LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
CALIFORNIA INSTITUTE OF TECHNOLOGY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Document Type	**LIGO-T000005-00 -**	**W**	12/13/99

2k FMy Pendulum Q

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Distribution of this draft:

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1 ABSTRACT

A measurement of the quality factor Q was made of the Folding Mirror at LHO. This measurement is important to LIGO because we want to verify that the LIGO optics have a high Q, or a large ringdown time, as prescribed in the COC Design Requirements (LIGO-TXXX). A large ringdown time also means a large ringup time, so if a frequency of motion were to transfer to the pendulum from external forces, it would take a long time before the optic started to swing with a large amplitude, detrimental to LIGO.

2 EQUATION VARIABLES

m	*mass*	ω_o	*angular frequency*
a	*acceleration*	γv	*resistive force*
b	*damping constant*	A	*amplitude*
v	*velocity*	t	*time*
k	*spring constant*	f	*frequency*
x	*position*	Q	*quality factor*

3 THEORY AND MEASUREMENT

In order to find the “quality factor” Q of the Folding Mirror (FMy) pitch and yaw motions, we looked at a segment of optical lever data that included an excitation and a subsequent ringdown damping of the mirror (see Figures 1 and 4). This excitation was due to an earthquake in California which sent the LIGO optics into oscillation. The electronic servo that signals the damping of the FMy was disabled at the time of the earthquake, so the optic was allowed to damp naturally, under no external forces.

To describe this damping motion, we began by looking at the fundamentals of Simple Harmonic Motion (SHM) in order to derive an expression to fit our ringdown. The Equation of Motion (EOM) to describe our system comes from Newton’s Second Law:

1.) $$\sum F = m\frac{\partial^2 x}{\partial t^2} = -b\frac{dx}{dt} - kx$$

2.) $$ma + bv + kx = 0$$

where,

3.)
$$\omega_\circ^2 = \frac{k}{m}$$

$$\gamma = \frac{b}{m}$$

After substitution, the EOM looks like this:

4.)
$$a + \gamma v + \omega_\circ^2 x = 0$$

Equation **4** has the solution of the form:

5.)
$$A(t) = A_\circ e^{-\gamma\frac{t}{2}}$$

A new parameter has been defined to facilitate a measurable quantity, the “quality factor” Q:

6.)
$$Q = (\omega_\circ / \gamma)$$

Substituting this into our solution, we get:

7.)
$$A(t) = A_\circ e^{-\omega_\circ\frac{t}{2Q}}$$

This is the equation we used to fit to our segment of data. We used the data viewer non-linear curve fitting program to fit a line to our data of the form:

8.)
$$y = A0e^{-\frac{x}{A1}} + A2$$

To give the program some starting estimates, we read the A0 and A2 values from the data segment graph, A0 being the initial amplitude of the curve, and A2 being the baseline amplitude shift value after ringdown (see Figure 2). After a hundred or more iterations of curve fitting, the program gave us the real values of A0, A1, and A2 and a nice line that fits the curve of our data segment. Setting equations 7 and 8 above equal to each other, we find that the value A1 will yield the calculation of Q:

9.)
$$A1 = 2\frac{Q}{\omega_o}$$

$$Q = \frac{A1\omega_o}{2}$$

4 RESULTS

We calculated the Q for both the minimum and maximum values of the yaw and pitch, in hopes that they would be the same. This was not the case for the yaw measurement as you can see from the calculated values in Table 1 below and by visual inspection of Figure 4. The minimum and maximum lines are not symmetric about the mean, and therefore would have different curve fitting parameters, giving a very different A1 value. The pitch min and max data appear to be fairly symmetric about the mean (Figure 1), so the A1 and Q values are at least in the same ballpark.

Table 1: Measured values from FMy with electronic local dampling disabled

	Pitch - MIN	**Pitch - MAX**	**Yaw - MIN**	**Yaw - MAX**
A0	-12.952	18.396	-16.123	147845
A1	37394 sec	35972 sec	24633 sec	14298 sec
A2	-0.141	-0.279	-0.315	-0.190
f	0.635 Hz	0.635 Hz	0.509 Hz	0.509 Hz
ω_o	4.000 Hz	4.000 Hz	3.200 Hz	3.200 Hz
Q measured	**74788**	**71943**	**39413**	**22877**
Q expected	**greater than $3x10^4$**	**greater than $3x10^4$**	**less than $3x10^4$**	**less than $3x10^4$**

The expected Q values and frequencies came from Nergis Malvala, LIGO-XXXXX.