Thermal noise in dielectric mirror coatings: calculation and optimization

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Outline

Sorts of phase noise in multilayer coating

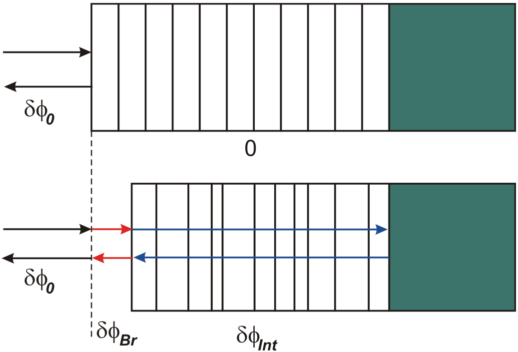
$$d\Phi = -s\,d\theta - u_{ik}\,d\sigma_{ik} + \mu\,dN + \dots$$

$$\frac{\partial u_{ik}}{\partial \theta}\delta\theta \qquad \frac{\partial n}{\partial \theta}\delta\theta \qquad \frac{\partial u_{ik}}{\partial \sigma_{ik}}\delta\sigma_{ik} \qquad \frac{\partial n}{\partial \sigma_{ik}}\delta\sigma_{ik}$$

$$\frac{\partial d}{\partial \theta}\delta\theta \qquad \frac{\partial n}{\partial \theta}\delta\theta \qquad \delta d \qquad \frac{\partial n}{\partial d}\delta d$$

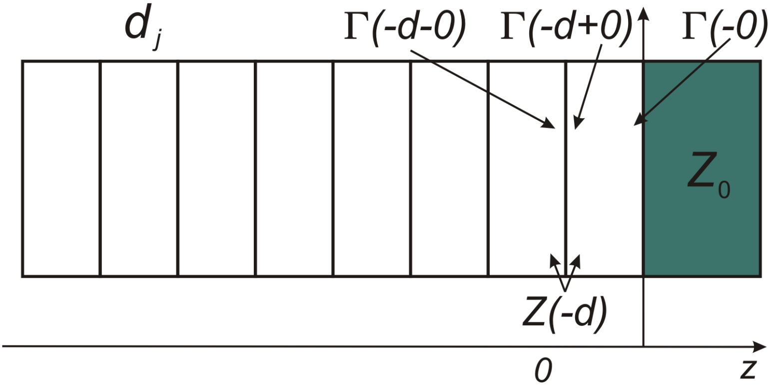
- Thermoelastic noise
- Thermorefractive noise
- Brownian (surface displacement) noise
- Photoelastis (Acoustooptic) noise
- Interference dephasing

Interference dephassing.



$$\delta\varphi = -2k_0 \sum_{j=1}^{N} [A_j n_j - n_e]\delta d_j - B_j \delta n_j$$

Method



- Impedances are continious on dielectric borders
- Reflection coefficient experiences jump on dielectric borders, but continious between borders

Noisy mirror: interferencial part.

- Temperature and mechanical stress influence reflection phase through layer thicknesses and refraction indexes.
- δd_j: $\varphi_i \to \varphi_i - 2k_0\delta n_i d_i - 2k_0 n_i \delta d_i$.
- δn_j: $\eta_i \to \eta_i(1 + \delta\eta_i)$
- Expand all formulas to the first order of δd_j and δn_j:

Mathematical results.

$$\Gamma'_m = \Gamma_m \left(1 + \sum_{j=1}^{m-1} \prod_{k=j+1}^{m} f_k z_{k-1} (i\Delta_j - \mu_j \frac{\delta n_j}{n_j}) - f_m \nu_m \frac{\delta n_m}{n_m} \right)$$

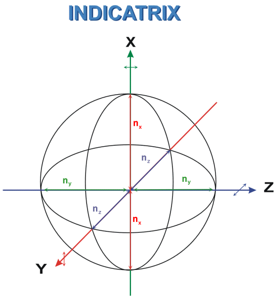
where

$$z_k = \frac{2\Gamma_k e^{i\varphi_k}}{1 - \Gamma_k^2 e^{i2\varphi_k}}; \qquad \nu_k = \frac{Z_{k-1}^2 + \eta_k^2}{Z_{k-1}^2 - \eta_k^2}; \ \nu_1 = 1;$$

$$f_k = \frac{2\eta_k Z_{k-1}}{Z_{k-1}^2 - \eta_k^2}; \qquad \mu_k = \left(f_k \nu_k + \frac{1}{z_k} \right);$$

$$\Delta_j = -2k_0 \delta n_j d_j - 2k_0 n_j \delta d_j; \tag{1}$$

Photoelastisity.



- $\sum B_{ij} x_i x_j = 1$

Optical indicatrix

$$\Delta B_\lambda = p_{\lambda\mu} u_\mu$$

Perpendicular refraction indexes

$$\delta n_x = -\frac{n_0^3}{2} p_{13} \frac{\delta d}{d}$$

$$\delta n_y = -\frac{n_0^3}{2} p_{23} \frac{\delta d}{d}$$

Physical results.

Perturbed reflection coefficient

$$\Gamma'_{N+e} \approx \Gamma_{N+e}\left(1 + \delta\Gamma_{Int}\right) e^{i\delta\varphi_{Int}}$$

Phase perturbation

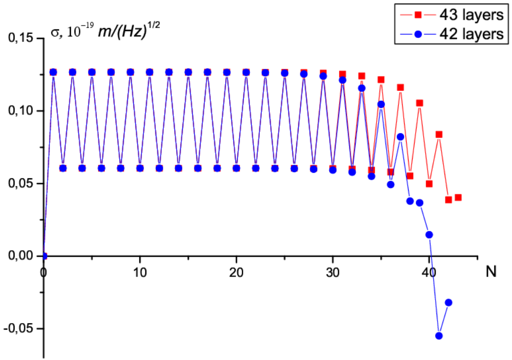
$$\delta\varphi_{Int} = \sum_{j=1}^{N} \Im\left[\alpha_j\right] \delta d_j,$$

Amplitude perturbation

$$\delta\Gamma_{Int} = \sum_{j=1}^{N} \Re\left[\alpha_j\right] \delta d_j,$$

$$\alpha_j = -2k_0 n_j \left(1 - \frac{n_j^2}{2} p_{13}\right) \prod_{k=j+1}^{N+e} f_k z_{k-1}\left(i + \mu_j\right)$$

Noise distribution through layers



- Interferencial part is significant on depth of penetration
- Photoelasticity acts as correction to refraction index.

Spectral density.

- Each layer's δd is independent, e.g.
  $< \delta d_{2j}^2 >= \sigma_{d0}^2, \quad < \delta d_{2j+1}^2 >= \sigma_{d1}^2, \quad < \delta d_j \delta d_k >= 0.$

One layer spectral density.

$$< \delta d_j^2 >= \sigma_{dj}^2 = \xi_j(\omega)\phi_j d_j$$

- where

$$\xi_j = \frac{4\theta}{\omega}\frac{1}{\pi R^2}\frac{E_j^2(1+\sigma_s)^2(1-2\sigma_s)^2 + E_s^2(1+\sigma_j)^2(1-2\sigma_j)}{E_s^2 E_j(1-\sigma_j^2)},$$

Yu. Levin, *Phys.Rev. D* **57**, 659-663 (1998),

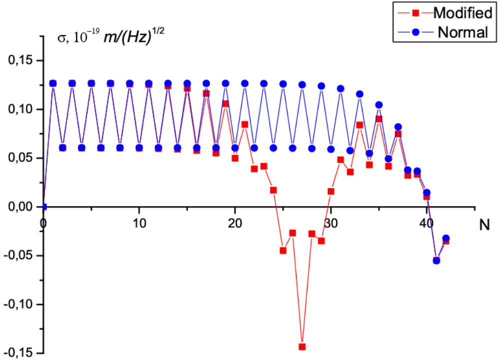
Results

Quarts-tantala mirror noises

Type	42 Layers	"'Standart"'	43 Layers
Transmittance, ppm	2.28	1.08	0.54
Brownian $10^{-20}\mathrm{m}/\sqrt{\mathrm{Hz}}$	6.44	6.46	6.54
With interference	6.21%	6.81%	5.5%
With photoelasticity	7.45%	5.42%	4.28%
Modified cap	7.45%	−−	4.02%

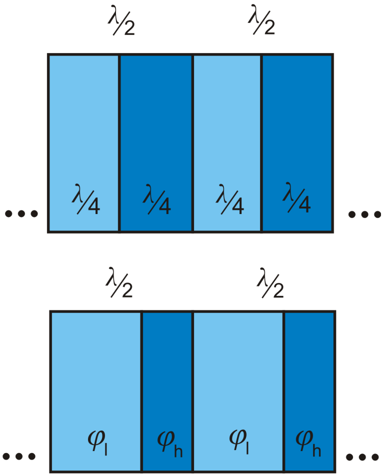
- Values relative in columns
- "'Standart"' here is $\lambda/4 + \lambda/2$cap mirror of 42 layers total.

Layer-corrector inside the mirror.



- H. J. Kimble, Phys. Rev. Lett. **101**, 260602 (2008).
- Noise suppression by 15%, increasing transmittanse.
- Attempt to add layers for restoring transmittance neglects the effect.

Modifying silica-tantala ratio.

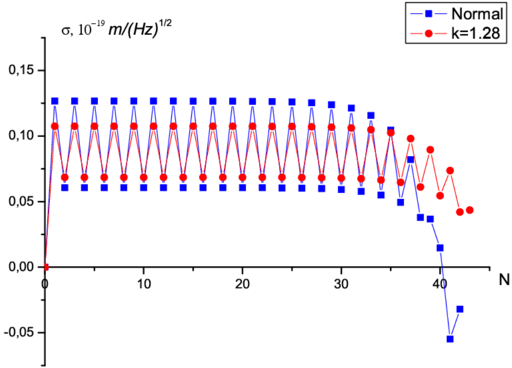


- Tantala is more "‘noisy"’

$$\gamma = \frac{\sigma_{ta}^2}{\sigma_{si}^2} = \frac{\xi(Y_{ta}, \sigma_{ta})\phi_{ta} n_{si}}{\xi(Y_{si}, \sigma_{si})\phi_{si} n_{ta}}$$

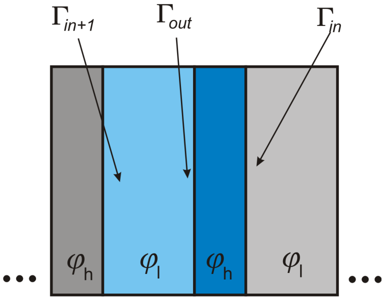
- Reduce tantala thickness, preserving "‘$d_{si} + d_{ta} = \lambda/2$"’ ($\varphi_l + \varphi_h = 2\pi$).
- Increase number of layers preserving transmittance.

Modifying silica-tantala ratio.



- Suppression efficiency is lower 5% for LIGO parameters from GWINC. ($\gamma = 4.6$)
- For $\gamma = 7$ efficiency is 8.6%, which coinsides with experiment of A. V. Villar (*Phys. Rev. D* **81**, 122001).

Optimal coating.



- $\Gamma_{in} = \Gamma_0 e^{i\varphi_0}$
$$\varphi_0 \approx \frac{\pi - \varphi_h}{2} - g^2 \sin(\varphi_h)$$
- $\arg[\Gamma_{in+1}] = \varphi_0$ for max gain
- "'$d_{si} + d_{ta} = \lambda/2$"' is not optimum.

$$\boxed{\varphi_l = 2\pi - \varphi_h - 2\sin(\varphi_h) g^2}$$

$$g = \frac{n_h - n_l}{n_h + n_l}; \quad \varphi = 2k_0 nd$$

Analitical approach.

- Beginning from $N > 3$ $T = \beta(\varphi_h, \varphi_c)\alpha^N(\varphi_h)\epsilon(\varphi_e)$
- First 3 pairs do not work in interferencial part.
- Full brownian noise can be obtained in form

Full Brownian noise.

$$\frac{\sigma^2_{Full}}{A} = \frac{\ln T_0 - \ln\beta - \ln\epsilon}{\ln\alpha}(\gamma\varphi_1 + \varphi_2) + \varphi_c - \varphi_2 +$$
$$+ E(\gamma\varphi_\epsilon + \varphi_{\epsilon 2}) + \varphi_0 - \varphi_{0\epsilon} + \frac{\sigma^2_{Int}}{A}$$

Optimization results.

Quarts-tantala mirror efficiencies

Type	Standart	A. Villar	Our method
Transmittance, ppm	277.6	277.7	276.8
Brownian (displacement)	0%	8.27	8.36%
With interference	11.1%	17.3%	17.4%
With photoelasticity	8.3%	16.2%	16.3
Relative	0	8.6%	8.7%

- Values relative to standart $\lambda/4 + \lambda/2$ cap mirror Brownian (displacement) noise
- $\sigma_{Standart} = 144 \times 10^{-20} \text{m}/\sqrt{\text{Hz}}$ ($\omega = 2700$ Hz)

Conclusion.

- Interference and photoelastic correction is 4 to 7 %
- Amplitude noise is small
- Double layers sould be less than half-wavelength ($\varphi_l + \varphi_h = 2\pi - 2g^2 \sin(\varphi_h)$)
- Optimisation efficiency is 5 to 8.7% depending on γ