

TITLE: - **SPRING STEEL WIRE FLEXURE POINT**  
 - based on calculation / equations used by CAC on 27 JAN 2004, see picture 1  
 WHO: - CIT, NAR, CAC, JHR, MPL  
 LIGO DCC NUMBER: - **D040183 -04** DATE: - 29th April 2004



**SECTION I SUMMARY OF USE WITH THE STAGES IN A CPTYPE QUADRUPLE PENDULUM**

NOTE: - Using the radii supplied by NAR on Mon 09 FEB 2004 The following calculations should be edited for actual radii of wires  
 Mass, (kg)

Top Mass **22**  
 Upper Intermediate Mass **22**  
 Penultimate Mass **38.4** (previously 40kg, see numbers for flexure point that are scored out)  
 Test Mass **39.6** (previously 40kg, see numbers for flexure point that are scored out)

**FLEXURE POINT, all in mm**

<u>A. CALCULATION OF FLEXURE POINT USING THE WIRE RADIUS FROM THE MATLAB MODEL</u>			with masses 38.4 and 39.6 kg
Top Wire	5.50E-04	<del>4.0</del>	5.14
Upper Int Wire	3.50E-04	<del>3.2</del>	3.25
Penultimate Wire	3.10E-04	<del>2.85</del>	2.89
Test Mass Wire	2.20E-04	<del>2.03</del>	2.04 (2.07 with test mass = 38.4 kg)

<u>B. CALCULATION OF THE FLEXURE POINT USING THE ACTUAL WIRE RADIUS. SEE SECTION III.</u>			with masses 38.4 and 39.6 kg
Top Wire	546	<del>5.02</del>	5.07
Upper Int Wire	355	<del>3.3</del>	3.35
Penultimate Wire	317	<del>2.98</del>	3.02
Test Mass Wire	228	<del>2.48</del>	2.19 (2.23 with test mass = 38.4 kg)

**PICTURE (1): - FLEXURE POINT CALCULATION BY CARLOINE CANTLEY**

FLEXURE POINT CALCULATION - ADLIGO RIBBONS 21/1/04

REF: Cayrol et al "Phys Lett A" 272(2000)39-45  
 pendulum swings with effective length  $l$  smaller than real length by amount  $Y\lambda$

$$\lambda = \sqrt{\frac{T}{E_0 I}}$$

$$Y\lambda = \left(\frac{E_0 I}{T}\right)^{1/2}$$

AdLIGO ribbon  $L = 600 \text{ mm}$   
 $w = 1.13 \text{ mm}$   
 $t = 113 \text{ }\mu\text{m}$

silica  $E_0 = 7 \times 10^{10} \text{ N/m}^2$   
 $\rho = 2200 \text{ kg/m}^3$   
 $\sigma = 767 \text{ MPa}$

Final mass = 40 kg sapphire  
 Penultimate mass = 40 kg heavy glass

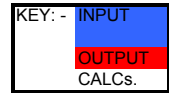
Steel wires (4 off)  $T = \frac{(m_2 + m_3)g}{4}$   
 $= \frac{80 \times 9.81}{4} = 196.2 \text{ N}$

Steel wire parameters from Conceptual DD T010103-03-D  
 $r_2 = 3.5 \times 10^{-4} \text{ m}$   
 (number of wires)  $nw_2 = 4$   
 $Y_2 = 2.2 \times 10^{11} \text{ N/m}^2$

Rectangle  $I = \frac{wt^3}{12}$   
 RIBBON  $\frac{Y\lambda}{\lambda} = \left[ \frac{7 \times 10^{10} \times 1.13 \times 10^{-3} \times (113 \times 10^{-6})^3}{12 \times 196.2} \right]^{1/2}$   
 $= 0.311 \text{ mm}$

Circle  $I = \frac{\pi r^4}{4}$   
 WIRE  $\frac{Y\lambda}{\lambda} = \left[ \frac{2.2 \times 10^{11} \times \pi \times (3.5 \times 10^{-4})^4}{4 \times 196.2} \right]^{1/2}$   
 $= 3.635 \text{ mm}$

**SECTION II USING THE CALCULATOR**



**1. QUAD TOP WIRE 22, 22, 40, 40 kg**

radius	5.46E-04 m		0.55 mm		
diameter	1.09E-03 m		1.09 mm		
mass	122 kg	Tension, T	598.41 kg*m/s^2		
no of wires	2	arera moment	6.98008E-14		
Youngs Modulus	2.20E+11 Pa	lamda =	0.005065731 m	<b>Flexure Point =</b>	<b>5.07 mm</b>

**2. QUAD CP UPPER INT WIRE 22, 22, 40, 40 kg**

radius	3.55E-04 m		0.36 mm		
diameter	7.10E-04 m		0.71 mm		
mass	100 kg	Tension, T	245.25		
no of wires	4	arera moment	1.24739E-14		
Youngs Modulus	2.20E+11	lamda	0.003345095 m	<b>Flexure Point =</b>	<b>3.35 mm</b>

**3. QUAD CP PENULTIMATE WIRE 22, 22, 40, 40 kg**

radius	3.17E-04 m		0.32 mm		
diameter	6.34E-04 m		0.63 mm		
mass	78 kg	Tension, T	191.295		
no of wires	4	arera moment	7.93098E-15		
Youngs Modulus	2.20E+11	lamda	0.003020112 m	<b>Flexure Point =</b>	<b>3.02 mm</b>

**4. QUAD CP FINAL WIRE 22, 22, 40, 40 kg**

radius	2.28E-04 m		0.23 mm		
diameter	4.56E-04 m		0.46 mm		
mass	39.6 kg	Tension, T	97.119		
no of wires	4	arera moment	2.12241E-15		
Youngs Modulus	2.20E+11 *	lamda	0.00219 m	<b>Flexure Point =</b>	<b>2.19 mm</b>

\* check?

**SECTION III: - ACTUAL DIAMETERS OF WIRE**

MUSIC WIRE AVAILABLE FROM CALIFORNIA FINE WIRE COMPANY

NOTE: - <http://www.calfinewire.com/>  
 GEO 600 wire bought from <http://www.knight-group.co.uk/>

	DIAMETER of Wire ENGLISH (thou)	RADIUS of Wire METRIC (microns)	WIRE RADIUS FROM NAR MODEL
6thou diam	6	76.2	
8thou diam	8	101.6	
10thou diam	10	127	
	12	152.4	
	14	177.8	
	16	203.2	
	17	215.9	
	18	228.6	220
	20	254	
	22	279.4	
	24	304.8	
	25	317.5	310
	26	330.2	
	27	342.9	
	28	355.6	350
	29	368.3	
	30	381	
	31	393.7	
	32	406.4	
	33	419.1	
	34	431.8	
	37	469.9	
	38	482.6	
	39	495.3	
	40	508	
	41	520.7	
	42	533.4	
	43	546.1	540

**SECTION IV: - OTHER EXAMPLES** FOR INTEREST ONLY

**5 MODE CLEANER INTERMEDIATE WIRE, 3, 3, 3 kg**

radius	1.00E-04 m		0.10 mm		
diameter	2.00E-04 m		0.20 mm		
mass	6 kg				
no of wires	4				
t	14.715				
Youngs Modulus	2.20E+11	*	lamda	0.001084 m	
arera moment	7.85398E-17		Flexure Point =		1.08 mm
				NOT INCLUDED!	

**6 MODE CLEANER UPPER WIRE, 3, 3, 3 kg**

radius	1.80E-04 m		0.18 mm		
diameter	3.60E-04 m		0.36 mm		
mass	9 kg				
no of wires	2				
t	44.145				
Youngs Modulus	2.20E+11	*	lamda	0.002027 m	
arera moment	8.2448E-16		Flexure Point =		2.03 mm
				NOT INCLUDED!	

**7 GEO 600. UPPER WIRE 4.5, 4.5, 4.5 kg**

radius	2.50E-04 m		0.25 mm		
diameter	5.00E-04 m		0.50 mm		
mass	18 kg				
no of wires	2				
t	88.29				
Youngs Modulus	2.20E+11	*	lamda	0.002765 m	
arera moment	3.06796E-15		Flexure Point =		2.76 mm
				NOT INCLUDED!	

**8 MC LOWER WIRE**

diameter	1.50E-04				
mass	3		lamda	0.000862013 m	
t	7.36E+00			0.86 mm	
area moment	2.48505E-17				

THE FOLLOWING ASSUMPTION NEED TO BE CHECKED!

**SECTION V: - CONCLUSIONS / PARAMETERS FOR CP Type QUAD**

**1 FLEXURE POINTS**

Top wire at Top blades	5 mm
Top wire at Top mass	5 mm
Upper Intermediater wire at top mass	3.3 mm
Upper intermediate wire at upper intermediate mass	3.3 mm
Penultimate wiree at upper intermediate mass	3 mm
Penultimate wire at penultime mass	3 mm
Test mass wire at Penultimate mass	2 mm
Test mass wire at test mass	2 mm

NB: FLEXURE POINT CHANGE FROM 39.6 kg to 38.4 kg for reaction chain final stage is small and therefore design left at 2mm

**2 MASS**

Top mass	22 kg
Upper Intermediate Mas	22 kg
Penultimate Mass	38.4 kg
Test Mass	39.6 kg

**3 WIRE**

Top Wire	18 thou diameter
Upper Intermediate Wire	25 thou diameter
Penultimate Wire	28 thou diameter
Final Wire	43 thou diameter

TITLE Pitch adjustment of the ALIGO - MC test mass by adjustment carried out at the top mass  
 NAMES Calum I. Torrie Norma Robertson, Mike Perreur Lloyd, Alastair Grant  
 DATE June 19th 2004  
 LIGO-DCC D040183-04 page 2  
 OFFSET Cylinder ~ 1 inch diameter 50mm long with or without a set screw insert, LIGO-D020137 50 mm (long)  
 DISTANCE Experiment carried out at 10ft from the optic, distance = 3.048 m (10 ft)  
 MASS OFFSETS

Number	Name	mass of offset	mass of combination
1 # 7/8-9	SET SCREW	120 g	256g
2 #1/2-20	SET SCREW	40 g	266g
3 #1/2-13	SET SCREW	39 g	270g
4 -	SOLID ROD	270 g	270g

NOTE: - 1.3 with original d's  
 0.87 with new d's including flexure points

\*\* EMAIL FROM NAR on 17 JUNE 2004  
 Xfer fn at DC: Test mass pitch/torque applied at top mass = 0.87 rad per Nm  
 Torque needed to pitch test mass 2.5mrad, torque = 0.002874 Nm  
 2.5 mrad  
 gravity = 9.81 m/s^2

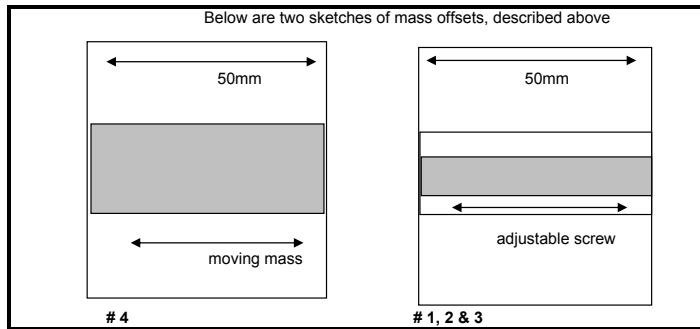
Sketch explaining detail wrt leverarm and mass from email by NAR

Compare this to 8gms at lever arm of 25 mm ( corresponds to moving previously centred screw 50 mm long by 10 mm, so that at one end we gain 10 mm of rod at lever arm centred at 30mm and at other end we "lose" 10mm of rod centred at 20 mm - thus equiv to gaining 20mm of rod, or 8 gms, at an average lever arm of 25mm)

This gives torque of  $0.008 \times 9.8 \times 0.025 = 1.96e-3$  Nm

FROM NAR EMAIL

movement of offset	2x vertical distance, Z	Vertical distance, Z	alpha	equiv moving mass	average leverarm	Torque	Theory Torque
mm	mm	mm	mrad	g	mm	Nm	Nm
<b>1 #7/8-9 120 g</b>							
20	95	47.5	15.6	48	25	1.18E-02	1.79E-02
10	45	22.5	7.4	24	25	5.89E-03	8.48E-03
0	0	0	0.0	-	-	-	-
-10	-45	-22.5	-7.4	24	25	-5.89E-03	-8.48E-03
-20	-85	-42.5	-13.9	48	25	-1.18E-02	-1.60E-02
<b>2 # 1/2-20 40 g</b>							
20	30	15	4.9	16	25	3.92E-03	5.66E-03
10	15	7.5	2.5	8	25	1.96E-03	2.83E-03
0	0	0	0.0	-	-	-	-
-10	-15	-7.5	-2.5	8	25	-1.96E-03	-2.83E-03
-20	-30	-15	-4.9	16	25	-3.92E-03	-5.66E-03
<b>3 #1/2-13 39 g</b>							
20	30	15	4.9	15.6	25	3.83E-03	5.66E-03
10	15	7.5	2.5	7.8	25	1.91E-03	2.83E-03
0	0	0	0.0	-	-	-	-
-10	-15	-7.5	-2.5	7.8	25	-1.91E-03	-2.83E-03
-20	-30	-15	-4.9	15.6	25	-3.83E-03	-5.66E-03
<b>4 SOLID ROD 270 g</b>							
10	100	50	16.4	54	25	1.32E-02	1.89E-02
5	50	25	8.2	27	25	6.62E-03	9.43E-03
0	0	0	0.0	-	-	-	-
-5	-45	-22.5	-7.4	27	25	-6.62E-03	-9.43E-03
-10	-100	-50	-16.4	54	25	-1.32E-02	-1.89E-02



MC FELXURE POINTS TOP 2mm  
 June 19th 2004, CIT MIDDLE 1mm  
 BOTTOM 0.8mm \* can we do without?

EMAIL: - AG, NAR, MPL, CIT  
 NOTES  
 1 QUAD on hold for now  
 2 TIE up experiment with NAR (MATLAB) and AG EXCEL CALCULATOR  
 - use email from NAR  
 3 # from NAR for QUAD

CIT

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