possible to conclude that suspension noises don't prevent from obtaining the sensitivity $\xi \lesssim 0.1$.

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