Cross-Correlation Searches for GWs
Practical Considerations
Applications and Outlook

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Cross-Correlation Methods in Continuous Gravitational-Wave Searches

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Outline

1. Cross-Correlation Searches for GWs
   - Application to Stochastic Background
   - Application to Periodic Gravitational-Wave Signals

2. Practical Considerations
   - Managing Computational Cost w/Semicoherent Methods
   - Tuning Search by Choice of Data Segments to Correlate

3. Applications and Outlook
   - Directed Search for Young Neutron Stars
   - Accreting Neutron Stars in Low-Mass X-Ray Binaries

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Gravitational-Wave Correlations

- Noisy data from GW Detector:

$$x(t) = n(t) + h(t) = n(t) + \overset{\leftrightarrow}{h}(t) : \overset{\leftrightarrow}{d}$$

- Correlate data btwn detectors (Fourier domain)

$$\langle \tilde{x}_1^*(f)\tilde{x}_2(f')\rangle = \langle \tilde{h}_1^*(f)\tilde{h}_2(f')\rangle = \overset{\leftrightarrow}{d}_1 : \langle \overset{\leftrightarrow}{\tilde{h}}_1^*(f) \otimes \overset{\leftrightarrow}{\tilde{h}}_2(f')\rangle : \overset{\leftrightarrow}{d}_2$$

- For stochastic backgrounds

$$\langle \tilde{h}_1^*(f)\tilde{h}_2(f')\rangle = \delta(f - f')\gamma_{12}(f)\frac{S_{\text{gw}}(f)}{2}$$

$S_{\text{gw}}(f)$ encodes spectrum; $\gamma_{12}(f)$ encodes geometry

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Detection Statistic

- Optimally filtered cross-correlation statistic

$$Y = \int df \, \tilde{x}_1^*(f) \, Q(f) \, \tilde{x}_2(f)$$

- Filter encodes expected spectrum & spatial distribution (isotropic, pointlike, spherical harmonics ...)

$$Q(f) \propto \frac{\gamma_{12}^*(f) S_{\mathrm{gw}}^{\mathrm{exp}}(f)}{S_{n1}(f) S_{n2}(f)}$$

- "Radiometer" search for ptlike srcs incl targeting Sco X-1: known sky location, unknown frequency
Ballmer, *CQG* **23**, S179 (2006); LSC, *PRD* **76**, 082003 (2007)

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Periodic GWs from Continuous Wave Sources

- Sco X-1 is Low-Mass X-Ray Binary:
  accreting neutron star in orbit w/companion
- Rotating NS w/deformation emits nearly sinusoidal signal

$$\overleftrightarrow{h}(t) = h_0 \left[\frac{1 + \cos^2 \iota}{2} \cos \Phi(\tau(t)) \overleftrightarrow{e}_+ + \cos \iota \sin \Phi(\tau(t)) \overleftrightarrow{e}_\times \right]$$

- $\Phi(\tau)$: phase evolution in rest frame;
- $\tau(t)$: Doppler mod from detector motion (& binary orbit)
- Features of signal model missing from stoch search:
  - Doppler shift @ each detector:
    correlations peaked @ different freqs
  - Long-term coherence:
    can correlate data @ different times

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Cross-Correlation of Continuous GW Signals

- Cross-correlation of signal w/intrinsic frequency f_0:

$$\langle \tilde{x}_I^*(f_I)\,\tilde{x}_J(f_J)\rangle = \tilde{h}_I^*(f_I)\,\tilde{h}_J(f_J)$$
$$= h_0^2\,\tilde{\mathcal{G}}_{IJ}\,\delta_{\Delta T}(f_0 - f_I - \delta f_I)\,\delta_{\Delta T}(f_0 - f_J - \delta f_J)$$

  - $\tilde{h}_I(f)$ is Short Fourier Transform, duration ΔT
  - $\delta_{\Delta T}(f - f') = \int_{-\Delta T/2}^{\Delta T/2} dt\, e^{i2\pi(f-f')t}$
  - $\tilde{h}_I$ & $\tilde{h}_J$ can be same or different times or detectors
  - δf_I is relevant Doppler shift

- For given set of params, can add products of all SFT pairs

$$Y = \sum_{IJ} Q_{IJ}\tilde{x}_I^*(f_0 - \delta f_I)\tilde{x}_J(f_0 - \delta f_J) \qquad Q_{IJ} \propto \frac{\tilde{\mathcal{G}}_{IJ}^*}{S_{n,I}(f_0)S_{n,J}(f_0)}$$

Dhurandhar, Krishnan, Mukhopadhyay & JTW *PRD* **77**, 082001 (2008)

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Computational Cost
Choice of SFT Pairs for Correlation

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Computational Costs and Frequency Resolution

- If freq, sky pos etc known, can do most sensitive fully coherent search (correlate all data)
- If some params unknown, have to search over them
- Long coherent observation → fine resolution in freq etc → need too many templates → computationally impossible

$$\text{e.g.} \qquad N_{\text{tmplts}} \sim \frac{1}{\Delta f}\frac{1}{\Delta \dot{f}}\frac{1}{\Delta \text{sky}} \sim T \cdot T^2 \cdot (fT)^2$$

- Most CW searches semi-coherent: deliberately limit coherent integration time & param space resolution to keep number of templates manageable

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Fully Coherent Search

	$x_1(t_0)$	$x_2(t_0)$	$x_1(t_1)$	$x_2(t_1)$	$x_1(t_2)$	$x_2(t_2)$	$x_1(t_3)$	$x_2(t_3)$
$x_1(t_0)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_2(t_0)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_1(t_1)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_2(t_1)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_1(t_2)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_2(t_2)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_1(t_3)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_2(t_3)$	Y	Y	Y	Y	Y	Y	Y	Y

Combine all SFT pairs; as with standard $\mathcal{F}$-statistic,
quadratic combination of all SFTs

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Synchronous Cross-Correlation Search

	$x_1(t_0)$	$x_2(t_0)$	$x_1(t_1)$	$x_2(t_1)$	$x_1(t_2)$	$x_2(t_2)$	$x_1(t_3)$	$x_2(t_3)$
$x_1(t_0)$	N	Y	N	N	N	N	N	N
$x_2(t_0)$	Y	N	N	N	N	N	N	N
$x_1(t_1)$	N	N	N	Y	N	N	N	N
$x_2(t_1)$	N	N	Y	N	N	N	N	N
$x_1(t_2)$	N	N	N	N	N	Y	N	N
$x_2(t_2)$	N	N	N	N	Y	N	N	N
$x_1(t_3)$	N	N	N	N	N	N	N	Y
$x_2(t_3)$	N	N	N	N	N	N	Y	N

"Stochastic-style": correlate data @ same time, diff detectors

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Excess Power Search

	$x_1(t_0)$	$x_2(t_0)$	$x_1(t_1)$	$x_2(t_1)$	$x_1(t_2)$	$x_2(t_2)$	$x_1(t_3)$	$x_2(t_3)$
$x_1(t_0)$	Y	N	N	N	N	N	N	N
$x_2(t_0)$	N	Y	N	N	N	N	N	N
$x_1(t_1)$	N	N	Y	N	N	N	N	N
$x_2(t_1)$	N	N	N	Y	N	N	N	N
$x_1(t_2)$	N	N	N	N	Y	N	N	N
$x_2(t_2)$	N	N	N	N	N	Y	N	N
$x_1(t_3)$	N	N	N	N	N	N	Y	N
$x_2(t_3)$	N	N	N	N	N	N	N	Y

Only consider "diagonal" auto-correlations

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Semi Coherent Search

	$x_1(t_0)$	$x_2(t_0)$	$x_1(t_1)$	$x_2(t_1)$	$x_1(t_2)$	$x_2(t_2)$	$x_1(t_3)$	$x_2(t_3)$
$x_1(t_0)$	Y	Y	Y	Y	N	N	N	N
$x_2(t_0)$	Y	Y	Y	Y	N	N	N	N
$x_1(t_1)$	Y	Y	Y	Y	N	N	N	N
$x_2(t_1)$	Y	Y	Y	Y	N	N	N	N
$x_1(t_2)$	N	N	N	N	Y	Y	Y	Y
$x_2(t_2)$	N	N	N	N	Y	Y	Y	Y
$x_1(t_3)$	N	N	N	N	Y	Y	Y	Y
$x_2(t_3)$	N	N	N	N	Y	Y	Y	Y

Coherently combine within epochs

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Lag-Limited Cross-Correlation Search

	$x_1(t_0)$	$x_2(t_0)$	$x_1(t_1)$	$x_2(t_1)$	$x_1(t_2)$	$x_2(t_2)$	$x_1(t_3)$	$x_2(t_3)$
$x_1(t_0)$	Y	Y	Y	Y	N	N	N	N
$x_2(t_0)$	Y	Y	Y	Y	N	N	N	N
$x_1(t_1)$	Y	Y	Y	Y	Y	Y	N	N
$x_2(t_1)$	Y	Y	Y	Y	Y	Y	N	N
$x_1(t_2)$	N	N	Y	Y	Y	Y	Y	Y
$x_2(t_2)$	N	N	Y	Y	Y	Y	Y	Y
$x_1(t_3)$	N	N	N	N	Y	Y	Y	Y
$x_2(t_3)$	N	N	N	N	Y	Y	Y	Y

"Sliding" semi-coherent search

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Astrophysical Search Targets for Cross-Corr Search

- Intended for directed searches:
  know some phase params, search over others

- Young neutron stars w/rapid spindown:
  - Search over frequency, GW & EM spindown params
  - PhD thesis by Christine Chung (Uni Melbourne);
  - Methods paper by Chung, Melatos, Krishnan & JTW submitted to MNRAS

- Low-mass X-ray binaries:
  - Assume $\dot{f} = 0$; search over freq & uncertain orbital params
  - e.g., Scorpius X-1: $1.4 M_\odot$ NS w/$0.4 M_\odot$ companion unknown params are f_0, $a \sin i$, orbital phase
  - Search under development

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Supernova 1987A Remnant

Credit: NASA/ESA, P. Challis, R. Kirshner (Harvard-Smithsonian Center for Astrophysics) and B. Sugerman (STScI)

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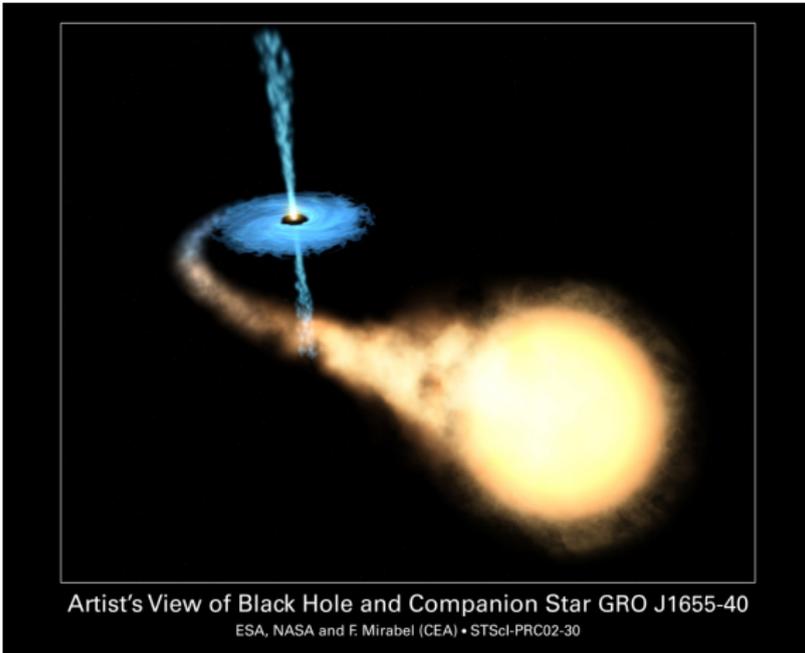Practical Considerations
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Searching for Young Neutron Stars

- Young ($\lesssim 100\,\text{yr}$) NSs should be spinning rapidly
  LIGO/Virgo band $50\,\text{Hz} \lesssim f_{\text{GW}} \lesssim 1500\,\text{Hz}$
- Look in likely sky locations for NSs not seen as pulsars:
  SN1987A should have one; galactic ctr could have $\mathcal{O}(1)$
- Spinning down rapidly; inefficient to search over f, $\dot{f}$, $\ddot{f}$, ...
  Phase model: GW spindown $\propto f^5$; EM spindown $\propto f^{\approx 3}$

$$\frac{df}{d\tau} = Q_{\text{GW}} \left(\frac{f}{f_{\text{ref}}} \right)^5 + Q_{\text{EM}} \left(\frac{f}{f_{\text{ref}}} \right)^n$$

Search over f_0, Q_{GW}, Q_{EM}, n

- PhD thesis by Christine Chung (Uni Melbourne);
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  submitted to MNRAS

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Low-Mass X-Ray Binary

Artist's View of Black Hole and Companion Star GRO J1655-40
ESA, NASA and F. Mirabel (CEA) • STScI-PRC02-30

Compact object accreting mass from companion star

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Searching for Neutron Stars in LMXBs

- LMXB: BH/NS/WD accreting mass from companion star
- Accretion spinup may be balanced by GW spindown [Bildsten *ApJL* **501**, L89 (1998)] → no $\dot{f}$
- Scorpius X-1: $1.4 M_\odot$ NS w/$0.4 M_\odot$ companion unknown params are f_0, $a \sin i$, orbital phase
- LSC searches for Sco X-1:
  - Coherent search w/6 hr of S2 data *PRD* **76**, 082001 (2007)
  - Directed stochastic cross-corr ("radiometer") search w/simultaneous S4 H1 & L1 data *PRD* **76**, 082003 (2007)
- Can use improved cross-corr method to search including wider range of correlated segments