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How to measure Braginskii's photo-thermal effect

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ABSTRACT

Two new mechanisms of thermal noise have been predicted by Braginskii and his group: photo-thermal noise, which comes from the Poissonian absorption of photons by the surface of a mirror and the subsequent heating of the mirror's surface; and thermo-elastic damping noise, which comes from thermal expansion of the mirror surface in response to random temperature fluctuations. Both effects are expected to make significant contributions to thermal noise in the sapphire mirrors of a second generation LIGO.

We would like to measure these predicted effects at the Thermal Noise Interferometer. A measurement of the thermo-elastic dampling noise is likely to require the complete Thermal Noise Interferometer, which is not finished as of this writing. The photo-thermal effect, however, can probably be measured with a simpler setup. This document describes how the photo-thermal effect might be measured, and outlines an experiment for doing so.

KEYWORDS

Thermal noise; photo-thermal noise

1 Introduction

Braginskii has predicted that whenever a photon is absorbed near the surface of a mirror, a very small area of the mirror heats up and expands [1]. Because photons are absorbed in a random, Poissonian process, this absorption and expansion shows up as a sort of frying of the surface of the mirror. Transient blisters appear wherever a photon has been absorbed, leading to an apparent frying, or bubbling, of the surface of the mirror. The thing that drives this frying is shot noise in the incident laser beam. Because the amplitude of the shot noise is relatively small, Braginskii has predicted a relatively small effect, although a significant one for gravitational-wave detectors.

We would like to test his prediction and actually measure this predicted effect. A measurement of the shot-noise induced frying would require an interferometer of unprecedented sensitivity. Our Thermal Noise Interferometer should be more than adequate for the job, but it is not finished yet. Fortunately, there may be an easier way to measure the photo-thermal effect.

Because the photo-thermal effect is driven by amplitude fluctuations (shot noise in Braginskii's paper), we should be able to enhance the effect by enhancing the amplitude fluctuations. If we were to modulate the intensity of a beam incident on a mirror, we might see measurable expansion and contraction of the mirror's surface at the same frequency as the beam's amplitude modulation.

2 An estimate of the size of the effect

2.1 A transfer function

Let's do a quick and dirty estimate of the size of the effect in a mirror with an amplitude modulated incident beam. Braginskii predicts a shot-noise induced photo-thermal effect of

$$S_{TS}(\omega) = 2\alpha^2 (1+\sigma)^2 \frac{\hbar\omega_0 W_0}{\left(\rho C \pi r_0^2\right)^2} \frac{1}{\omega^2}, \tag{1}$$

where α is the thermal expansion coefficient of the mirror material, σ is its Poisson ratio, ρ is its density, and C is its specific heat. W_0 is the power absorbed from the beam, r_0 is the beam radius, and ω_0 is the (angular) frequency of the photons in the beam.

The drive for this effect is shot noise, given by

$$S_{shot} = 2\hbar\omega_0 W,$$

where W is the total power in the beam. We may estimate the transfer function of the photo-thermal effect by simply dividing Braginskii's predicted effect by the shot noise. This yields

$$G(\omega) = \frac{W_0}{W} \frac{\alpha^2 (1+\sigma)^2}{\left(\rho C \pi r_0^2\right)^2} \frac{1}{\omega^2},$$

or in terms of length per watt of incident power modulation,

$$G^{1/2}(\omega) = \frac{\alpha(1+\sigma)}{\rho C \pi} \frac{1}{r_0^2} \sqrt{\frac{W_0}{W}} \frac{1}{\omega}. \tag{2}$$

Here I have split the transfer function up into a purely material-dependent part, the square root of the absorption (or very nearly that), a term involving the laser spot size, and one over the measurement frequency squared.

2.2 Material dependence

Let's calculate the material dependent term for sapphire and for fused silica. For fused silica, the material parameters are [1]

$$\begin{aligned} \alpha &= 5.5 \times 10^{-7} K^{-1} \\ \sigma &= 0.17 \\ \rho &= 2.2 \times 10^{3} kg/m^{3} \\ C &= 6.7 \times 10^{2} J/kgK. \end{aligned}$$

For fused silica, then,

$$\frac{\alpha\left(1+\sigma\right)}{\rho C \pi} = 1.4 \times 10^{-13} m^{3}/J.$$

For sapphire, the material parameters are [1]

$$\begin{aligned} \alpha &= 5.0 \times 10^{-6} K^{-1} \\ \sigma &= 0.29 \\ \rho &= 4.0 \times 10^{3} kg/m^{3} \\ C &= 7.9 \times 10^{2} J/kgK. \end{aligned}$$

The material dependent part of the response for sapphire is not very different from that of fused silica. Sapphire's response is about five times as large as fused silica's.

$$\frac{\alpha\left(1+\sigma\right)}{\rho C \pi} = 6.5 \times 10^{-13} m^{3}/J$$

2.3 Spot size and measurement frequency

Now we need to estimate the other parameters. A spot size of $250 \mu m$ seems reasonable for a tabletop experiment, so let's use that. (Remember, the smaller the spot size, the larger the effect.) Furthermore, let's assume that we want to do the measurement at $1kHz$, so $\omega = 2\pi \times 10^{3} Rad/s$. For these parameters,

$$\frac{1}{r_0^2 \omega} = 2.5 \times 10^{3} m^{-2} s.$$

2.4 Absorption

The only thing left to estimate is the absorption W_0/W. We should have some control over this, and I expect it is easy to make a mirror with a high absorption just by introducing impurities into the material near the surface. (We could bombard it with ions or something. Making a high absorption layer should be one of the things we look into first in this experiment.) Just guessing, I would expect an absorption of $W_0/W = 100ppm$ to be reasonable. This gives

$$\sqrt{W_0/W} = 10^{-2}.$$

2.5 Magnitude of the driven photo-thermal effect

The above estimates lead to a photo-thermal effect in fused silica of about

$$G_{F.S.}^{1/2} = 3.5 \times 10^{-12} m/W,$$

and for sapphire of about five times as much,

$$G_{Sap.}^{1/2} = 1.6 \times 10^{-11} m/W.$$

These are the expected motions of the surface of a mirror per Watt of modulation on the incident beam. If the mirror in question is part of a high-finesse Fabry-Perot cavity, then we could easliy generate up to several hundred Watts of amplitude modulation, and the response of the mirror could be as much as a few Angstroms in both sapphire and fused silica. Of course, we would want to keep the response of the mirror small enough that the cavity is not driven out of resonance, or even out of the linear regime, but it's nice to know we have plenty of signal to work with.

(If our mirror losses are $100ppm$, then we might reasonable expect to have a reflection of $1000ppm$ to keep the cavity close to critically coupled. This would lead to a finesse of

$$\mathcal{F} = 3 \times 10^3.$$

This would give us a circulating power of at least a kilo-Watt, if we were to use our $500mW$ NPRO, and a reasonable amplitude modulation inside the cavity would be $500W$.)

3 The basic idea for the experiment

Figure 3 shows a schematic of the experiment. The basic idea is to pump the photo-thermal effect at a definite frequency using a high-intensity, amplitude modulated beam, and to lock-in detect the length of the cavity from a Pound-Drever-Hall error signal. This same error signal would be used to frequency lock the laser to the cavity. Since both the pump and probe beams are generated simultaneously by the same laser, both should resonate simultaneously in the cavity. We would only need to lock the laser to the probe beam to get the pump beam resonant.

To separate the pump and probe beams, the main beam of the laser is split using a 10% beamsplitter. 90% of the beam passes through an acousto-optic modulator and becomes the pump beam. It then is redirected into a circulator using a 90% beamsplitter, after which it enters the cavity. Any return pump beam passes through the circulator and is eventually stopped by a beam dump (not shown, but it would sit just above the 10% in the picture) and the Faraday Isolator. The pump beam is vertically polarized.

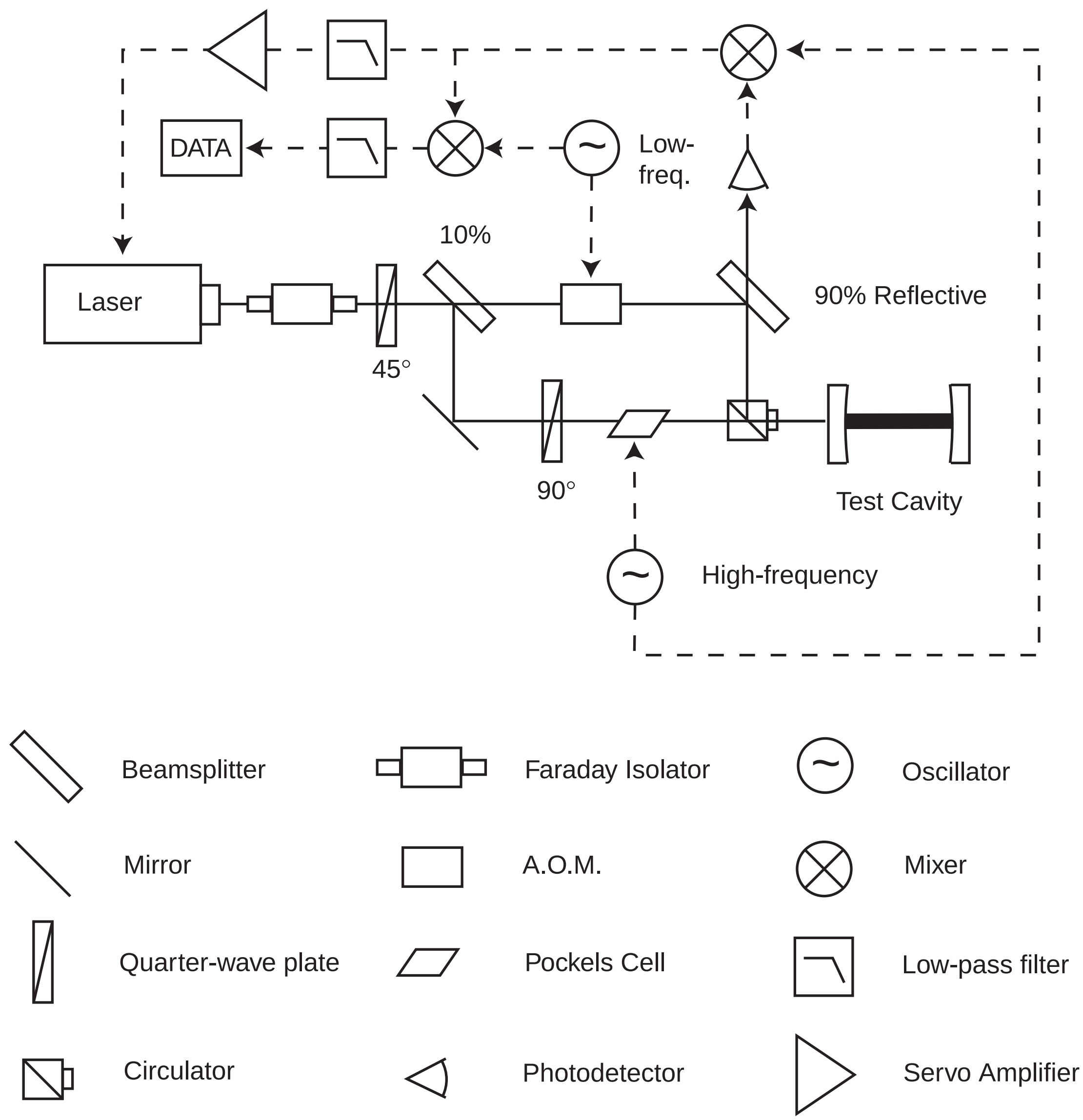

Figure 1: The basic layout for measuring the photo-thermal effect. Here two beams are employed with orthogonal polarizations: a high intensity pump beam, which excites the photo-thermal effect, and a low intensity probe beam, which measures the length of the cavity. The laser is frequency locked to the cavity, which ensures that both the pump and the probe beams are resonant.

The 10% beam tapped by the first beamsplitter bounces off a mirror and then goes through a half-wave plate, which rotates its polarization by $90°$. The now horizontally-polarized beam goes through a Pockels cell to get sidebands placed on it, through the circulator, and into the cavity. The sidebands and any carrier not resonant in the cavity return and are redirected by the circulator to the 90% beamsplitter. The 10% of the beam that passes through this beamsplitter then goes into a photodetector. The 90% of the probe beam that does not go into the photodetector is trapped by a beam dump (not shown, but it's the same one that traps the returning pump, just above the 10% beamsplitter) and the Faraday Isolator.

The signal from the photodetector is demodulated at the high frequency at which the Pockels cell is driven. The resulting signal is then low-pass filtered and fed back to the laser to lock it to the cavity. A copy of this signal, before it is low-pass filtered, is demodulated at the (relatively low) pump beam's amplitude modulation frequency. This signal, after being low-pass filtered, provides a measure of the length response of the cavity to the pump beam's amplitude modulation.

Note that if any of the pump beam leaks into the photodetector (due to the finite extinction ratio of the circulator, for example), that is not a problem. Only the probe beam will be register after the signal is demodulating at the Pockels cell's driving frequency.

4 A quantitative model

What will the error signal look like? First let's see what the power falling on the photodetector looks like. The power reflected off the cavity in the probe beam will be

$$P = \left| E_{carrier} + E_{sidebands} \right|^2,$$

where $E_{carrier}$ is the electric field of the reflected carrier, and $E_{sidebands}$ is the electric field of the reflected sidebands. Near resonance, these will be given by [2]

$$E_{carrier} \approx \sqrt{P_{probe}} \frac{4L\mathcal{F}}{\lambda} \left(\frac{\delta f}{f} + \frac{\delta L}{L} \right),$$

and

$$E_{sidebands} = -\sqrt{P_{probe}} 2J_1 \left(\beta \right) \sin \Omega_{PC} t.$$

Here P_{probe} is the power in the probe beam, L is the length of the test cavity, $\mathcal{F}$ is the finesse of the cavity, λ is the wavelength of the laser light ($1.064 \mu m$), and f is the frequency of the laser light ($300THz$). The quantities δf and δL are the deviations in laser frequency and cavity length from resonance, as detailed in [2]. Ω_{PC} is the sideband frequency, at which the Pockels cell is driven.

Let's assume the pump beam modulates the cavity's length so that

$$\delta L = \delta L_0 \sin \Omega_{AOM} t.$$

Then the power falling on the photodiode will be

$$\begin{aligned} P \quad = \quad & 4P_{probe} \left(10\% \right) \left[\frac{4L^2\mathcal{F}^2}{\lambda^2} \left\{ \frac{\delta f^2}{f^2} + 2\frac{\delta f \delta L_0}{fL} \sin \Omega_{AOM} t + \frac{\delta L_0^2}{L^2} \sin^2 \Omega_{AOM} t \right\} \right. \\ & \left. - \frac{4L\mathcal{F}}{\lambda} \left\{ \frac{\delta f}{f} + \frac{\delta L_0}{L} \sin \Omega_{AOM} t \right\} J_1 \left(\beta \right) \sin \Omega_{PC} t + J_1^2 \left(\beta \right) \sin^2 \Omega_{PC} t \right]. \end{aligned}$$

The factor of 10% accounts for the beamsplitter between the circulator and the photodiode. The electrical signal coming out of the photodetector will be proportional to this power, and the first mixer will isolate the term proportional to $\sin\Omega_{PC}t$, or

$$V \propto 0.4P_{probe}\left[-\frac{2L\mathcal{F}}{\lambda}J_1\left(\beta\right)\left\{\frac{\delta f}{f}+\frac{\delta L_0}{L}\sin\Omega_{AOM}t\right\}\right]. \tag{3}$$

If we low-pass filter this signal, then we get a measure of the frequency of the laser, relative to the average length of the cavity. This filtered signal can then be fed back into the laser to lock it to the cavity.

$$V_{servo} \propto 0.4P_{probe}\left[-\frac{2L\mathcal{F}}{\lambda}J_1\left(\beta\right)\frac{\delta f}{f}\right].$$

We can get a measure of the modulated length of the cavity by mixing V with the AOM drive signal, effectively lock-in detecting the response to the pump beam's amplitude modulation. This mixing will pull out the term in Equation 3 proportional to $\sin\Omega_{AOM}t$, or

$$V_{data} \propto 0.4P_{probe}\left[-\frac{2L\mathcal{F}}{\lambda}J_1\left(\beta\right)\frac{\delta L_0}{L}\right].$$

5 Sensitivity

How sensitive will this setup be to changes in the cavity length? The main source of noise will probably be laser frequency noise, since it competes with length in Equation 3. In this setup, we want to lock the laser to the cavity at low frequencies, well below Ω_{AOM}, and use the error signal at high frequencies as a data channel. We should then assume that the laser is essentially free-running at Ω_{AOM}. The free-running frequency noise of a lightwave NPRO is around $100Hz/\sqrt{Hz}$ at $100Hz$, decreasing roughly as f^{-1} at higher frequencies [3]. Our minimum length sensitivity will then be, if we drive the pump beam at $1kHz$,

$$\delta L_{0,min} \approx \left(10^{-14}\frac{m}{\sqrt{Hz}}\right)\frac{L}{30cm}\frac{\delta f}{10Hz/\sqrt{Hz}}\frac{300THz}{f}.$$

This should be entirely adequate for measuring the effect.

6 Conclusions

Based on the above considerations, Braginskii's photo-thermal effect looks easily measurable in either sapphire or fused silica. The effect looks to be quite large, provided we excite it with an amplitude-modulated pump beam, and the experimental setup outlined above seems easily capable of measuring the effect. I recommend that we make it a priority to measure the effect in sapphire, since that is the material we are most worried about for LIGO II. We could start out with Aluminum, which may have an even bigger photo-thermal signature [4], and then move on to sapphire, but I strongly urge that sapphire be our ultimate goal in this experiment.

I am somewhat worried about my estimation of the size of the effect. I have done a very naive thing here, dividing Braginskii's noise term by the shot noise to get a transfer function, and I would feel much better if we could carry out a more careful calculation to estimate the size of the response to an amplitude-modulated beam. Such a calculation could be carried out simultaneously with construction of the experiment.

References

[1] V. B. Braginskii, M.L. Gorodetsky and S. P. Vyatchanin, *Thermodynamical fluctuations and photo-thermal shot noise in gravitational wave antennae*, In preparation

[2] Eric D. Black, *An introduction to Pound-Drever-Hall laser frequency stabilization*, submitted to the American Journal of Physics

[3] A. Abramovici, R. Savage, *NPRO-PSL Conceptual Design*, LIGO Technical Note T960089-00-D

[4] Shanti Rao, private communication.