Eigen Modes and Frequencies of Aligo MC Optics

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The Eigenmodes and frequencies of the Aligo MC mirrors are analyzed using Comsol 3.3a.

The optic was modeled as a cylinder with a wedge. One face is perpendicular to the cylindrical axis; the other face is wedged 2 degrees with respect to the cylindrical axis. The diameter is 0.150m, and thickness is 0.075m at the thickest portion of the wedge. The real optics' also have a chamfer of 2mm X 45 degrees on each edge, but this was neglected for the FEA model. The effect of the optical coatings is also neglected.

The material is fused silica, Corning code 7980. Corning provides material constants for the mechanical properties of the glass, which are listed in Table 1.

PROPERTY	VALUE
Young's Modulus	72.7 GPa
Poisson's Ratio	0.16
Density	2201 kg / m^3

Table 1. Material Constants

Table 2 gives the initial mesh statistics, with 6443 tetrahedral elements. The first legitimate eigenfrequency calculates to 12486 Hz with this mesh.

Number of degrees of freedom	29283
Number of mesh points	1379
Number of elements	6443
Tetrahedral	6443
Prism	0
Hexahedral	0
Number of boundary elements	1122
Triangular	1122
Quadrilateral	0
Number of edge elements	84
Number of vertex elements	8
Minimum element quality	0.451
Element volume ratio	0.096

Table 2. Initial Mesh Statistics

In order to be certain that the mesh is sufficiently fine, the mesh is refined to 22417 elements. The first legitimate eigenfrequency with this new mesh is 12483 Hz. This is within a tenth of a percent of the original value, so the mesh is sufficiently fine. Table 3 gives the new mesh statistics. This new mesh is used for all of the following analysis.

Number of degrees of freedom	98169
Number of mesh points	4648
Number of elements	22417
Tetrahedral	22417
Prism	0
Hexahedral	0
Number of boundary elements	2022
Triangular	2022
Quadrilateral	0
Number of edge elements	142
Number of vertex elements	8
Minimum element quality	0.283
Element volume ratio	0.061

Table 3. New Mesh Statistics

The solution is run to solve for 21 eigenfrequencies. The first six frequencies are neglected because they represent free body motion, not vibration modes. Several modes appear at two or three frequencies spaced a few Hz apart. These multiple modes are identical in shape, except that one usually aligns with the wedge, and the others are a distinct rotation of the first.

The figures on the following pages show the eigenmodes and list their frequencies. In all of the figures the wedged face is on top, with the thick side to the left.

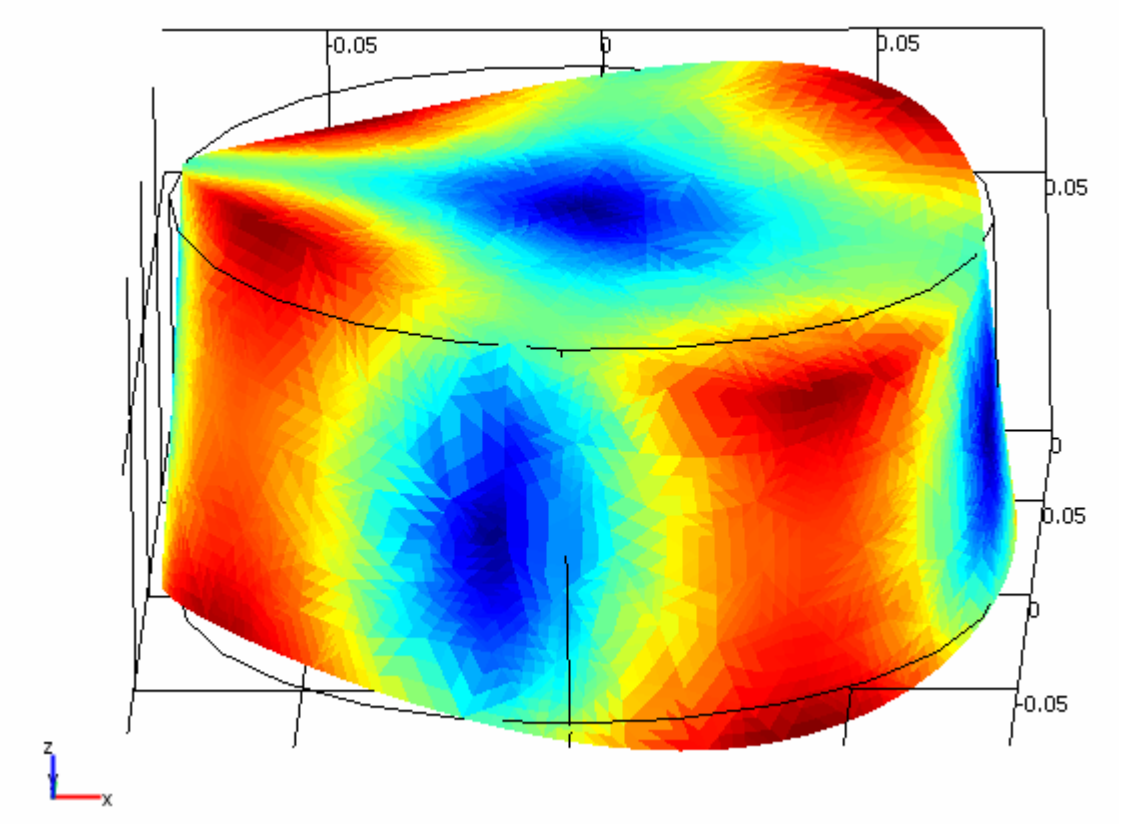

Figure 1. First Mode 12483 HZ

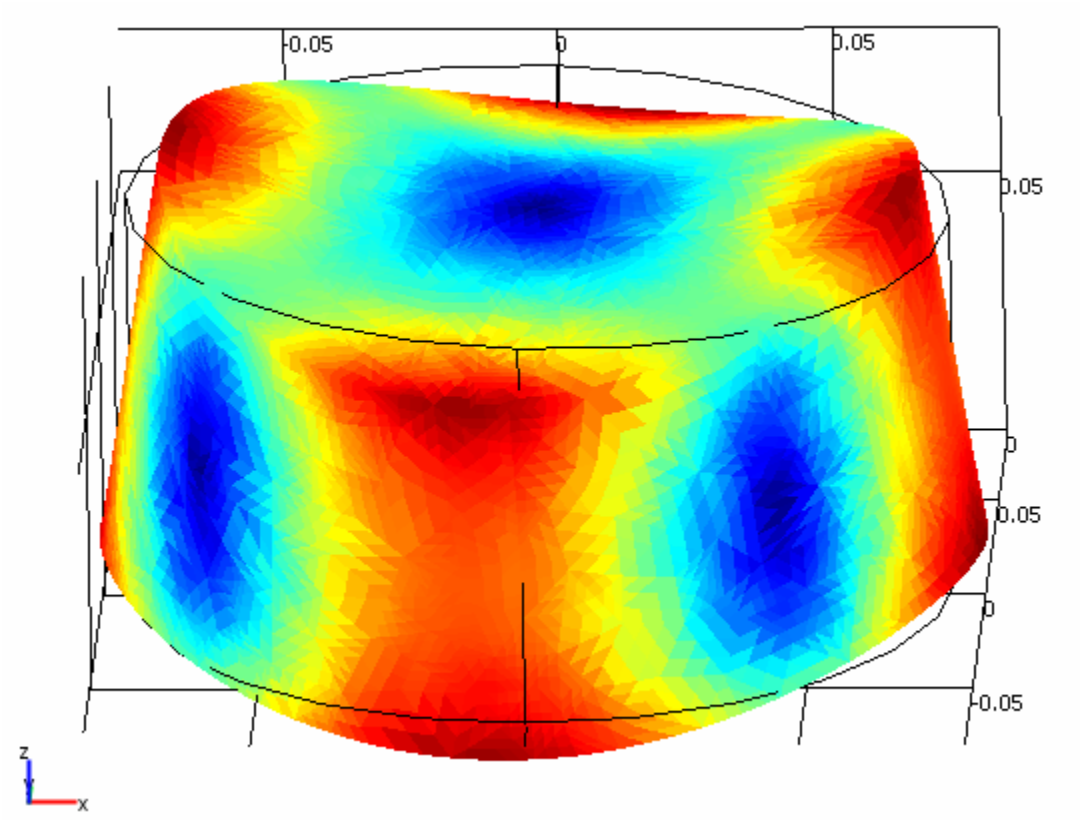

Figure 2. Second Frequency of First Mode 12485 Hz

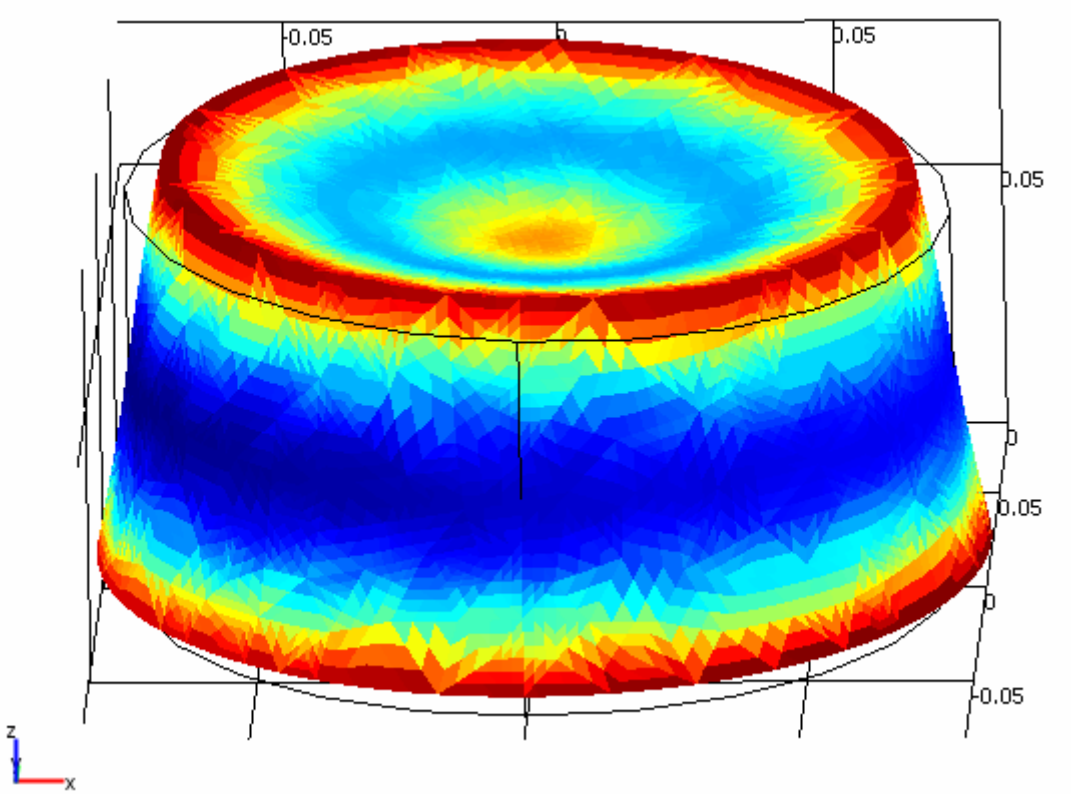

Figure 3. Second Mode 17122 Hz

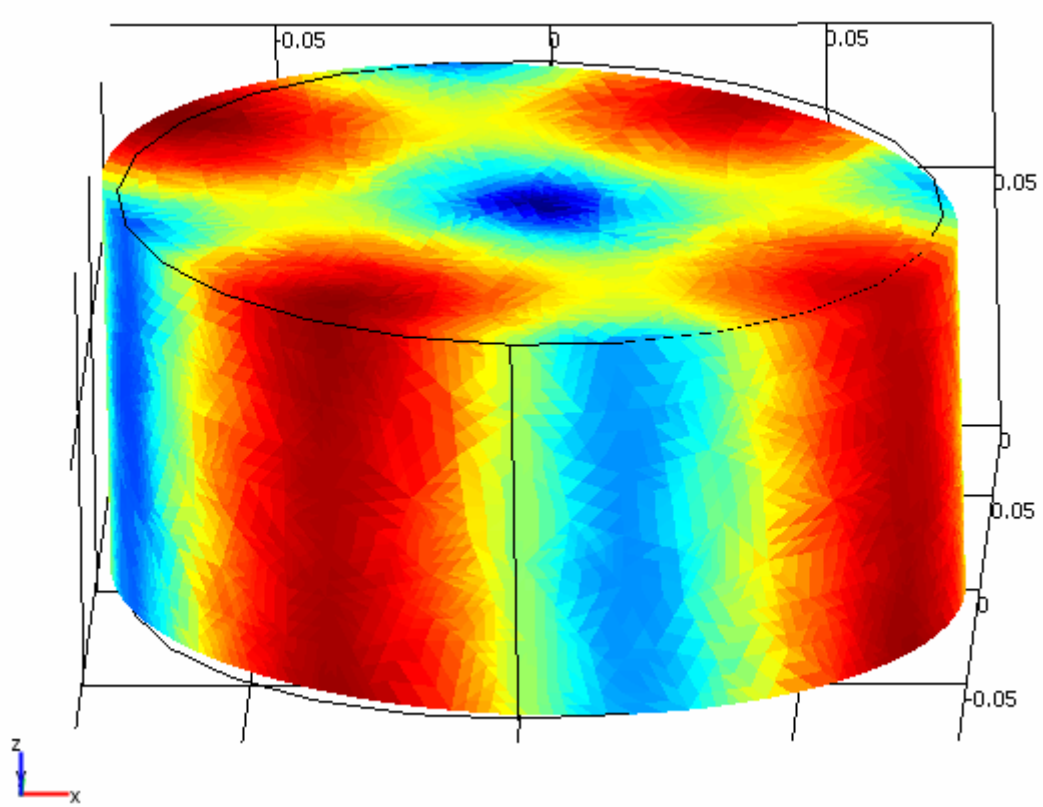

Figure 4. Third Mode 18747Hz (also has an identical duplicate solution)

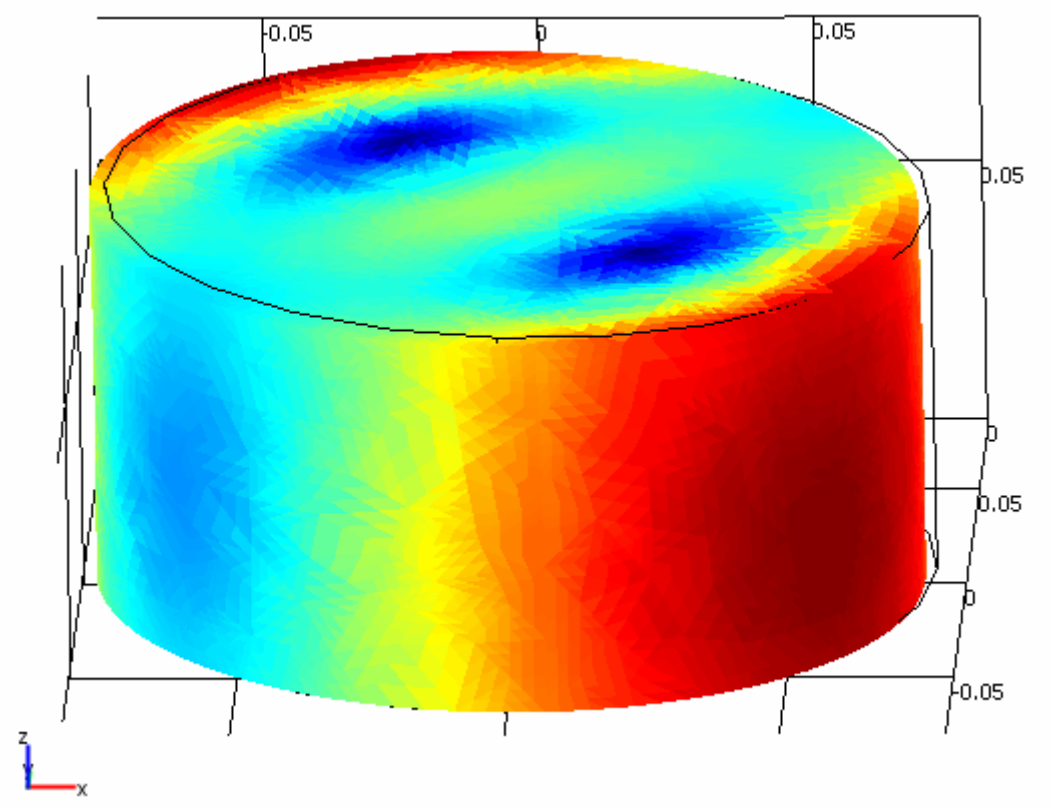

Figure 5. Fourth Mode 20948 Hz

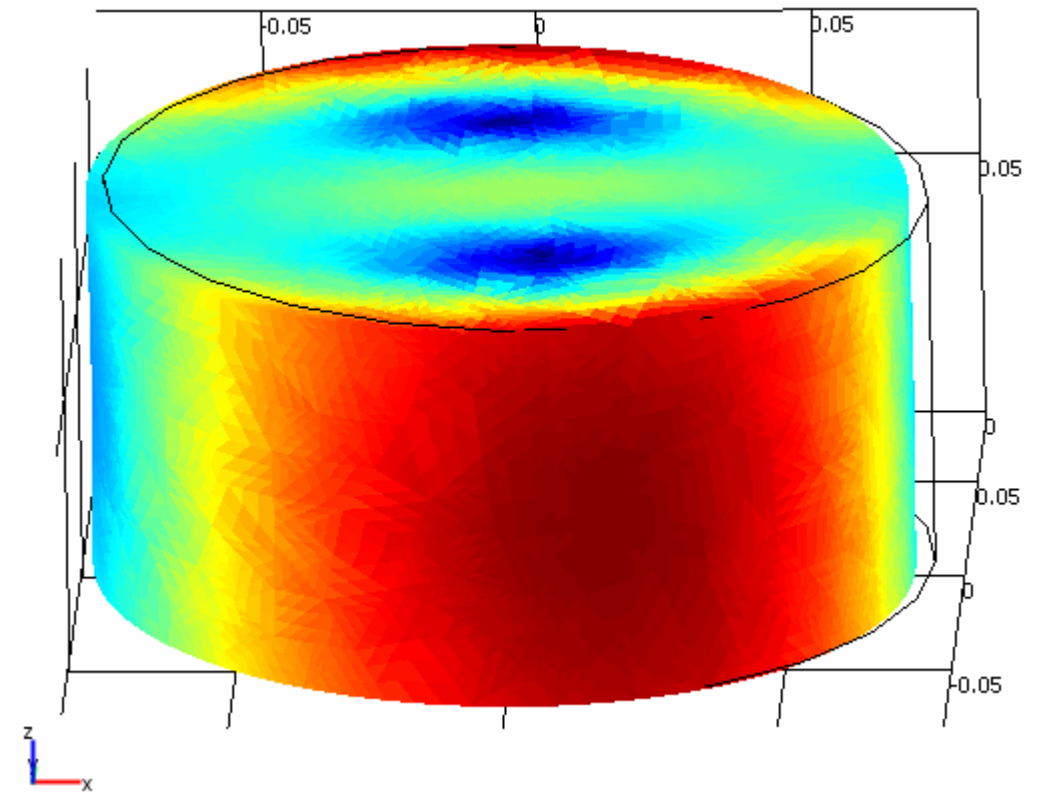

Figure 6. Second Frequency of Fourth Mode 20958 Hz

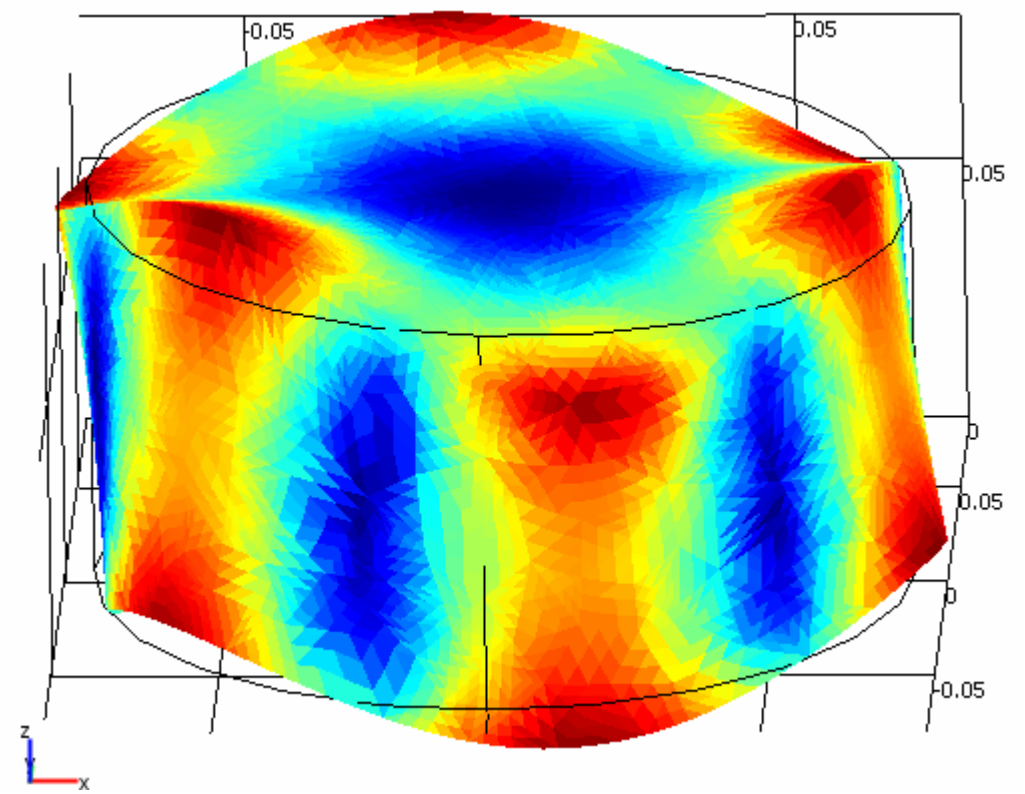

Figure 7. Fifth Mode 22130 Hz

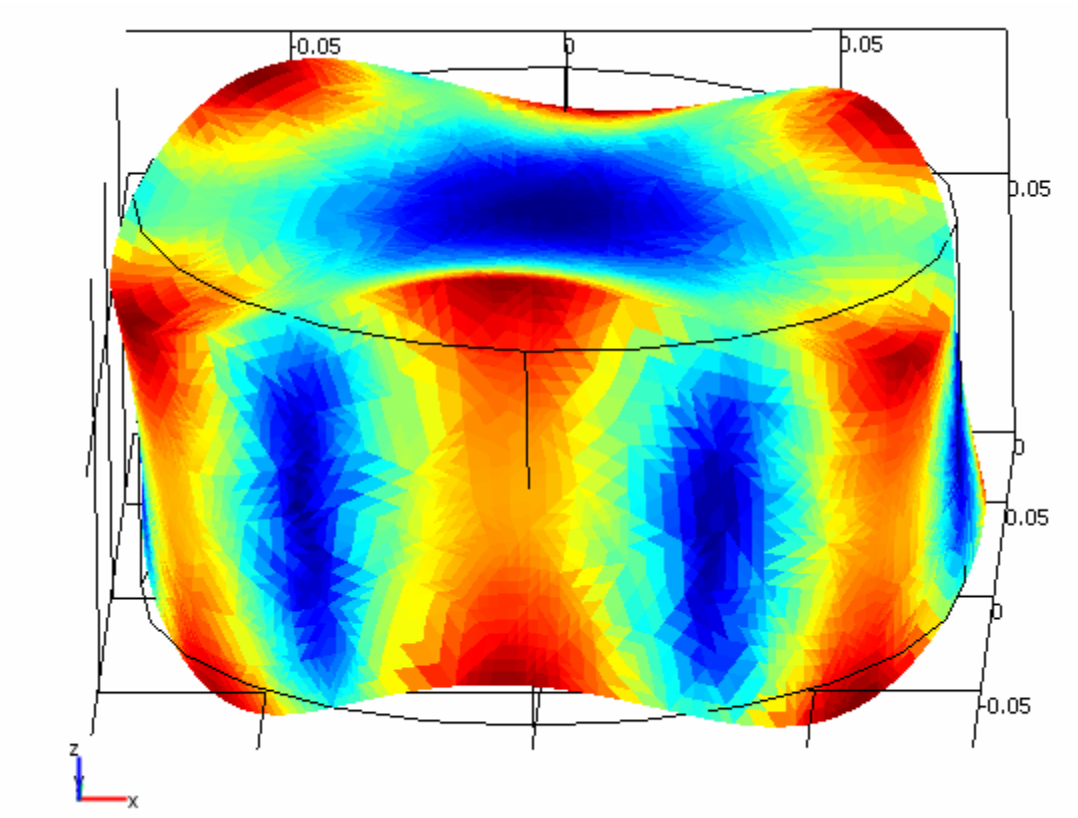

Figure 8. Second Frequency of Fifth Mode 22137 Hz

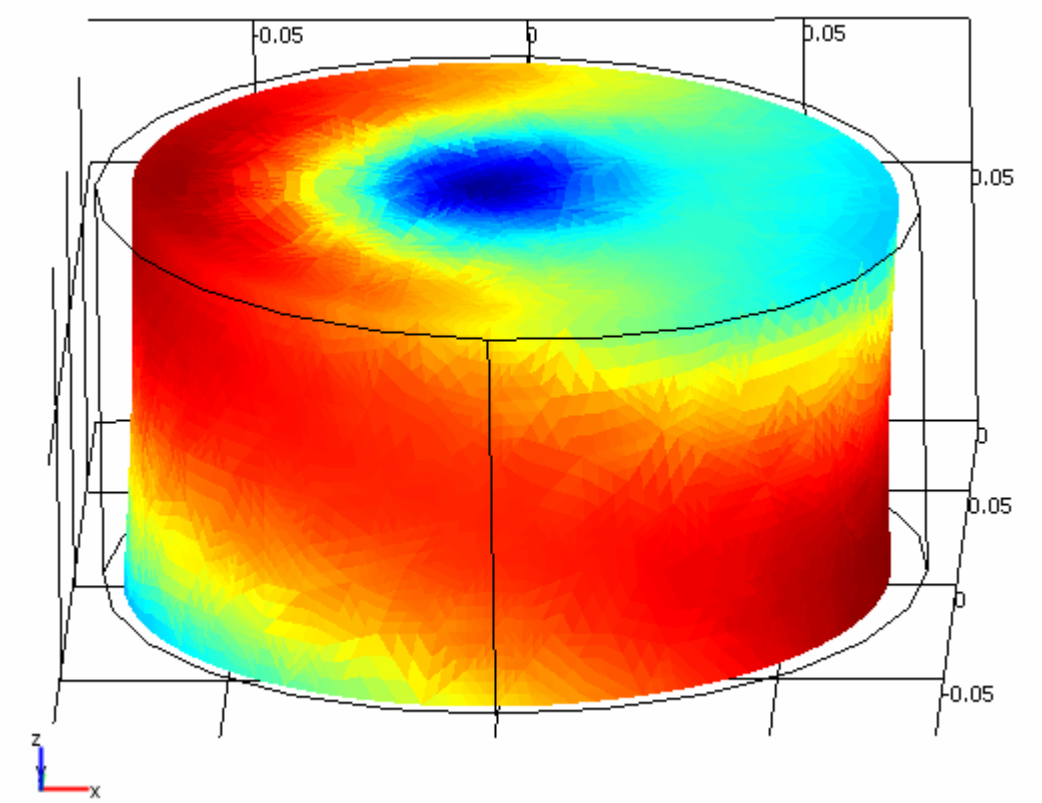

Figure 9. Sixth Mode 23853 Hz

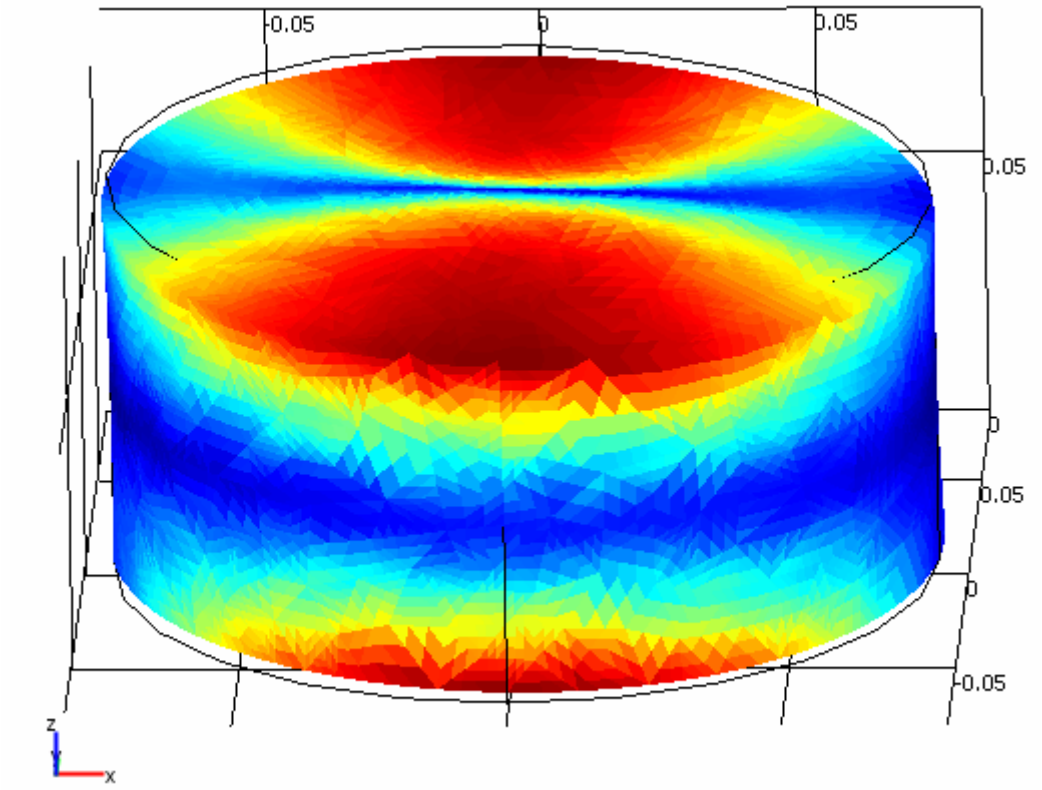

Figure 10. Second Frequency of Sixth Mode 24264 Hz

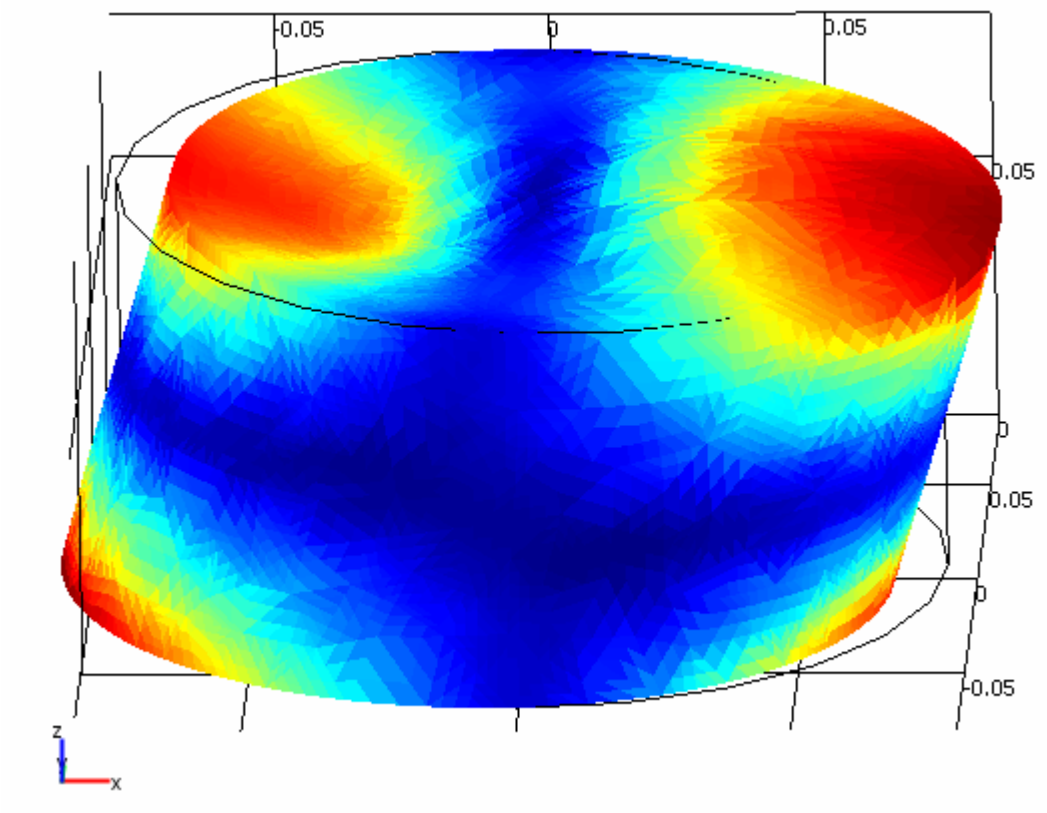

Figure 11. Third Frequency of Sixth Mode 24271 Hz

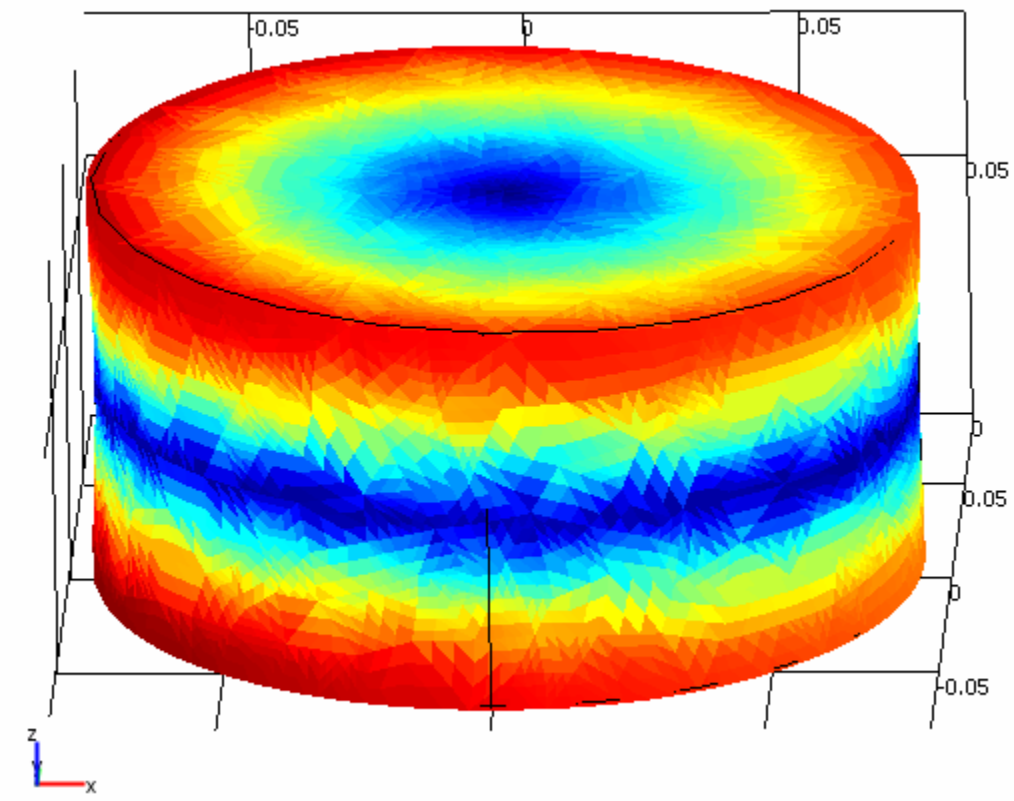

Figure 12. Seventh Mode 26057 Hz

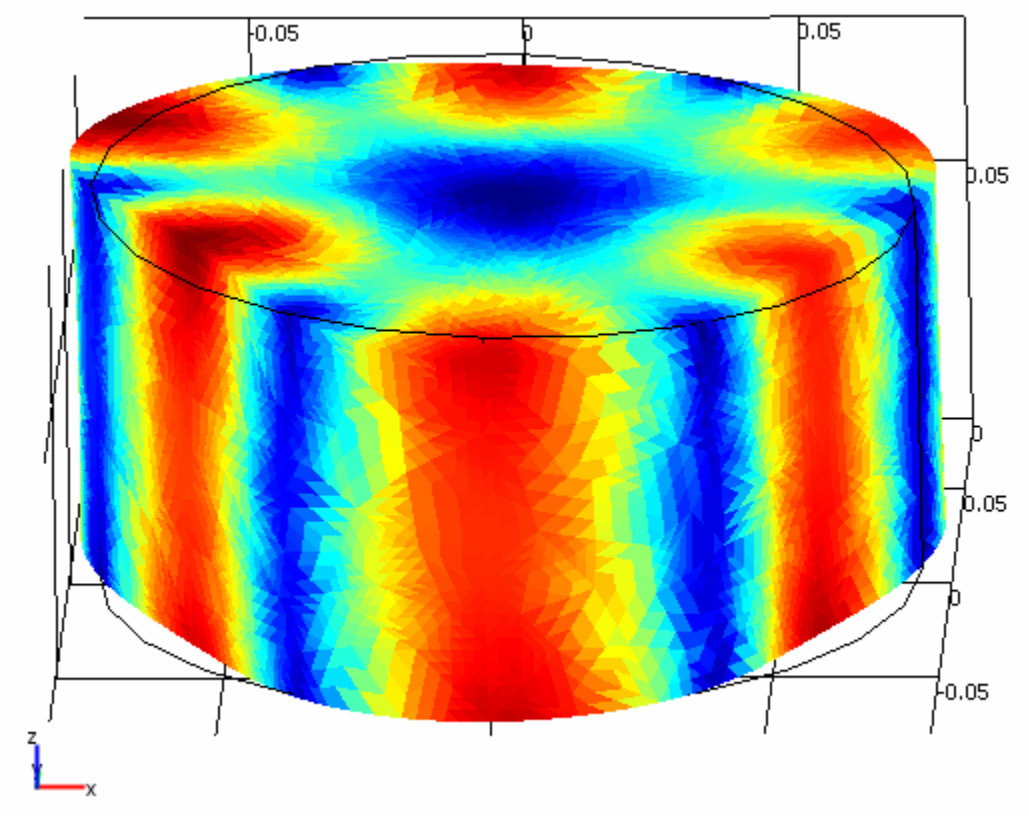

Figure 13. Eighth Mode 28628 Hz

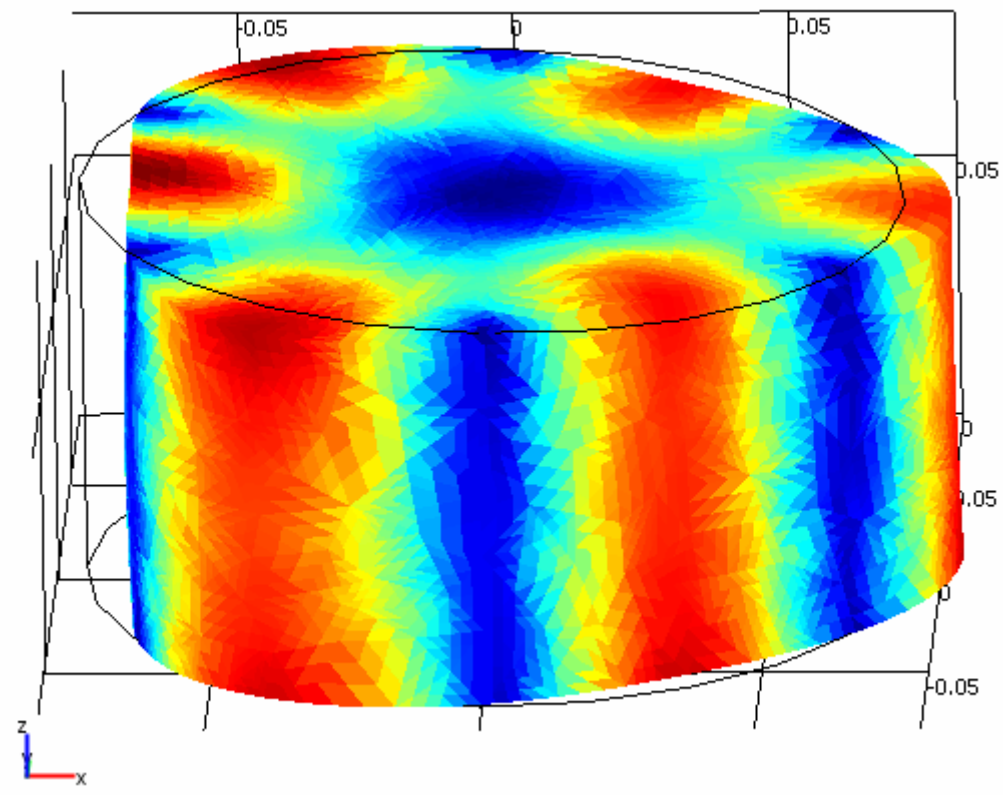

Figure 14. Second Frequency of Eighth Mode 28638 Hz

Table 4 sumarizes the modes and their frequencies, including the first six free body motion solutions.

MODE	FREQUENCY (Hz)	TYPE
X	0	Free Body Motion
X	0	Free Body Motion
X	0	Free Body Motion
X	0	Free Body Motion
X	0	Free Body Motion
X	0	Free Body Motion
1	12483	Butterfly (axial) 4-fold
1 (second)	12485	Butterfly (axial) 4-fold
2	17122	Asymmetric Drumhead
3 (primary and duplicate)	18747	Breathing (radial) 4-fold
4	20948	Cardioid
4 (second)	20958	Cardioid
5	22130	Butterfly (axial) 6-fold
5 (second)	22137	Butterfly (axial) 6-fold
6	23853	Shear
6 (second)	24264	Shear
6 (third)	24271	Shear
7	26057	Symmetric Drumhead
8	28628	Breathing (radial) 6-fold
8 (second)	28638	Breathing (radial) 6-fold

Table 4. Summary of Modes and Frequencies

More modes and frequencies can be calculated if needed, but the accuracy of the solution becomes more suspect as the mode number increases.