TITLE: -	**SPRING STEEL WIRE FLEXURE POINT**		
	- based on calculation / equations used by CAC on 27 JAN 2004, see picture 1		
WHO: -	CIT, NAR, CAC, JHR, MPL		
LIGO DCC NUMBER: -	**D040183** **-03**	DATE: -	29th April 2004

SECTION I **SUMMARY OF USE WITH THE STAGES IN A CPTYPE QUADRUPLE PENDULUM**

NOTE: - Using the radii supplied by NAR on Mon 09 FEB 2004 — The following calculations should be edited for actual radii of wires

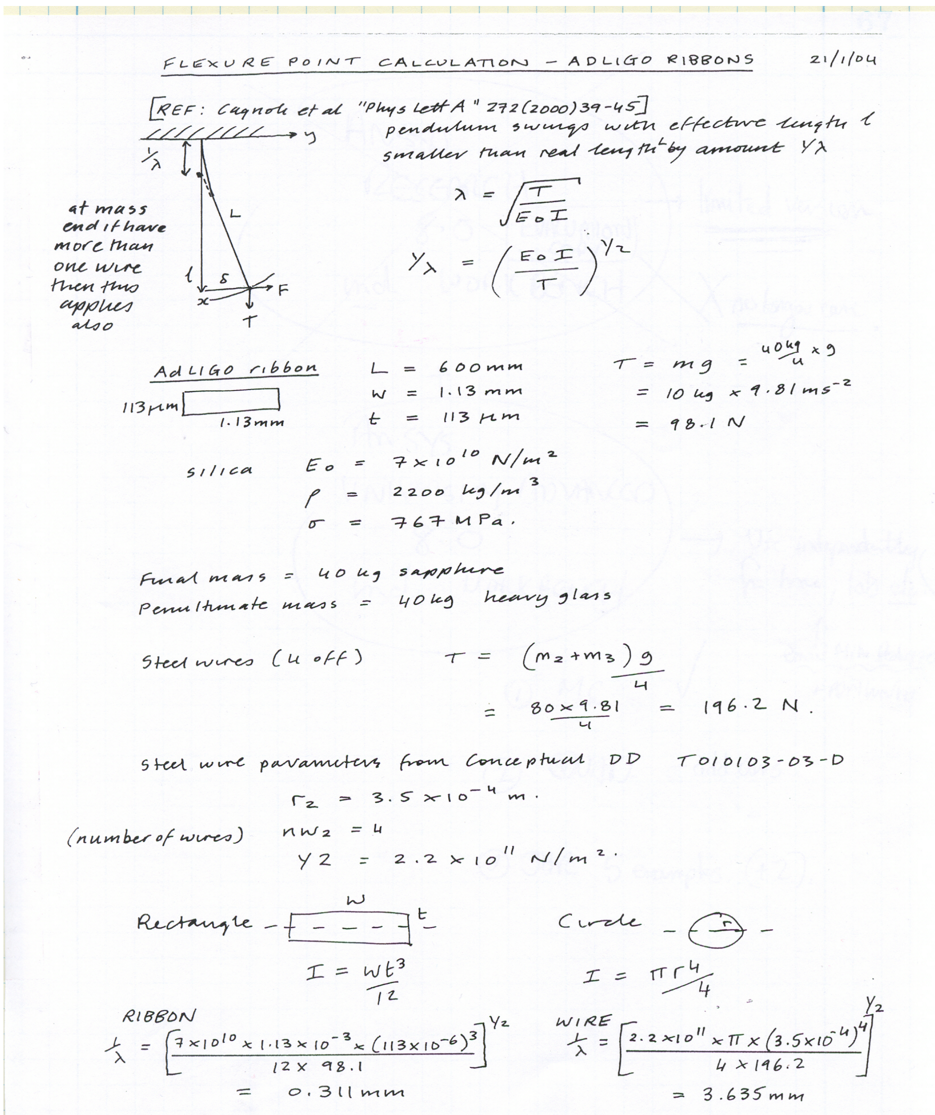
	Mass, (kg)	
Top Mass	**22**	
Upper Intermediate Mass	**22**	
Penultimate Mass	**38.4**	(previously 40kg, see numbers for flexure point that are scored out)
Test Mass	**39.6**	(previously 40kg, see numbers for flexure point that are scored out)

FLEXURE POINT, all in mm

A. CALCULATION OF FLEXURE POINT USING THE WIRE RADIUS FROM THE MATLAB MODEL			with masses 38.4 and 39.6 kg	
Top Wire	5.50E-04	~~4.9~~	5.14	
Upper Int Wire	3.50E-04	~~3.2~~	3.25	
Penultimate Wire	3.10E-04	~~2.85~~	2.89	
Test Mass Wire	2.20E-04	~~2.03~~	2.04	(2.07 with test mass = 38.4 kg)

B. CALCULATION OF THE FLEXURE POINT USING THE ACTUAL WIRE RADIUS, SEE SECTION III.			with masses 38.4 and 39.6 kg	
Top Wire	546	~~5.02~~	5.07	
Upper Int Wire	355	~~3.3~~	3.35	
Penultimate Wire	317	~~2.98~~	3.02	
Test Mass Wire	228	~~2.18~~	2.19	(2.23 with test mass = 38.4 kg)

PICTURE (1): - FLEXURE POINT CALCULATION BY CARLOINE CANTLEY



FLEXURE POINT CALCULATION – ADLIGO RIBBONS 21/1/04

[REF: Cagnoli et al "Phys Lett A" 272 (2000) 39-45]

pendulum swings with effective length l smaller than real length L by amount $1/\lambda$

at mass end if have more than one wire then this applies also

$$\lambda = \sqrt{\frac{T}{E_0 I}}$$

$$1/\lambda = \left(\frac{E_0 I}{T}\right)^{1/2}$$

AdLIGO ribbon: 113 μm × 1.13 mm

L = 600 mm
w = 1.13 mm
t = 113 μm

$T = mg = \frac{40\,kg}{4} \times g = 10\,kg \times 9.81\,ms^{-2} = 98.1\,N$

silica
$E_0 = 7\times10^{10}\,N/m^2$
$\rho = 2200\,kg/m^3$
$\sigma = 767\,MPa$.

Final mass = 40 kg sapphire
Penultimate mass = 40 kg heavy glass

Steel wires (4 off) $T = \frac{(m_2+m_3)g}{4} = \frac{80\times9.81}{4} = 196.2\,N$.

steel wire parameters from Conceptual DD T010103-03-D

$r_2 = 3.5\times10^{-4}\,m$.
(number of wires) $nw_2 = 4$
$Y2 = 2.2\times10^{11}\,N/m^2$.

Rectangle – w, t — $I = \frac{wt^3}{12}$

Circle – r — $I = \frac{\pi r^4}{4}$

RIBBON
$$\frac{1}{\lambda} = \left[\frac{7\times10^{10}\times1.13\times10^{-3}\times(113\times10^{-6})^3}{12\times98.1}\right]^{1/2} = 0.311\,mm$$

WIRE
$$\frac{1}{\lambda} = \left[\frac{2.2\times10^{11}\times\pi\times(3.5\times10^{-4})^4}{4\times196.2}\right]^{1/2} = 3.635\,mm$$

KEY: -	INPUT
	OUTPUT
	CALCs.

SECTION II **USING THE CALCULATOR**

1. QUAD TOP WIRE 22, 22, 40, 40 kg

radius	5.46E-04 m			0.55 mm		
diameter	1.09E-03 m			1.09 mm		
mass	122 kg	Tension, T		598.41 kg*m/s^2		
no of wires	2	arera moment		6.98008E-14		
Youngs Modulus	2.20E+11 Pa	lamda =		0.005065731 m	Flexure Point =	5.07 mm

2. QUAD CP UPPER INT WIRE 22, 22, 40, 40 kg

radius	3.55E-04 m		0.36 mm		
diameter	7.10E-04 m		0.71 mm		
mass	100 kg	Tension, T	245.25		
no of wires	4	arera moment	1.24739E-14		
Youngs Modulus	2.20E+11	lamda	0.003345095 m	Flexure Point =	3.35 mm

3. QUAD CP PENULTIMATE WIRE 22, 22, 40, 40 kg

radius	3.17E-04 m		0.32 mm		
diameter	6.34E-04 m		0.63 mm		
mass	78 kg	Tension, T	191.295		
no of wires	4	arera moment	7.93098E-15		
Youngs Modulus	2.20E+11	lamda	0.003020112 m	Flexure Point =	3.02 mm

4. QUAD CP FINAL WIRE 22, 22, 40, 40 kg

radius	2.28E-04 m			0.23 mm		
diameter	4.56E-04 m			0.46 mm		
mass	39.6 kg		Tension, T	97.119		
no of wires	4		arera moment	2.12241E-15		
Youngs Modulus	2.20E+11	*	lamda	0.00219 m	Flexure Point =	2.19 mm

* check?

SECTION III: - **ACTUAL DIAMETERS OF WIRE**

MUSIC WIRE AVALIABLE FROM CALIFORNIA FINE WIRE COMPANY
http://www.calfinewire.com/
NOTE: - GEO 600 wire bought from
http://www.knight-group.co.uk/

	DIAMETER of Wire ENGLISH (thou)	RADIUS of Wire METRIC (microns)		WIRE RADIUS FROM NAR MODEL
6thou diam	6	76.2		
8thou diam	8	101.6		
10thou diam	10	127		
	12	152.4		
	14	177.8		
	16	203.2		
	17	215.9		
	18	228.6		220
	20	254		
	22	279.4		
	24	304.8		
	25	317.5		310
	26	330.2		
	27	342.9		
	28	355.6		350
	29	368.3		
	30	381		
	31	393.7		
	32	406.4		
	33	419.1		
	34	431.8		
	37	469.9		
	38	482.6		
	39	495.3		
	40	508		
	41	520.7		
	42	533.4		
	43	546.1		540

SECTION IV: - **OTHER EXAMPLES** FOR INTEREST ONLY

5 MODE CLEANER INTERMEDIATE WIRE, 3, 3, 3 kg

radius	**1.00E-04** m		0.10 mm			
diameter	**2.00E-04** m		0.20 mm			
mass	**6** kg					
no of wires	**4**					
t	14.715					
Youngs Modulus	**2.20E+11**	*	lamda	0.001084 m		
arera moment	7.85398E-17		Flexure Point =			**1.08** mm
					NOT INCLUDED!	

6 MODE CLEANER UPPER WIRE, 3, 3, 3 kg

radius	**1.80E-04** m		0.18 mm			
diameter	**3.60E-04** m		0.36 mm			
mass	**9** kg					
no of wires	**2**					
t	44.145					
Youngs Modulus	**2.20E+11**	*	lamda	0.002027 m		
arera moment	8.2448E-16		Flexure Point =			**2.03** mm
					NOT INCLUDED!	

7 GEO 600, UPPER WIRE 4.5, 4.5, 4.5 kg

radius	**2.50E-04** m		0.25 mm			
diameter	**5.00E-04** m		0.50 mm			
mass	**18** kg					
no of wires	**2**					
t	88.29					
Youngs Modulus	**2.20E+11**	*	lamda	0.002765 m		
arera moment	3.06796E-15		Flexure Point =			**2.76** mm
					NOT INCLUDED!	

THE FOLLOWING ASSUMPTION NEED TO BE CHECKED!

SECTION V: - **CONCLUSIONS / PARAMETERS FOR CP Type QUAD**

1 FLEXURE POINTS

Top wire at Top blades	5 mm
Top wire at Top mass	5 mm
Upper Intermediater wire at top mass	3.3 mm
Upper intermediate wire at upper intermediate mass	3.3 mm
Penultimate wiree at upper intermediate mass	3 mm
Penultimate wire at penultime mass	3 mm
Test mass wire at Penultimate mass	2 mm
Test mass wire at test mass	2 mm

NB: FLEXURE POINT CHANGE FROM 39.6 kg to 38.4 kg for reaction chain final stage is small and therefore design left at 2mm

2 MASS

Top mass	22 kg
Upper Intermediate Mas	22 kg
Penultimate Mass	38.4 kg
Test Mass	39.6 kg

3 WIRE

Top Wire	18 thou diameter
Upper Intermediate Wire	25 thou diameter
Penultimate Wire	28 thou diameter
Final Wire	43 thou diameter