Using a Single Pipe for the Distribution of Hydraulic Fluid in the LLO Corner Station

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LIGO-E030347-00-L

Summary

Using a single pipe for the distribution of fluid to the chambers in the corner station is acceptable. The dynamics of 7 pipes in parallel is slightly preferable to dynamics of a single line, but the performance difference is small, and both cases are acceptable.

For a single pipe:

flow is 8.53 gpm (5.39e-4 m^3/s)
inner diameter = 33.6 mm (1.32 inches)
Reynolds number = 182

For 2 pipes in parallel

inner diameter = 28.3 mm (1.11 inches)
Reynolds number = 107

For 7 pipes in parallel

inner diameter = 20.7 mm (0.81 inches)
Reynolds number = 43

Calculation parameters

We consider a 60 meter long line feeding 7 chambers.

Dynamic viscosity $\mu = .1\ kg*m^{-1}*s^{-1}$ = 100 centipoise.

Density $\rho = 900\ kg/m^3$

bulk modulus, $E = 1e9\ Pa/m^2$

Thus, the sound speed, a, is 1.0541e3 m/s.

We require a 1.03e5 Pa (15 psi) pressure drop along the set of distribution lines.

The chamber is modeled as 8 resistors of 5e10 $Pa*s*m^{-3}$ in parallel, with an accumulator at the inlet and an accumulator at the outlet.

The accumulator is 0.5 pints of air, 85 psi at chamber supply, 15 psi and chamber return, with a feedline 1 cm in diameter and 8 cm long.

The lines from the distribution manifold (at the accumulators) to the resistors are ignored in this calculation (set to length 0), since they add resonances which make the results difficult to present, but do not change the conclusions.

Calculations

The scaling of the pipes can be seen in figure 1. This shows how the Reynolds number and pipe diameter scale with the number of pipes in parallel which feed the 7 chambers.

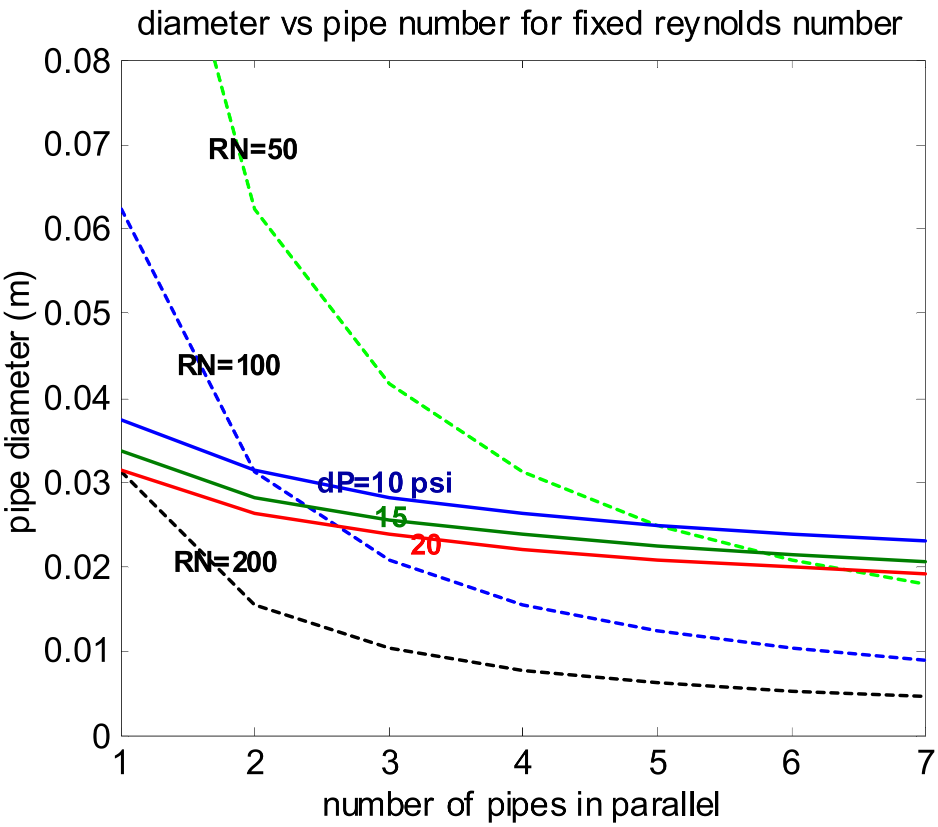

Figure 1. Scaling of pipe diameter with the number of parallel pipes feeding 7 chambers. The dashed lines show how the Reynolds number scales for a fixed total flow. A Reynolds number of less than 200 should be acceptable for a long pipeline (theoretical flow is turbulent at Reynolds numbers above ~2300). The solid lines show the required pipe diameter to have a given pressure drop from one end of the pipe to the other (blue is 10 psi, green is 15 psi, and red is 20 psi). The pressure drop scales as the 1/pipe diameter^4. Calculated by pipe_calcs.m, BTL June 19 2003.

Changing the pipe diameter also changes the viscous damping of standing waves in the distribution line. Standing waves are present because the distribution accumulator is effectively a short circuit at the end of the distribution line (the line acts as a transmission line, not a lumped element). The characteristic impedance of the line is of order 1e9 Pa*s*m^{-3}, and the impedance of the accumulator is of order 1e3 Pa*s*m^{-3}. Fortunately, the high viscosity of the fluid should be effective at damping the standing waves. The damping is characterized by α, the "viscosity factor," $\alpha = 32*\mu/(D^2*\rho)$.

The calculations are done with an m-file called LIGO_distribution_system_fancy.m. This code is based on the pipeline dynamic calculation done in Taco Viersma's book, "Analysis, Synthesis and Design of Hydraulic Servosytems and Pipelines" and coded by Dan DeBra and others. The code has been modified to use the good quality expansion for the viscosity effects shown on page 143, namely

```
soa = s/alpha;
N = 1 + alpha/s + .1918/(1+.2496*soa) + .0948/(1+ .0352*soa) + .0407/(1+.0024*soa);
```

This series should be good up to frequencies abs(s)<1000 alpha, which is about 1 kHz in our case.

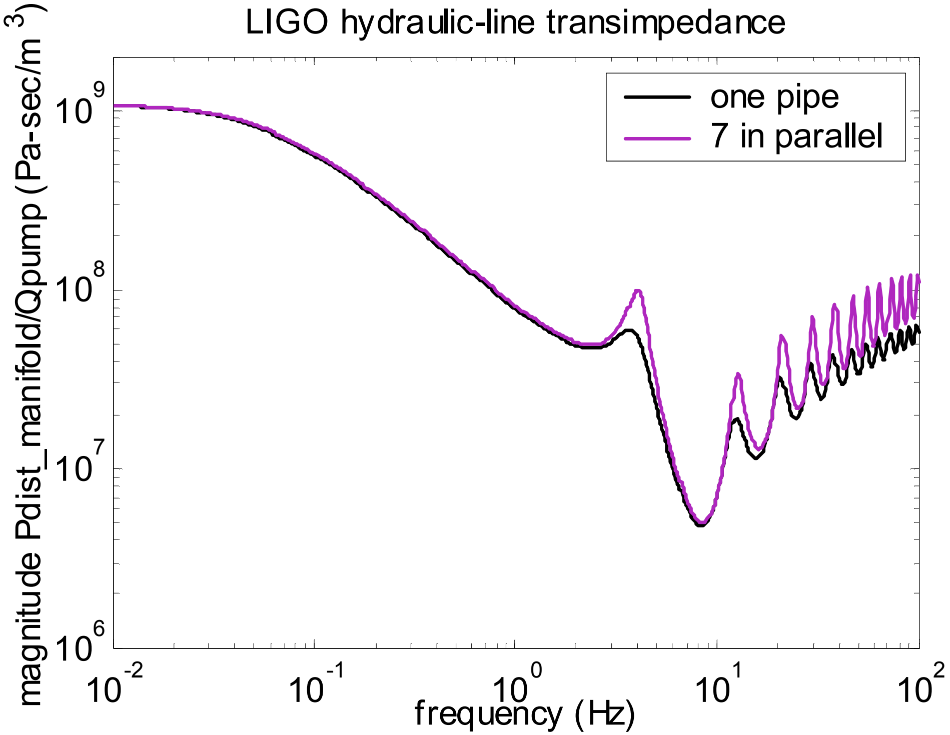

Figure 2. The characteristic impedance of LIGO 1 distribution system. The system modeled is a bundle of supply pipes, 7 chambers in parallel, and a bundle of return pipes. The curves show the transimpedance of the supply bundle, which is the transfer function of pressure-at-supply-pipe-outlet over flow-at-supply-pipe-inlet. The resonances are from the standing waves in the pipeline. The purple curve is from a single supply line, and the black curve, which is better damped, is from a bundle of 7 pipes in parallel. The performance of the 7 parallel pipes is slightly better, but both are acceptable. Calculated by LIGO_simple_distribution.m, BTL, June 20, 2003.

Scaling of a Single Pipe

Moving to a single pipe of diameter smaller than 33.6 mm ID will lead to much ligher pressure loss in the distribution system which is inefficient. However, moving to a larger diameter is more expensive and gives less viscous damping. For a 44.5 mm (1.75 inch) ID pipe, we have

Pipe ID: 44.5 mm (1.75 inch)
Reynolds number: 139
pressure drop: 4.9 psi

as compared with

Pipe ID: 33.6 mm (1.32 inch)
Reynolds number: 182
pressure drop: 15 psi

The damping is not quite as good in a 44.5 mm pipe as a 33.6 mm pipe, as can be seen below in figure 3.

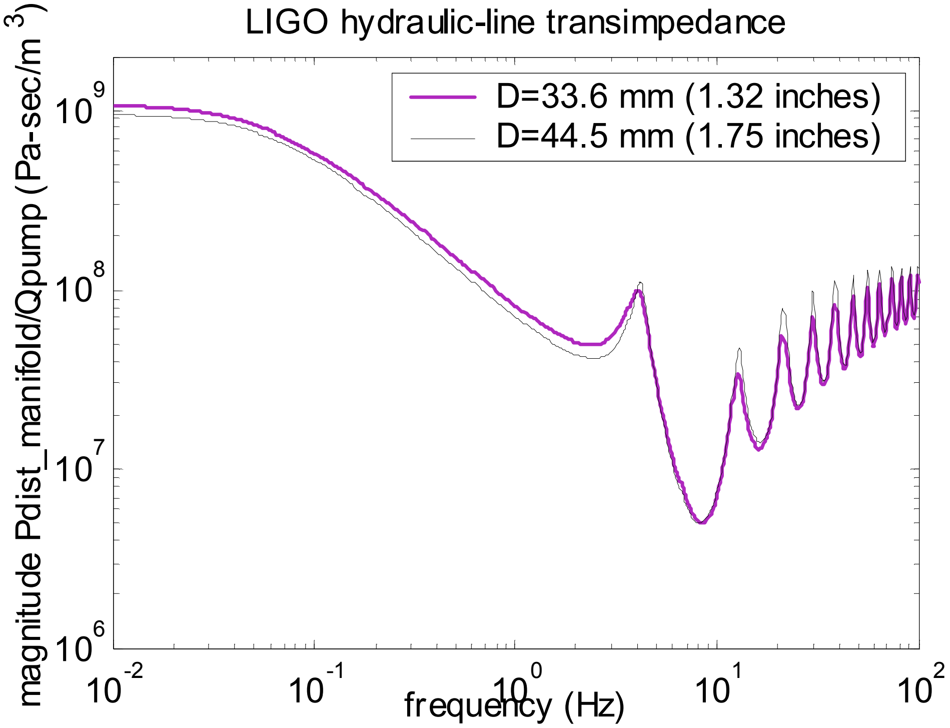

Figure 3. Comparison of the damping for two different diameters of supply pipe. Both cases use a single pipe, the purple curve is the single line with a 33.6 mm diameter (the baseline 15 psi drop). The thin black curve is a slightly larger diameter (44.5 mm = 1.75 inches). The larger diameter gives decreased damping.

Conclusion

The performance difference between a single line and a parallel bundle of distribution lines is small, seven pipes in parallel give slightly better performance than a single, larger pipe, but the both have acceptable performance. Increasing the diameter above that required for a 15 psi drop is not recommended, but slight increases do not impact the performance very much.

A zipped verison of all the Matlab files has been submitted to the DCC as E030366-00-L

```
conversion_factors  % load some useful conversion factors

total_flow = 7*8*9.7e-6;        % m^3/sec flow for 7 chambers, 8 actuators, 10ml/sec per actuator
L = 60; % pipe is 60 meters long
mu = .1;        % viscosity of fluid is 100 centipoise  = .1 SI
rho = 900;      % kg/m^3
% Res = (8*viscosity*length)/(pi*rad^4);
%Reyn = (4/pi)*(flow_rate*density)/(diameter*viscosity);

% calc based on the reynolds number

n=[1,2,3,4,5,6,7];
flow_per_pipe = total_flow./n;

% calculate the pipe diameter needed to achieve a given reynolds number
% given the total flow and number of parallel pipes
RN = 200;
D200 = (4/pi) * (flow_per_pipe*rho)/(mu*RN);

RN = 100;
D100 = (4/pi) * (flow_per_pipe*rho)/(mu*RN);

RN = 50;
D50 = (4/pi) * (flow_per_pipe*rho)/(mu*RN);

figure
pp = plot(n,D200,'k',n,D100,'b',n,D50,'g');
title('diameter vs pipe number for fixed reynolds number')
xlabel('number of pipes in parallel')
ylabel('pipe diameter (m)')
set(pp(1),'LineWidth',1.5)
set(pp(2),'LineWidth',1.5)
set(pp(3),'LineWidth',1.5)
text(n(1),D50(1),' RN=50');
text(n(1),D100(1),' RN=100');
text(n(1),D200(1),' RN=200');

legend('RN = 200','RN = 100','RN = 50')


% allow a fixed pressure drop, of 15 psi
P = 15 * psi2SI
R_total = P/total_flow
R_pipe = n*R_total;

% Res = (128*viscosity*length)/(pi*D^4)
% D = ((128*viscosity*length)/(pi*Res))^.25

D_res_15 = ((128*mu*L)./(pi*R_pipe)).^.25;

% allow a drop of 10 psi
P = 10 * psi2SI
R_total = P/total_flow
R_pipe = n*R_total;
D_res_10 = ((128*mu*L)./(pi*R_pipe)).^.25;

% allow a drop of 20 psi
P = 20 * psi2SI
R_total = P/total_flow
R_pipe = n*R_total;
D_res_20 = ((128*mu*L)./(pi*R_pipe)).^.25;
```

```
hold on
p2 = plot(n,D_res_10,n,D_res_15,n,D_res_20);
```

```
% LIGO_simple_distribution.m is used to model the distribution system for LLO
% based on code by Joshua Phinney
% originally by Dan DeBra
%
%   THIS USES THE FANCY DEFN OF DAMPING IN MPIPE, WHICH GIVES MUCH MORE DAMPING AT HIGH FREQ RES.
%
%  in part to answer questions about the benefits of mutliple parallel pipes
%
% you need mpipe2_fancy.m for this to run
% mpipe2 is the same as mpipe, but allows multiple parallel pipes and improved high freq damping
%
% BTL June 23, 2003


% parts are:
% distribution line bundle
% load = chambers in parallel
% return line bundle
%
% each chamber is
% accumulator at distribution manifold
% 8 parallel runs of pipe - actuator - pipe
% accumulator at return side manifold

format short e
clear
disp(' ')
disp(' ')
%close all

f1 = .01;
f2 = 100;
points = 1000;

j=sqrt(-1);
psi2SI = 6894.757;

%----------------------------------------------------------------------------
%   FLUID PROPERTIES

%   oil
beta=  1e9;
rho=900 ;
mu=.1   ;
a=sqrt(beta/rho);

%   air
gamma=1.4;



%----------------------------------------------------------------------------
%   NETWORK PARAMETERS

num_chambers = 7    % number of chambers fed by a distribution line
num_pipes = 1

L.dist = 60;    % distance from second accumulator to load side accumulator
L.branch=0;     % distance between the load accumulator and the load
L.feed = .08;   % length of accumulator feed line, meters

D.branch = (1/2)*25.4e-3;   % diameter of the line between the load side accumulator and the load, ↙
meters
```

```
D.feed = 10e-3;     % diameter of accumulator feedline meters

resistance_bridge = 5e10/(8*num_chambers);


% put the resistance into the pipes before the RC's accumulator
p_bridge=70*psi2SI;
Qss=p_bridge/resistance_bridge; %3gpm
Qperpipe = Qss/num_pipes;

p_dist = 15*psi2SI;     % allow a 15 psi drop in the distribution lines
resistance_dist = num_pipes*p_dist/Qss      % resistance per pipe

% the zero freq res of a pipe is: R = (128*viscosity*length)/(pi*diameter^4);
D.dist = ((128*mu*L.dist)/(pi*resistance_dist))^.25
%D.dist = 1.75 * 25.4e-3;   % fixed diameter
RN_dist = pipeReynolds(Qperpipe,D.dist,rho,mu)
% ounce * 29.57 = cubic cm (CRC)
% pint * 16 * 29.57 * 1e-6 = m^3 (4.7e-4)
pint2SI = 16 * 29.57 * 1e-6;
vol_Cman  = .5*pint2SI; % air volume of the accumulator for the distribution manifold (15 and 20)



%----------------------------------------------------------------------------
%   PIPE AND LOAD IMPEDANCES

A.dist      = pi*D.dist^2/4 ;       %   m^2 crossectional area
T.dist      = L.dist/a ;            %   sec sound propagation time
Z.dist      = rho*a/A.dist ;        %   Pa-sec/m^3 characteristic impedance
alpha.dist  = 32*mu/(D.dist^2*rho); %   1/sec viscosity frequency
R.dist      = 128*mu*L.dist/(pi*D.dist^4);  %   Pa-sec/m^3 low frequency flow resistance of 1 of th ↙
e pipes

A.feed      = pi*D.feed^2/4 ;       %   m^2 crossectional area
T.feed      = L.feed/a ;            %   sec sound propagation time
Z.feed      = rho*a/A.feed ;        %   Pa-sec/m^3 characteristic impedance
alpha.feed  = 32*mu/(D.feed^2*rho); %   1/sec viscosity frequency
R.feed      = 128*mu*L.feed/(pi*D.feed^4);  %   Pa-sec/m^3 low frequency flow resistance

A.branch    = pi*D.branch^2/4 ;         %   m^2 crossectional area
T.branch    = L.branch/a ;          %   sec sound propagation time
Z.branch    = rho*a/A.branch ;      %   Pa-sec/m^3 characteristic impedance
alpha.branch= 32*mu/(D.branch^2*rho);   %   1/sec viscosity frequency
R.branch    = 128*mu*L.branch/(pi*D.branch^4);  %   Pa-sec/m^3 low frequency flow resistance

R.all_dist  = R.dist/num_pipes;
R.all_supply_branches = R.branch/(num_chambers*8);      % DC resistance of all the branch lines in ↙
parallel
R.all_actuators = resistance_bridge;        %   bridge resistance
R.all_return_branches = R.branch/(num_chambers*8);      % DC resistance of all the branch lines in ↙
parallel


%----------------------------------------------------------------------------
%   DC LOAD FLOW

%   the steady state pressure at each accumulator
%   Given the bridge supply pressure=70 psi
disp(sprintf('flow is %g m^3/sec (%g gpm)',Qss, Qss*15840))
R.tot=R.all_dist + R.all_return_branches + R.all_actuators + R.all_return_branches + R.all_dist;
p.supply_pipe_inlet  =R.tot*Qss;            % pump average output pressure
p.dist_manifold_supply  =(R.tot-R.all_dist)*Qss;
```

```
p.actuator_supply  =(R.all_actuators + R.all_return_branches + R.all_dist)*Qss;
p.actuator_return =(R.all_return_branches + R.all_dist)*Qss;
p.dist_manifold_return =(R.all_dist)*Qss;

disp(sprintf('pressures are: \npipe inlet:     %g psi\ndist manifold:  %g psi\nbefore load:    %g p ↙
si\nafter load:     %g psi\nreturn manifold:%g psi', ...
    p.supply_pipe_inlet/psi2SI,p.dist_manifold_supply/psi2SI,p.actuator_supply/psi2SI, ...
    p.actuator_return/psi2SI,p.dist_manifold_return/psi2SI))
%-----------------------------------------------------------------------------

%  monitors outlet of pipe:
% PoQ_distac - pressure after the distribution accumulator

PoQ_distac = [];
wv=[];

%-----------------------------------------------------------------------------
%   COMPUTE FREQUENCY RESPONSE
for w=logspace(log10(f1*2*pi), log10(f2 *2*pi), points)
    wv=[wv  w];
    s=j*w;
    m_feed=mpipe2_fancy(s,T.feed,alpha.feed,Z.feed,1);

    G.dist_accum    = s*vol_Cman/(gamma*p.dist_manifold_supply);
    G.return_accum  = s*vol_Cman/(gamma*p.dist_manifold_return);

    G.feed_dist_accum   = (m_feed(1,2)-G.dist_accum*m_feed(2,2))/(m_feed(2,1)*G.dist_accum-m_feed(1 ↙
,1));
    G.feed_return_accum = (m_feed(1,2)-G.return_accum*m_feed(2,2))/(m_feed(2,1)*G.return_accum-m_fe ↙
ed(1,1));

    m_dist_accum   =[1  -G.feed_dist_accum; 0   1];
    m_return_accum =[1  -G.feed_return_accum; 0     1];

    m_branch = mpipe2_fancy(s,T.branch,alpha.branch,Z.branch,1);

    m_supplyline = mpipe2_fancy(s,T.dist,alpha.dist,Z.dist,num_pipes);
    m_returnline = m_supplyline;
    m_actuator = [1 0; -5e10, 1];

    m_one_actuator_loop = m_branch * m_actuator * m_branch;     % actuator and lines to dist manifol↙
d
    m_all_actuator_loop = parallel_copies(m_one_actuator_loop,8);   % 8 acts in parallel
    m_one_chamber = m_return_accum * m_all_actuator_loop * m_dist_accum;    % 8 acts and the two ma ↙
nifolds w/ accumulators
    m_load = parallel_copies(m_one_chamber,num_chambers);



    % distribution manifold
    mline_dist   =  m_supplyline;
    mreturn_dist = m_returnline * m_load;
    Zreturn_dist = -mreturn_dist(2,1)/mreturn_dist(2,2);
    PoQ_distac_n = (Zreturn_dist)/(mline_dist(2,2) - Zreturn_dist*mline_dist(1,2) );
    PoQ_distac = [PoQ_distac PoQ_distac_n];


end %   for w



%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%                               plotting                                    %%%%%
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


freq = wv/2/pi;

plmag=1;
if plmag==1
    figure(1)
    h=loglog(freq,abs(PoQ_distac),'k');
    axis([f1 f2 1e6 2e9]);
    set(h,'LineWidth',1.5);

    xlabel('frequency (Hz)')
    ylabel('magnitude Pdist\_manifold/Qpump (Pa-sec/m^3)')
    title('LIGO hydraulic-line transimpedance')
    hold on
end

plphase=0;
if plphase==1
    figure
    h=semilogx(freq,angle(PoQ_distac)*180/pi,'k');
    set(h,'LineWidth',1);
        hold on
    xlabel('frequency (Hz)')
    ylabel('phase pdist\_manifold/Qpump (deg)')
    title('LIGO hydraulic-line transimpedance')
end
```

```
function M=mpipe2_fancy(s,T,alpha,Z,np)
%mpipe2_fancy Calculate the elements of the four terminal network for a set of parallel pipes
%   This function needs s=j*w, T=L/a, alpha=32mu/D^2rho, Z=rho*a/A,
%   and np, the number of equally sized pipes
%   M=mpipe2_fancy(s,T,alpha,Z,np)
%
%   uses a more elaborate method for computing N which gives more accurate
%     response for pipeline resonances at high frequency see pg 143, eqn 6e
%
%   DeBra 1988 may 5 ref Taco Viersma pg 146

%N=1+alpha/s;
soa = s/alpha;
N = 1 + alpha/s + .1918/(1+.2496*soa) + .0948/(1+ .0352*soa) + .0407/(1+.0024*soa);

sqn=sqrt(N);
tsn=T*s*sqn;
A=cosh(tsn);
zn=Z*sqn;
B=-np*sinh(tsn)/zn;
C=-sinh(tsn)*zn/np;
D=A;

M=[A B; C D];
```