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Effect of Seismic Cutoff on Observations of BH/BH Mergers with Intermediate Mass

Kip S. Thorne, Caltech

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1 Introduction

Peter Fritschel has asked what science will be lost if the seismic cutoff for LIGO-II is changed from 10 Hz to something larger — say, as large as 20 Hz.

The most serious loss may be in the mass range of BH/BH binaries that can be detected. In this note I try to quantify that loss.

2 BH/BH Mergers

I have computed the cosmological redshift z out to which LIGO-II can see the waves from BH/BH mergers, under the following assumptions:

1. A LIGO network of two 4km interferometers and one 2 km interferometer, with noise curves as communicated to me by Ken Strain in early August (Fig. 1), and with seismic cutoffs at 10Hz and at 20Hz. For each cutoff there are two noise curves, one assuming sapphire test masses; the other, fused silica.[1]

2. Optimal signal processing by the method of matched filters. This presumes that by the LIGO-II time frame numerical relativity will have produced a sufficiently reliable family of search templates for the late inspiral and merger phases.

3. A network amplitude signal to noise ratio of 8.

4. Equal-mass black holes with signal strengths, as a function of frequency, as estimated by Flanagan and Hughes [2]. These signal strengths presume that the holes are rapidly spinning and have spins roughly aligned with each other and with their orbital angular momentum vector.

[1] The Sapphire interferometer's poorer performance at low frequencies results from quantum correlations between radiation pressure noise and shot noise [1]: The Sapphire optimization entails adjusting these correlations so as to lower the noise sharply around 200 Hz, at the price of the correlations' raising the noise below about 50 Hz. The worse thermal noise in fused Silica prevents one from gaining by this type of optimization, so there is less price paid at low frequencies but also less improvement around 200 Hz.

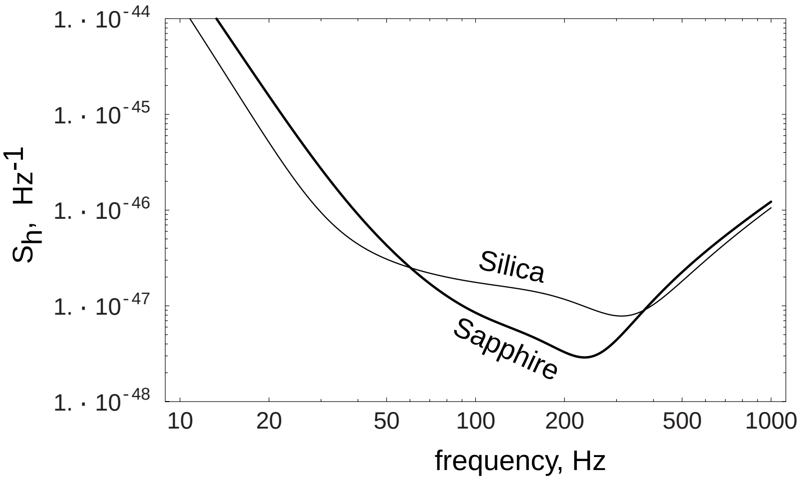

Figure 1: Spectral density of the noise in one 4-km interferometer, $S_h(f)$. The thick curve is for a configuration with Sapphire test masses, an RF output, and parameters optimized for detection of NS/NS inspirals; the thin curve is the same, but for fused Silica test masses. The seismic noise is not included in these curves; in this note we take it into account by a seismic wall at some cutoff frequency $f_{\rm cutoff}$. (These noise curves are from Ken Strain, private communication; they include the Buonanno-Chen quantum analysis of shot noise, radiation-pressure noise, and their correlations [1].)

5. A cosmological model that is spatially flat and has $H_o = 65$ km/s/Mpc, cold matter density in units of critical $\Omega_M = 0.4$, and vacuum polarization density (cosmological-constant density) $\Omega_\Lambda = 0.6$ [3].

While some of these assumptions may be questionable, changing them in plausible ways would likely just rescale the various numbers in my conclusions by amounts that are sufficiently modest as to not be important for a decision about the cutoff frequency.

Figure 2 shows the redshift to which the BH/BH mergers can be seen, under the above assumptions, as a function of the binary's total mass $M = M_1 + M_2 = 2M_1$. Several features of this figure deserve note:

1. LIGO-II can see out to redshifts z of order unity or larger for binary masses throughout the range $M \sim 50 M_\odot$ up to a cutoff frequency $M_{\rm cutoff} \sim 1000 M_\odot$ [i.e., individual hole masses $M_1 = M_2$ from $M_1 \sim 25 M_\odot$ up to $M_{\rm cutoff}/2$].

2. The cutoff mass scales inversely with the seismic cutoff frequency

$$M_{\rm cutoff} \simeq 2000 M_\odot \left(\frac{10{\rm Hz}}{f_{\rm cutoff}} \right) . \tag{1}$$

   This cutoff mass arises from the fact that, for $M > M_{\rm cutoff}$, the binary's waves have been pushed down to frequencies $f < f_{\rm cutoff}$.

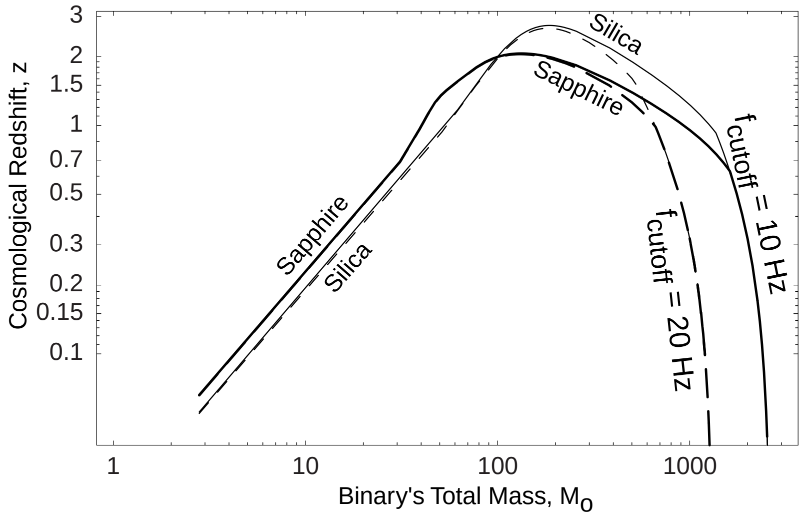

Figure 2: The redshift z to which LIGO-II can see the gravitational waves from BH/BH mergers for Wide-Band, RF-Output, Sapphire detectors with seismic cutoffs at 10 Hz (thick solid curve) and 20 Hz (thick dashed curve), and for Wide-Band, fused Silica detectors with seismic cutoffs at 10 Hz (thin solid curve) and 20 Hz (thick solid curve).

3. The redshift to which the waves can be seen peaks at $z \sim 2$ for sapphire and 3 for silica, and at binary masses $M \sim 100$ to 200 $M_\odot$. This peak occurs because at these masses, the binary's strongest waves are concentrated at frequencies near the minimum of the noise curves, $f \sim 200$ Hz.

Unfortunately we know very little about the event rate for BH/BH mergers with *intermediate-mass* black holes, $M \sim 1000 M_\odot$. There is recent observational evidence that such holes may exist in the universe, and it is very plausible that they might form in galactic nuclei; but it may well be that LIGO's observations will provide the first definitive information about such systems.

This lack of event-rate information makes it very hard to estimate the value of keeping $f_{\rm cutoff} \simeq 10$ Hz. The fact that the redshift to which the binaries can be seen is falling, with increasing M, at $M \gtrsim 200$ Hz, suggests that holding to $f_{\rm cutoff} \sim 10$ Hz may have less value than otherwise would be the case. Nevertheless, it is conceivable that the mass range between 1000 and $2000 M_\odot$ will turn out to be a key one and we could someday regret it if we wiped out the possibility of seeing BH/BH mergers, in LIGO-II, in this mass range by increasing $f_{\rm cutoff}$ from 10 to 20 Hz.

In any attempt to evaluate the consequences of changing $f_{\rm cutoff}$ for BH/BH merger waves, it may be helpful to know the estimates of how the event rate scales with the redshift z to which the waves can be seen. It is plausible that the rate at fixed mass M and per comoving volume in the expanding universe is proportional to the birth rate for massive stars, per comoving volume, which has been estimated by Madau [4] and others. Folding Madau's massive-star birth rate into the cosmological mode $H_o = 65$ km/s/Mpc, $\Omega_M = 0.4$, $\Omega_\Lambda = 0.6$, I obtain that the rate of

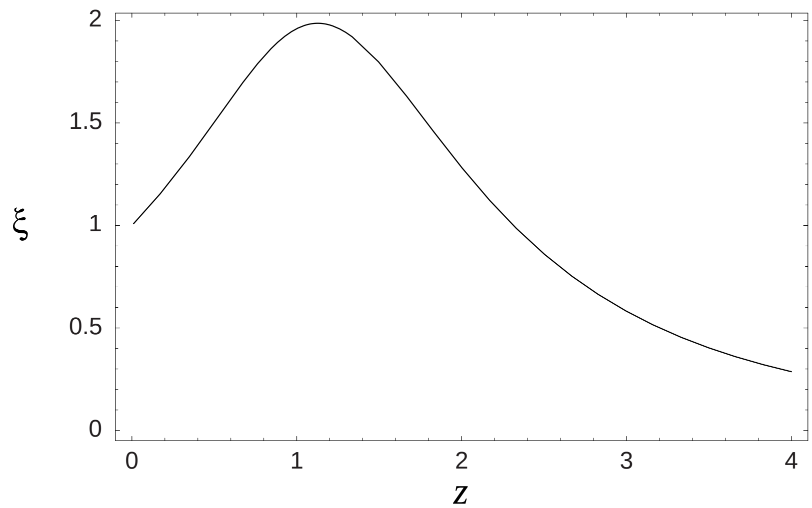

Figure 3: The factor $\xi(z)$ in the formula $\mathcal{R} = \xi(z)z^3$ for the BH/BH merger rate inside redshift z, assuming that the comoving rate is proportional to the comoving rate of massive star formation as estimated by Madau [4].

events (per unit earth time) inside redshift z scales as

$$\mathcal{R} \propto \xi(z)z^3 \tag{2}$$

where $\xi(z)$ is plotted in Fig. 3. This says that the event rate at fixed M inside $z = 2$ is $2^3/2 \sim 4$ times higher than inside $z = 1$, and inside $z = 3$ is $3^3/4 \sim 7$ times higher than inside $z = 1$. Of course this estimate could be wrong, and it says nothing about absolute event rates or how the rate scales with M.

If the numbers and curves in this paper come to play a significant role in any decision about the choice of $f_{\rm cutoff}$, then my computations should be checked by somebody before a final decision is made.

3 My Personal Prejudices

I suspect that among all sources that we know of, the BH/BH mergers discussed in this note are the ones that will be most seriously affected by a change of seismic cutoff. However, it is important that a careful estimate be made of the sensitivity to stochastic background as a function of the cutoff; that might turn out to be more important.

Of course, unknown sources could also be sensitive to the cutoff.

Assuming the BH/BH mergers are the dominant issue, my own prejudice is that it would be unwise to pull all the way back to a cutoff of 20 Hz; but that pulling back from 10 to 12 or 13 Hz could be justified by the factors cited in Fritschel's email of 4 October 2000. But this is only a prejudice.

Much more important, I think, than the seismic cutoff is seeking good sensitivity and noise-curve configurability in the band from 200 Hz to 1000 Hz. This is likely to be of great importance for Low Mass X-Ray Binaries and other spinning neutron stars, and for the final merger waves from NS/BH binaries and modest-mass BH/BH binaries — sources that are likely to be plentiful and to send us rich information in the 200 to 1000 Hz band.

References

[1] A. Buonanno and Y. Chen, Phys. Rev. Lett., submitted; gr-qc/0010011.

[2] E.E. Flanagan and S.A. Hughes, Phys. Rev. D **57**, 4535 (1998); gr-qc/9701039.

[3] M.S. Turner, in *Proceedings of Particle Physics and the Universe (Cosmo-98)*, ed. D.O. Caldwell (AIP, Woodbury NY, 1999); astro-ph/9904051.

[4] Eq. (4) of P. Madau, astro-ph/9902228.