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Calculation of the Photothermal Effect

■ Thermal Expansion Noise

Thermal expansion noise, in which absorbed light heats a mirror and causes it to expand, was brought to our attention by Braginsky et al [1]. We then had an idea for an experiment which could measure this effect. First, construct an interferometer from a metal mirror and a dielectric mirror. If the metal mirror has high thermal expansion, low heat capacity, and high thermal conductivity, then the effect will be noticeable. Then, we modulate the intensity of the laser into the IFO. This will heat the surface of the metal mirror, which will change the length of the cavity. This, we can measure either with this modulated beam or with another probe beam. And, since we can do the modulation at a known frequency, we can make a lock-in measurement of the thermal expansion effect.

To analyze this effect, we model the mirror as an infinite half-space of material. For the simple, 1-d calculation, pretend that the mirror is illuminated by a uniform light whose intensity varies, perhaps as $\frac{P}{2} (1 - \cos[\omega t])$. The heat equation for this system is [2]:

$$\left[-\left(\frac{d}{dx}\right)^2 + \sigma \frac{d}{dt} \right] u(x, t) = \sigma f(x, t)$$

$$f(x, t) = \delta(x) \sin[\omega t]$$

And the displacement of the surface of the mirror is

$$d(t) = \int_0^\infty \frac{\alpha}{\rho C} u(x, t) dx$$

where $u(x,t)$ is the thermal energy density of the material above equilibrium, $f(x,t)$ is the thermal forcing function, and the constant $\sigma = a^{-1} = \rho C/\lambda$. We shall solve this in the steady-state general case of an oscillating beam intensity.

The standard Green's function solution to the diffusion equation in D dimensions is:

$$G(x, x'; s) = \frac{1}{i 2 \sigma \sqrt{s/a}} e^{-\sqrt{s/a} |x-x'|}$$

$$G(x, t; x', t') = (4 \pi a |t - t'|)^{-D/2} e^{-\frac{(x-x')^2}{4a(t-t')}}$$

Then, invoking the magic of Green's functions,

$$u(x, t) = \int_0^t dt' G(x, t; x', t') h(x', t')$$

■ 1 dimension

For a step function perturbation,

$$h(x', t') = \delta(x') \Theta(t') P(\text{energy/area}),$$

$$u(x, t) = P \int_0^t dt' (4 \pi a |t - t'|)^{-1/2} e^{-\frac{(x-x')^2}{4a(t-t')}}$$

$$d(t) = \frac{P\alpha}{\rho C} \int_0^\infty dx \int_0^t dt' (4 \pi a |t - t'|)^{-1/2} e^{-\frac{(x-x')^2}{4a(t-t')}}$$

Integrating first in x, then in t', we get

$$d(t) = \frac{P\alpha}{2\rho C} t,$$

which corresponds to constant thermal expansion of the mirror, as expected.

For an oscillatory perturbation,

$$h(x', t') = \delta(x') \Theta(t') \sin(\omega t') P(\text{energy/area}),$$

$$u(x, t) = P \int_0^t dt' (4 \pi a |t - t'|)^{-1/2} e^{-\frac{(x-x')^2}{4a(t-t')}} \sin(\omega t')$$

$$d(t) = \frac{P\alpha}{\rho C} \int_0^\infty dx \int_0^t dt' (4 \pi a |t - t'|)^{-1/2} e^{-\frac{(x-x')^2}{4a(t-t')}} \sin(\omega t')$$

Note that $h(x', t' < 0) = 0$. This makes the integration easier, and it turns out that this doesn't affect the result for large t. This is only true in one dimension. Since we integrate thermal expansion over the variable x, all the time terms other than in the driving force disappear from the integral. The physical explanation is that since heat deposited in the mirror only diffuses in one direction, away from the mirror surface, and since thermal expansion is linear wrt thermal energy, the mirror is a perfect integrator of thermal energy.

Using the same trick, integrating first in x and then in t', we get

$$d(t) = \frac{1}{\omega} \frac{P\alpha}{2\rho C} \cos(\omega t)$$

Note the 90 degree phase shift in the displacement measurement relative to the input.

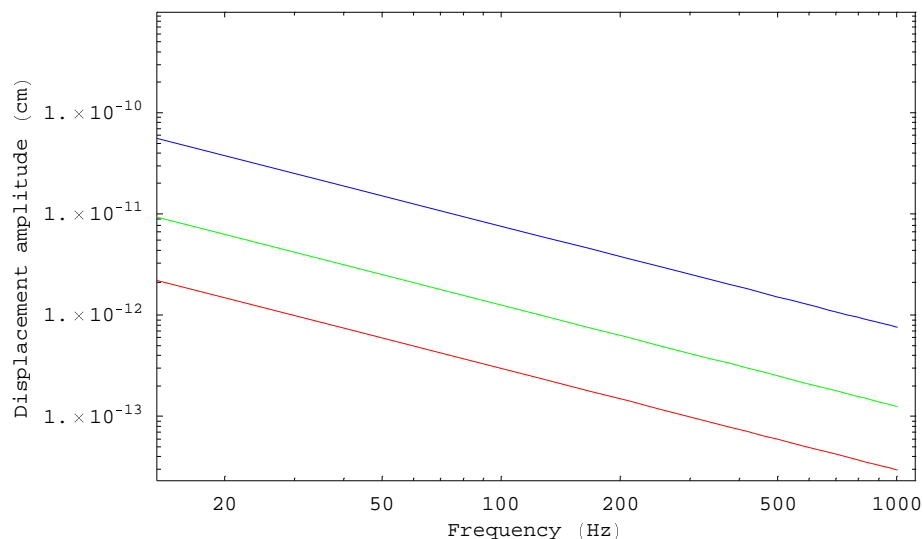
Here's a plot of the amplitude of the response of the mirror, in centimeters, for Aluminum, Sapphire, and Fused Silica for .001 watt/cm² of heat being absorbed by the mirror. All units are cgs.

```
<< Graphics`Animation`
<< Graphics`Graphics`
<< Graphics`Colors`

a =  $\frac{\lambda}{\rho C}$ ;
Aluminum = { $\alpha \rightarrow 23.1 * 10^{-6}$ ,  $\lambda \rightarrow 2.37 * 10^7$ ,  $\rho \rightarrow 2.7$ ,  $C \rightarrow .897 * 10^7$ };
Sapphire = { $\alpha \rightarrow 5 * 10^{-6}$ ,  $\lambda \rightarrow 0.4 * 10^7$ ,  $\rho \rightarrow 4$ ,  $C \rightarrow 0.79 * 10^7$ };
FusedSilica = { $\alpha \rightarrow 0.55 * 10^{-6}$ ,  $\lambda \rightarrow 0.014 * 10^7$ ,  $\rho \rightarrow 2.2$ ,  $C \rightarrow 0.67 * 10^7$ };

ResponseD[P_,  $\omega$ _, Material_] :=  $\frac{P \alpha}{2 \omega \rho C}$  /. Material;

LogLogPlot[{ResponseD[104, 2  $\pi$   $\nu$ , Aluminum],
  ResponseD[104, 2  $\pi$   $\nu$ , Sapphire], ResponseD[104, 2  $\pi$   $\nu$ , FusedSilica]}, { $\nu$ , 1, 1000},
  PlotStyle -> {RGBColor[0, 0, 1], RGBColor[0, 1, 0], RGBColor[1, 0, 0]},
  Frame -> True, FrameLabel -> {"Frequency (Hz)", "Displacement amplitude (cm)"}];
```



■ 3 dimensions

Same thing, but in 3-d. To do this, we add a gaussian beam profile to the thermal forcing function and the y and z coordinates to the Green's function. Let $x' = x1$ for *Mathematica's* benefit. To make the calculation simpler, I'm going to leave out the laser power and the material properties until then end. I'll assume that Poisson's ratio is zero. Later, we can handle nonzero σ with the transformation $\alpha \rightarrow \alpha(1+\sigma)$, and only be off by geometric factors of order unity for small perturbations.

$h(y,z,t)$ is the thermal forcing function -- the amount of power per area per time deposited on the mirror surface. Implicit in h is the laser power, which we will remember for later. We do not have to take $h(x',t' < 0) = 0$, as we did in the 1-D case. Later, we'll integrate the perturbation time, t' , from $-\infty$ to t .

$G(x,y,z,t;x',y',z',t')$ is the standard three-dimensional Green's function for heat diffusion. [2]

$$a = \frac{\lambda}{\rho C}.$$

$$h[y_-, z_-, t_-] = E^{-\frac{(y^2+z^2)}{4a^2(t-t_1)}} E^{I \omega t} \frac{1}{\pi r^2}; (* \text{ length}^{-2} *)$$

$$G[\mathbf{x}_-, \mathbf{y}_-, \mathbf{z}_-, t_-, \mathbf{x}1_-, \mathbf{y}1_-, \mathbf{z}1_-, t1_-] = (4 \pi a (t - t1))^{-3/2} E^{-\frac{(x^2+(y-y1)^2+(z-z1)^2)}{4a^2(t-t1)}}; (* \text{ time}^{-3/2} *)$$

The thermal energy density in the material is what we will use to calculate the motion of the mirror surface. By the standard Green's function equations,

$$u(x, y, z, t; t') = \int G(x, y, z, t; x', y', z', t') h(y, z, t) d(y', z')$$

and

$$u(x, y, z, t) = \int u(x, y, z, t; t') dt'$$

We should also integrate out t' , but it turns out that it will be easier to do this later. Remember: we're still missing the laser power term in the *Mathematica* equations.

$$u1[\mathbf{x}_-, \mathbf{y}_-, \mathbf{z}_-, t_-, t1_-] = \text{Integrate}[G[\mathbf{x}, \mathbf{y}, \mathbf{z}, t, \mathbf{x}1, \mathbf{y}1, \mathbf{z}1, t1] h[\mathbf{y}1, \mathbf{z}1, t1], \{\mathbf{y}1, -\infty, \infty\}, \{\mathbf{z}1, -\infty, \infty\}];$$

We now know the thermal energy density in the mirror material. We can integrate this energy density over depth to calculate the displacement of the surface of the mirror.

$$d(y, z, t; t') = \frac{\alpha}{\rho C} \int u(x, y, z, t) d(x)$$

$$d(y, z, t) = \int d(y, z, t; t') dt'$$

It turns out to be easier to first integrate the Green's function and the source function over x , then to integrate over the source variables y' and z' . This gives us the amount of heat energy below a point on the surface of the mirror. Multiplying by $\alpha/\rho C$ converts the heat energy into an actual distance.

The missing constants are now $\frac{\text{Plaser}\alpha}{\rho C}$.

$$d1[\mathbf{y}_-, \mathbf{z}_-, t_-, t1_-] = \text{Simplify}[\text{Integrate}[G[\mathbf{x}, \mathbf{y}, \mathbf{z}, t, \mathbf{x}1, \mathbf{y}1, \mathbf{z}1, t1] h[\mathbf{y}1, \mathbf{z}1, t1], \{\mathbf{x}, 0, \infty\}, \{\mathbf{y}1, -\infty, \infty\}, \{\mathbf{z}1, -\infty, \infty\}]] (* \text{ length} *)$$

\$Aborted

`d1[y, z, t, t1]`

$$\frac{C e^{\frac{i(4(t-t_1)t_1\lambda + C\rho(iy^2 + iz^2 + r^2t_1\omega))}{4t\lambda - 4t_1\lambda + Cr^2\rho}} \rho}{2\pi \sqrt{\frac{(t-t_1)\lambda}{C\rho}} \sqrt{\frac{C\rho}{t\lambda - t_1\lambda} (4t\lambda - 4t_1\lambda + Cr^2\rho)}}$$

Now, we can integrate over t' to calculate the displacement of a point on the mirror. Well, actually, we can't. We can solve it numerically, or integrate out y and z first, which we will do when we calculate the measured signal.

`SurfaceDisplacement[y_, z_, t_, r_, a_, ω_] :=`

$$\text{NIntegrate}\left[\frac{e^{\frac{y^2+z^2-i t_1 (r^2+4 a t-4 a t_1) \omega}{r^2+4 a (t-t_1)}}}{2 \pi (r^2+4 a (t-t_1))}, \{t_1, 0, t\}\right];$$

We can evaluate that integral numerically, but we're really interested in the signal we measure with an interferometer. With the Pound-Drever-Hall technique, we measure the amplitude, not the power, of the returned light. So, we want to average the surface displacement over the amplitude of the electric field at the mirror. This is different than the calculation that Braginsky made! Since the light amplitude falls off slower than the intensity, this actually leads to a smaller measurement, as we sample more of the region of mirror that is less displaced. This is what accounts for the $3r^2$ term

I am now assuming that the surface displacement changes are small compared to the dynamic range of the PDH error signal, and that the electric field is that of a Gaussian TEM00 mode.

$$f(\omega, t) = \int d(y, z, t; t') E(y, z) d(y, z, t')$$

The missing constants are still $\frac{P_0}{\rho C}$.

$$\text{Field}[y_, z_] = \frac{E e^{-\frac{y^2+z^2}{r^2}}}{\pi r^2}; \text{ (* normalized *)}$$

$$\text{f1} = \frac{e^{\frac{y^2+z^2-i t_1 (r^2+4 a t-4 a t_1) \omega}{r^2+4 a (t-t_1)}}}{2 \pi (r^2+4 a (t-t_1))} \text{Field}[y, z];$$

`Simplify[f1]`

$$\frac{C e^{-\frac{y^2+z^2}{r^2} + \frac{i(4(t-t_1)t_1\lambda + C\rho(iy^2 + iz^2 + r^2t_1\omega))}{4t\lambda - 4t_1\lambda + Cr^2\rho}} \rho}{2\pi^2 r^2 (4t\lambda - 4t_1\lambda + Cr^2\rho)}$$

`f2 = Integrate[f1, {y, -∞, ∞}, {z, -∞, ∞}];`

`Simplify[f2]`

$$\frac{C e^{i t_1 \omega} \rho}{8 \pi t \lambda - 8 \pi t_1 \lambda + 4 C \pi r^2 \rho}$$

We'll integrate this function by hand, since the computer doesn't do it very well. This is a tabulated integral, which *Mathematica* knows as

$$\int \frac{e^{ax}}{bx+c} dx = \frac{e^{\frac{ac}{b}}}{b} \text{ExpIntegralEi}\left[\frac{ac}{b} + ax\right]$$

So, integrating, we get...

$$\int_{-\infty}^t \frac{\rho C e^{i t_1 \omega}}{8 \pi t_1 \lambda + 4 C \pi r^2 \rho - 8 \pi t_1 \lambda} dt_1$$

$$= -\frac{e^{i \omega \frac{r^2}{2a}} e^{i \omega t}}{8 \pi a} \left(\text{ExpIntegralEi} \left[-i \omega \frac{r^2}{2a} \right] + i \pi \right)$$

What is that, you ask?

For $x \rightarrow \infty$, $\text{Ei}(\pm i x) \rightarrow \pm i \pi$.

For $x \gg 1$, $\text{Ei}(-i x) + i \pi = -x^{-1} e^{-ix}$

So, approximately, for large frequency,

$$f(t) = \frac{P \alpha}{\rho C} \frac{e^{i \omega t}}{8 a \pi} \frac{2 a}{r^2 \omega}$$

$$f(\omega) = \frac{P \alpha}{4 \pi r^2 \rho C \omega}$$

Or, in display form:

$$f(\omega) = \frac{P \alpha}{\rho C} \frac{e^{i \omega \frac{r^2}{2a}}}{8 \pi a} \left(\text{EI} \left[-i \omega \frac{r^2}{2a} \right] + i \pi \right)$$

Where $\text{EI}(z) = -\int_{-z}^{\infty} e^{-t} / t dt$

These are the results:

```
Response3DAprox[P_, r_, ω_, Material_] :=  $\frac{P \alpha}{\rho C} \frac{1}{8 a \pi} \frac{2 a}{r^2 \omega}$  /. Material;
Response3D[P_, r_, ω_, Material_] :=
 $\frac{P \alpha}{\rho C} \frac{1}{8 a \pi} e^{i \frac{r^2}{2a} \omega} \left( -\text{ExpIntegralEi} \left[ -\frac{i r^2 \omega}{2 a} \right] - I \pi \right)$  /. Material
```

For now, let's look at expected sensitivity plots for some materials. We'll take the spot radius to be 0.04 cm and the variation in the laser power absorbed by the surface to be 400 mW peak-peak. Remember, dimensions are in cgs. The x axis is the frequency at which the laser power varies, and the y axis is the distance in cm that will be measured by the PDH error signal.

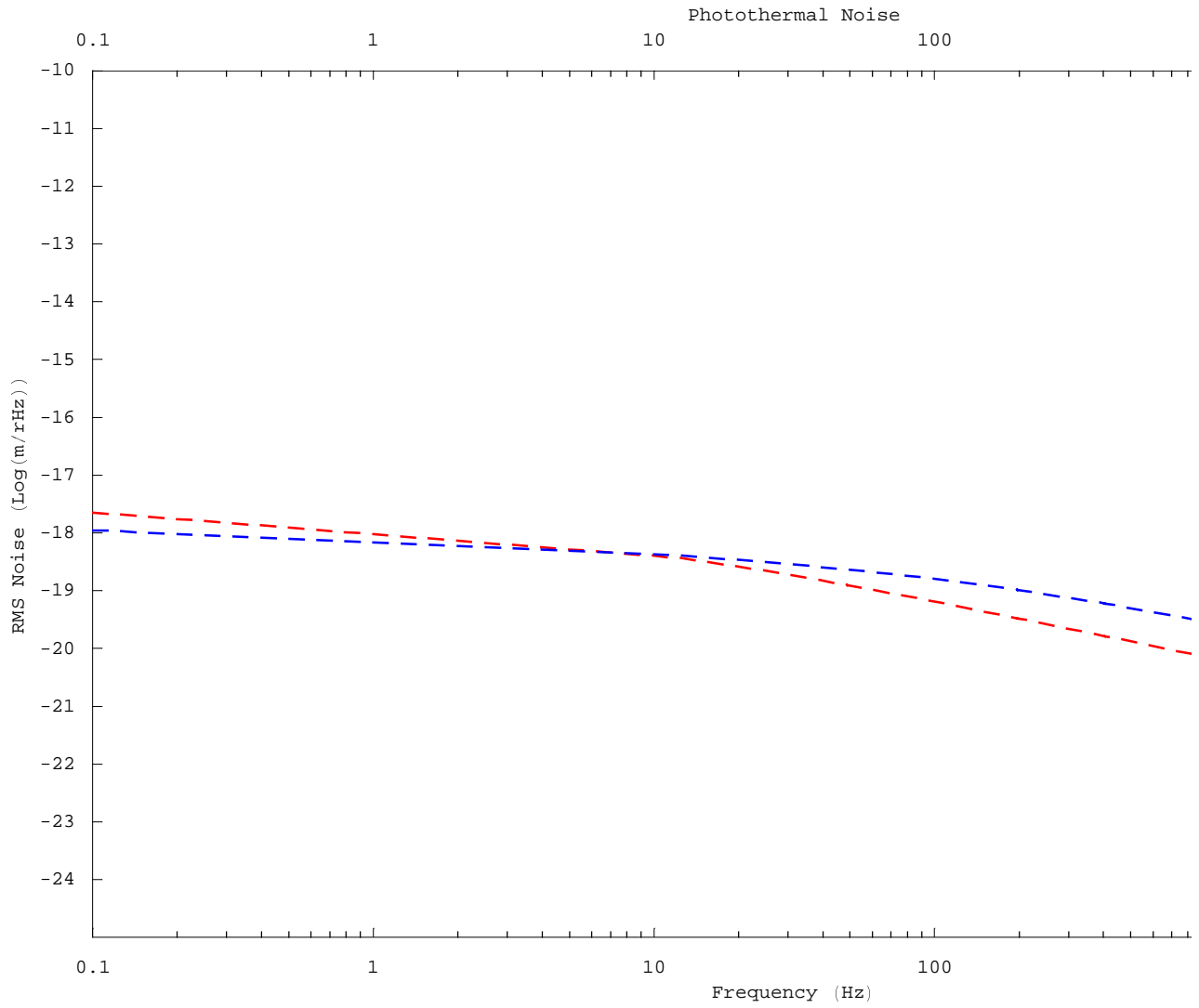
Note that in this formula, P is the square root of the spectral density of the absorbed power. For excitations driven by shot

noise, the variation in the thermal forcing function is $\frac{Phc}{\lambda}$ (Braginsty et al, 1999). For example, 5 mW of shot noise excited phonons should give a response of:

```

ThermalForcing = Sqrt[6.6 10^-27 3 10^10 5 10^4 / 10^-4];
LogLogPlot[{0.01 Abs[Response3D[ThermalForcing, 0.015, 2 π f, FusedSilica]],
  0.01 Abs[Response3D[ThermalForcing, 0.015, 2 π f, Sapphire]]},
  {f, 0.1, 10000}, GridLines → None,
  Ticks → None,
  PlotStyle → {{Red, Thickness[0.002], Dashing[{0.01, 0.01]}]},
    {Blue, Thickness[0.002], Dashing[{0.01, 0.01]}]},
  FrameTicks → {Automatic, Table[{10^-n, -n}, {n, 10, 25}]},
  PlotRange → {{10^-1, 10^4}, {10^-25, 10^-10}},
  ImageSize → {800, 600}, Frame → True, AspectRatioFixed → False,
  FrameLabel → {"Frequency (Hz)", "RMS Noise (Log(m/rHz))",
    "Photothermal Noise", "RMS Noise (Log(m/rHz))"}];

```



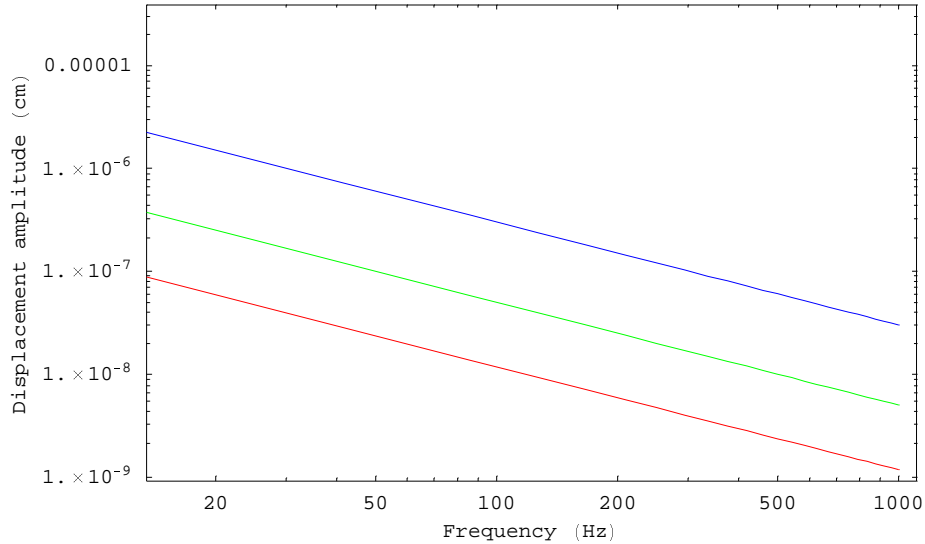
■ **Sine wave perturbation**

For an oscillatory perturbation, here's what you get ($1 \text{ W} = 10^7 \text{ erg/s}$):


```

LogLogPlot[{
  Response3DAprox[4 106, 0.04, 2 π ν, Aluminum],
  Response3DAprox[4 106, 0.04, 2 π ν, Sapphire],
  Response3DAprox[4 106, 0.04, 2 π ν, FusedSilica]},
{ν, 1, 1000}, PlotStyle → {RGBColor[0, 0, 1], RGBColor[0, 1, 0],
  RGBColor[1, 0, 0]}, Frame → True,
FrameLabel → {"Frequency (Hz)", "Displacement amplitude (cm)"}];

```



Now, the exact results for all three materials, using the same experiment conditions, with magnitude and phase. The black line indicates the approximate result from above.

```

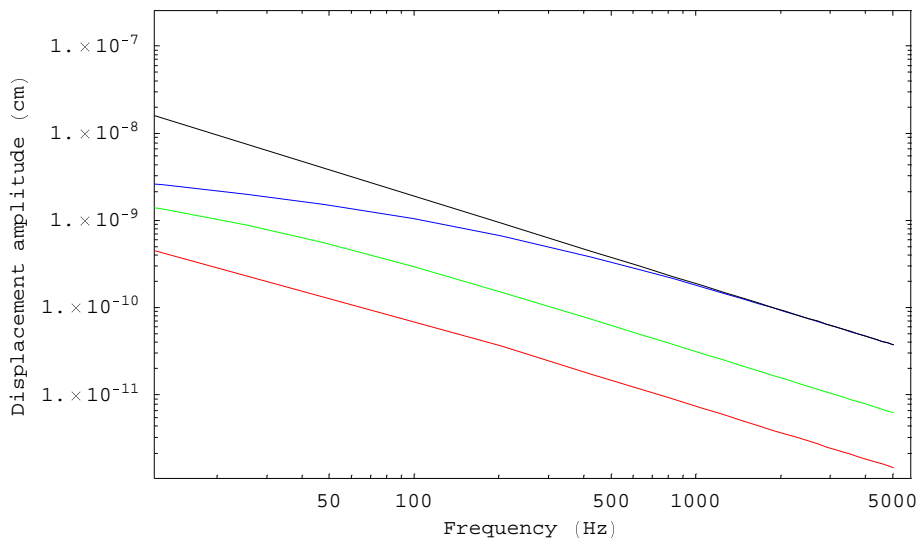
LaserPower = 2.5 106 * 0.01; (* 1% absorption is realistic for Aluminum *)
SpotSize = 0.04;
MaxFreq = 5000;

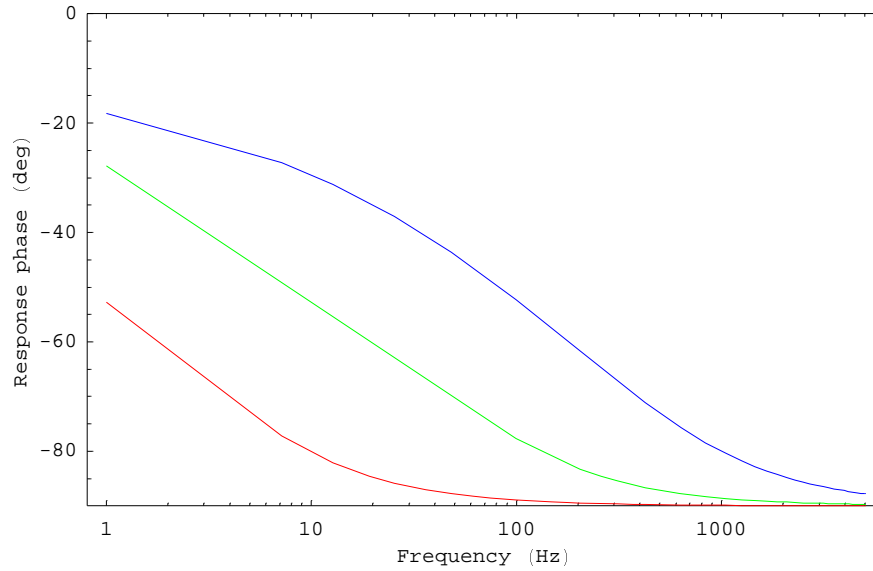
```

```

LogLogPlot[{
  Abs[Response3D[LaserPower, SpotSize, 2  $\pi$   $\nu$ , Aluminum]],
  Abs[Response3D[LaserPower, SpotSize, 2  $\pi$   $\nu$ , Sapphire]],
  Abs[Response3D[LaserPower, SpotSize, 2  $\pi$   $\nu$ , FusedSilica]],
  Response3DAprox[LaserPower, SpotSize, 2  $\pi$   $\nu$ , Aluminum]],
{ $\nu$ , 1, MaxFreq}, PlotStyle  $\rightarrow$  {RGBColor[0, 0, 1], RGBColor[0, 1, 0],
  RGBColor[1, 0, 0], GrayLevel[0]}, Frame  $\rightarrow$  True,
  FrameLabel  $\rightarrow$  {"Frequency (Hz)", "Displacement amplitude (cm)"}];
LogLinearPlot[{
   $\frac{180}{\pi}$  Arg[Response3D[LaserPower, SpotSize, 2  $\pi$   $\nu$ , Aluminum]],
   $\frac{180}{\pi}$  Arg[Response3D[LaserPower, SpotSize, 2  $\pi$   $\nu$ , Sapphire]],
   $\frac{180}{\pi}$  Arg[Response3D[LaserPower, SpotSize, 2  $\pi$   $\nu$ , FusedSilica]]},
{ $\nu$ , 1, MaxFreq}, PlotStyle  $\rightarrow$  {RGBColor[0, 0, 1], RGBColor[0, 1, 0],
  RGBColor[1, 0, 0]}, PlotRange  $\rightarrow$  {0, -90}, Frame  $\rightarrow$  True,
  FrameLabel  $\rightarrow$  {"Frequency (Hz)", "Response phase (deg)"}];

```





Now, compare what effect the spot size has on a mirror. For this example, take $P=1\text{mW}$ (absorbed power variation is 2 mW peak-peak), and spot sizes of .1mm, 0.5mm, 1mm, and 2mm on an aluminum mirror. The lighter colors indicate smaller spots.

```
Abs[Response3D[104, 0.01, 2 π 100, Aluminum]]
Abs[Response3D[104, 0.05, 2 π 100, Aluminum]]
Abs[Response3D[104, 0.1, 2 π 100, Aluminum]]
Abs[Response3D[104, 0.2, 2 π 100, Aluminum]]
```

1.2601×10^{-9}

3.18486×10^{-10}

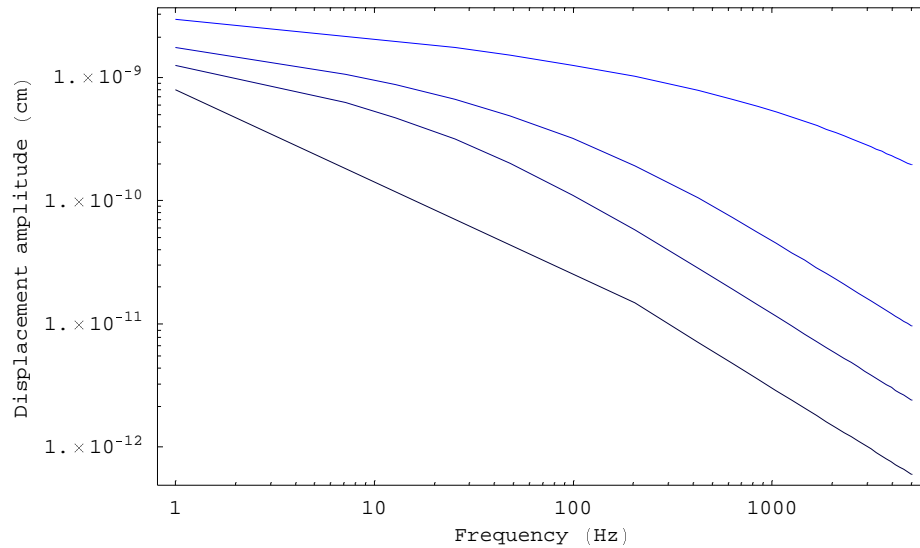
1.10598×10^{-10}

2.99431×10^{-11}

```

LogLogPlot[{Abs[Response3D[104, 0.01, 2 π ν, Aluminum]],
  Abs[Response3D[104, 0.05, 2 π ν, Aluminum]],
  Abs[Response3D[104, 0.1, 2 π ν, Aluminum]],
  Abs[Response3D[104, 0.2, 2 π ν, Aluminum]]}, {ν, 1, MaxFreq}, PlotStyle →
  {RGBColor[0, 0, 1], RGBColor[0, 0, .75], RGBColor[0, 0, .5], RGBColor[0, 0, .25]},
  Frame → True, FrameLabel → {"Frequency (Hz)", "Displacement amplitude (cm)"}];

```

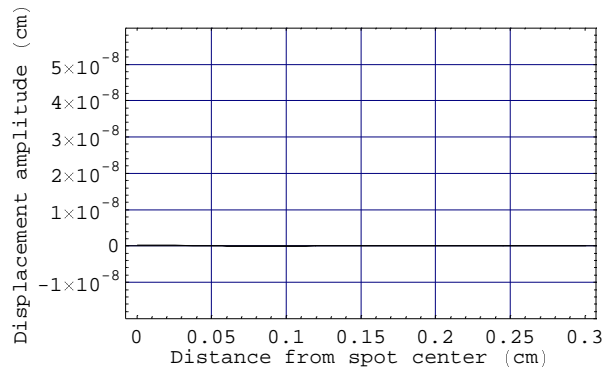


Neat, eh? The effect should be quite large. Here's a snapshot of what the mirror surface will look like for an Aluminum mirror driven at 400 Hz with 400 mW p-p power variation. The spot size is 0.04 cm. The x axis is the distance from the center of the spot, in cm.

```

A[y_, t_] := Re[LaserPower  $\frac{\alpha}{\rho C}$ 
  SurfaceDisplacement[y, 0, t, SpotSize, a /. Aluminum, 2 π 400] /. Aluminum];
Plot[A[y, 0.01], {y, 0, 0.3}, PlotPoints → 25,
  PlotRange → {6 10-8, -2 10-8}, GridLines → Automatic, Frame → True,
  FrameLabel → {"Distance from spot center (cm)", "Displacement amplitude (cm)"}];

```



■ Square wave perturbation

We may want to chop the pump beam intensity instead of varying it continuously. The fourier expansion of a square wave is

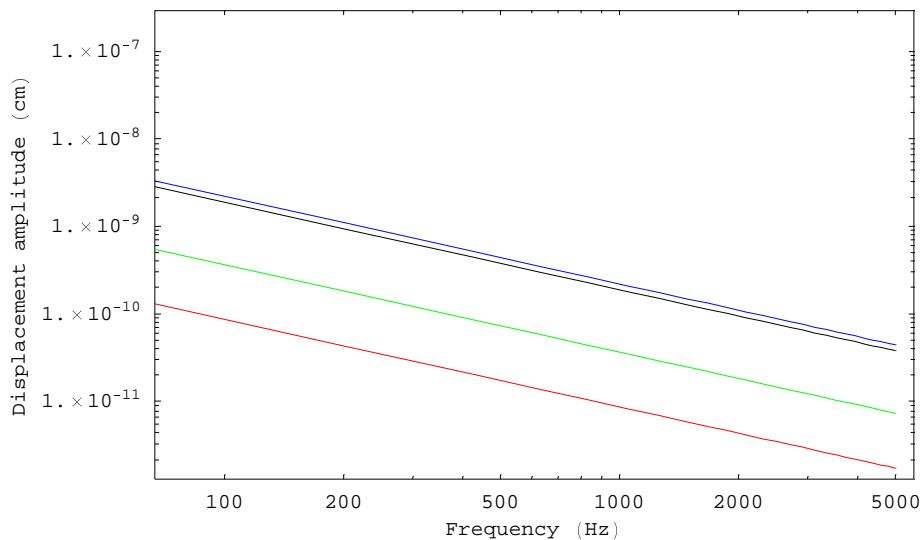
$$\frac{-4}{\pi} \operatorname{Re} \left[\sum_{n=0}^{\infty} \frac{e^{i(2n+1)\omega t + i\pi/2}}{(2n+1)} \right]$$

where ω is the angular frequency of the principal mode. To do this, we'll sum the series. So the obvious thing to do is to recast the displacement measures above in terms of this expansion. The frequency components that are included are clearly the odd harmonics.

```
SquareResponse3DApprox[P_, r_, ω_, Material_] :=
  Sum[ $\frac{4(-1)^n}{\pi(2n+1)}$  Response3DApprox[P, r, (2n+1) ω, Material], {n, 0, Infinity}];
```

Here's a plot of the approximate result for the three materials. The black line is the result from the sinusoidal power variation above, for Aluminum. As you can see, the effect we will see by using a square wave is greater than for a sine wave, by a factor of $4/\pi$.

```
LogLogPlot[{
  SquareResponse3DApprox[LaserPower, SpotSize, 2 π ν, Aluminum],
  SquareResponse3DApprox[LaserPower, SpotSize, 2 π ν, Sapphire],
  SquareResponse3DApprox[LaserPower, SpotSize, 2 π ν, FusedSilica],
  Response3DApprox[LaserPower, SpotSize, 2 π ν, Aluminum]},
{ν, 1, MaxFreq}, PlotStyle → {RGBColor[0, 0, 1], RGBColor[0, 1, 0],
  RGBColor[1, 0, 0], RGBColor[0, 0, 0]}, Frame → True,
FrameLabel → {"Frequency (Hz)", "Displacement amplitude (cm)"}];
```



The exact calculation is a little more involved. Here are magnitude and phase for the three materials, compared to the approximate response for sinusoidal and square wave perturbation.

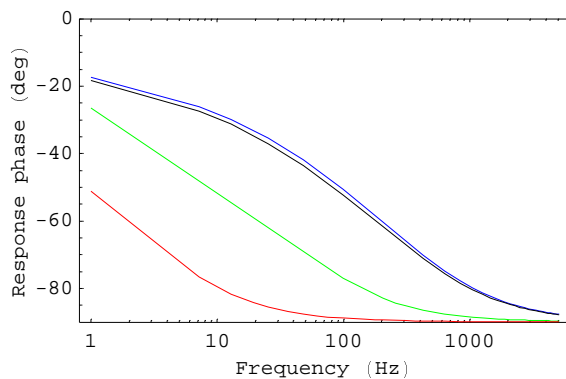
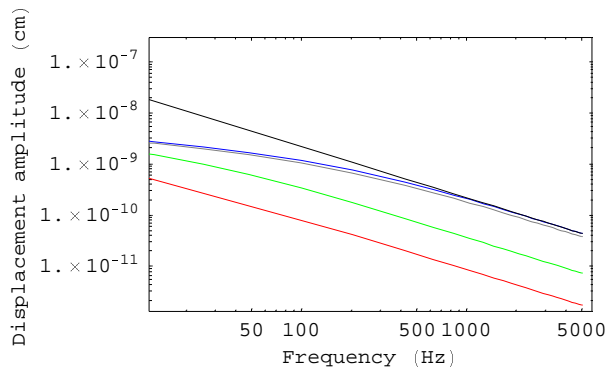
```

SquareResponse3D[P_, r_, ω_, Material_] :=
  Sum[ $\frac{4 (-1)^n}{\pi (2n + 1)}$  Response3D[P, r, (2n + 1) ω, Material], {n, 0, Infinity}];

LogLogPlot[{
  Abs[SquareResponse3D[LaserPower, SpotSize, 2 π ν, Aluminum]],
  Abs[SquareResponse3D[LaserPower, SpotSize, 2 π ν, Sapphire]],
  Abs[SquareResponse3D[LaserPower, SpotSize, 2 π ν, FusedSilica]],
  Abs[SquareResponse3DApprox[LaserPower, 0.04, 2 π ν, Aluminum]],
  Abs[Response3D[LaserPower, SpotSize, 2 π ν, Aluminum]]},
{ν, 1, MaxFreq}, PlotStyle → {RGBColor[0, 0, 1], RGBColor[0, 1, 0],
  RGBColor[1, 0, 0], GrayLevel[0], GrayLevel[.5]}, Frame → True,
  FrameLabel → {"Frequency (Hz)", "Displacement amplitude (cm)"}];

LogLinearPlot[{
   $\frac{180}{\pi}$  Arg[SquareResponse3D[LaserPower, SpotSize, 2 π ν, Aluminum]],
   $\frac{180}{\pi}$  Arg[SquareResponse3D[LaserPower, SpotSize, 2 π ν, Sapphire]],
   $\frac{180}{\pi}$  Arg[SquareResponse3D[LaserPower, SpotSize, 2 π ν, FusedSilica]],
   $\frac{180}{\pi}$  Arg[Response3D[LaserPower, SpotSize, 2 π ν, Aluminum]]},
{ν, 1, MaxFreq}, PlotStyle → {RGBColor[0, 0, 1], RGBColor[0, 1, 0],
  RGBColor[1, 0, 0], GrayLevel[0], GrayLevel[0.5]}, PlotRange → {0, -90},
  Frame → True, FrameLabel → {"Frequency (Hz)", "Response phase (deg)"}];

```



■ Damage threshold

How hot will the mirror spot get? For a rough estimate, take power * C / spot size² / skin depth

$$\text{SkinDepth} = \frac{10^{-4}}{2\pi}$$

$$\text{MaxTemp} = \text{LaserPower} / (\text{C SkinDepth SpotSize}^2) / .\text{Aluminum}$$

$$\frac{1}{20000\pi}$$

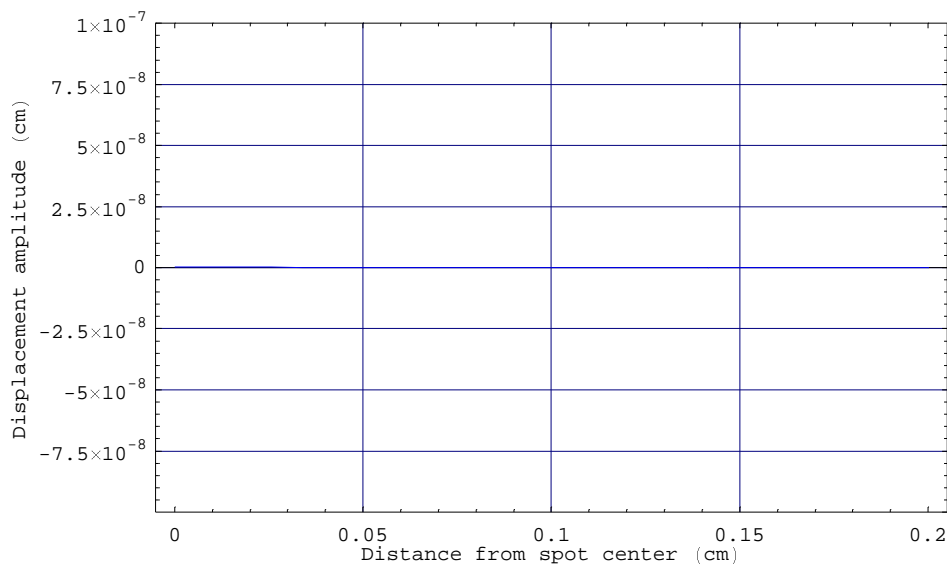
$$109448.$$

■ Movie

This is an animation of the surface of an aluminum mirror with a 400 mW variation in power at 400 Hz. The x-axis is the distance from the center of the spot in cm. The y-axis is the displacement of the mirror from equilibrium. The spot size is 0.04 cm.

```
MyMovie =
  Table[Plot[A[y, n/8000], {y, 0, 0.2}, PlotPoints -> 25, PlotRange -> {10^-7, -10^-7},
    GridLines -> Automatic, PlotStyle -> RGBColor[0, 0, 1], Frame -> True,
    FrameLabel -> {"Distance from spot center (cm)", "Displacement amplitude (cm)"},
    DisplayFunction -> Identity], {n, 19}];

ShowAnimation[MyMovie];
```



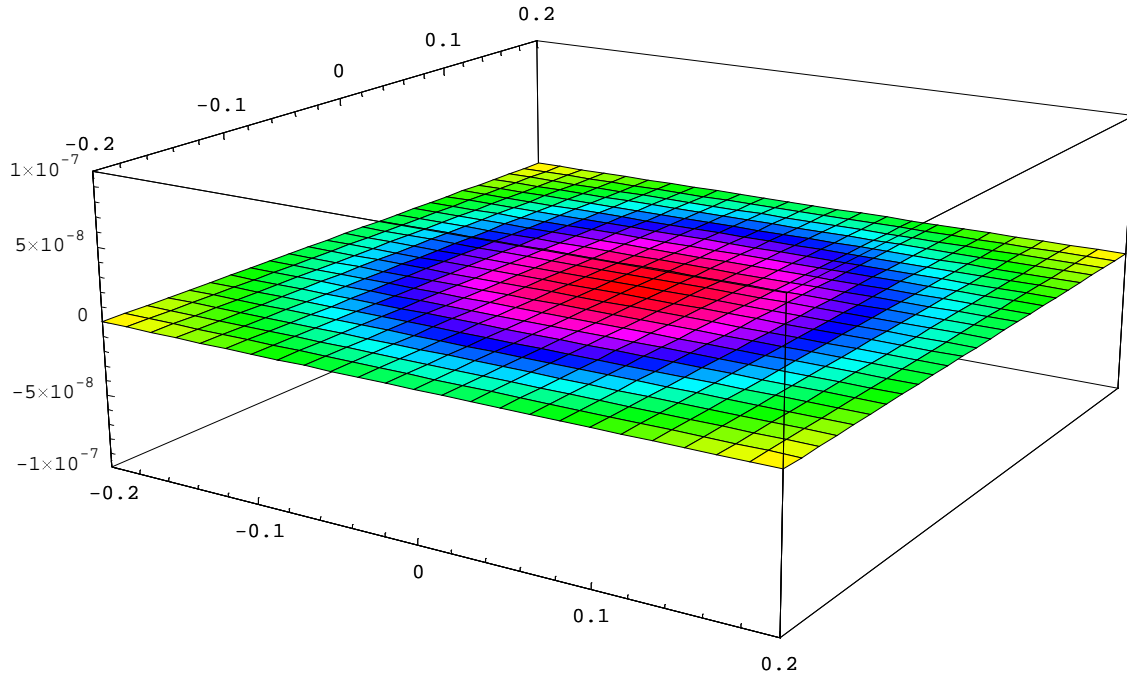
Same thing in 3-D

```

MyMovie3D = Table[Plot3D[{A[Sqrt[y^2 + x^2], n / 8000], Hue[E-(y^2+x^2)/spotSize]}],
  {x, -0.2, 0.2}, {y, -0.2, 0.2}, PlotPoints -> 25, PlotRange -> {10-7, -10-7},
  DisplayFunction -> Identity, ViewPoint -> {1.718, -2.747, 0.975}], {n, 19}];

ShowAnimation[MyMovie3D];

```



■ Footnotes

[1] Braginsky, Gorodetsky, Vyatchanin, *Thermodynamical fluctuations and photo-thermal shot noise in gravitational wave antennae*, Physics Letters A 264 (1999), pp 1-10.

[2] Baker, Sutlief, *Green's Functions in Physics*, personal communication, 1997