The Search for Gravitational Waves

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REU Lecture
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Outline

1. What are Gravitational Waves?
    - Motivation: Gravity + Relativity
    - General Relativity
    - Gravitational Waves
2. Gravitational Waves Searches w/LIGO & Virgo
    - Observations
    - Data Analysis

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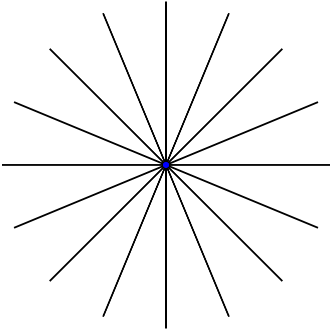
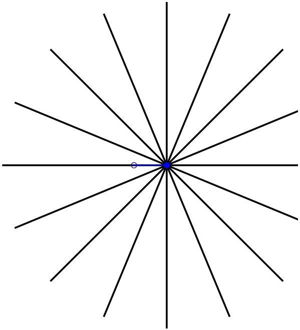
Action at a Distance

- Newtonian gravity: mass generates gravitational field
- Lines of force point towards object



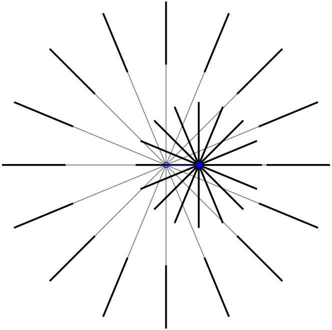
Issues with Causality

- Move object; Newton says: lines point to new location
- Relativity says:
  can't communicate
  faster than light
  to avoid paradoxes
- You could send me supraluminal messages via grav field



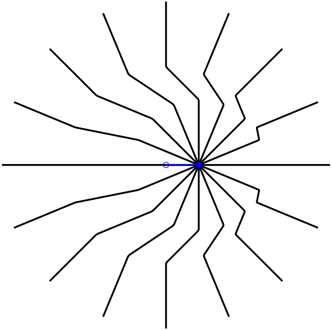
Gravitational Speed Limit

- If I'm 10 light years away,
  I can't know you moved
  the object 6 years ago
- Far away, gravitational field lines
  have to point to
  old location of the object



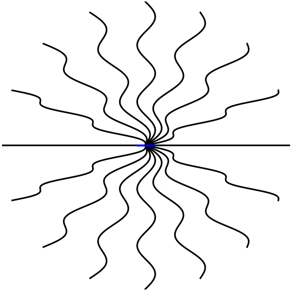
Gravitational Shock Wave

- Sudden motion (acceleration) of object generates gravitational shock wave expanding at speed of light



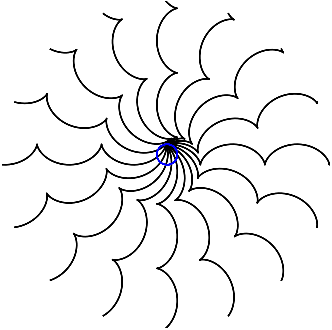
Ripples in the Gravitational Field

- Move object back & forth
  ⟶ gravitational wave
- Same argument applies to electricity:
  - can derive magnetism as relativistic effect
  - accelerating charges generate electromagnetic waves propagating @ speed of light



Gravitational Wave from Orbiting Mass?

- Move around in a circle
- Still get grav wave pattern, but looks a bit funny
- Time to move beyond simple pseudo-Newtonian picture



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The Equivalence Principle

- Funny thing about (Newtonian) gravitational forces: always proportional to an object's mass, something in a gravitational field undergoes the same acceleration, no matter what it is
- Fictitious forces (e.g., centrifugal force) in non-inertial (accelerating, rotating, etc) reference frames behave the same way
- In Einstein's general relativity, gravity is something like a fictitious force which only manifests itself because the reference frame is non-inertial
- The catch: **NO** (globally) inertial reference frames!

A Thought Experiment

- In a freely falling elevator: Can you tell you’re not in space?
- You, the elevator, and anything you drop are accelerating downwards at $9.8\,\mathrm{m}/\mathrm{s}^2 \longrightarrow$ no relative acceleration

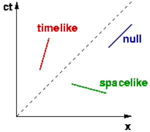
A Thought Experiment

- In a freely falling elevator: Can you tell you're not in space?
- You, the elevator, and anything you drop are accelerating downwards at $9.8\,\mathrm{m/s^2} \longrightarrow$ no relative acceleration
- Actually, you can tell if the elevator is big enough:
    - Top of elevator farther from Earth → grav field weaker → stuff accelerates less $\Longrightarrow$ accelerates up in elevator frame
    - Bottom of elevator closer to Earth → grav field stronger → stuff accelerates more $\Longrightarrow$ down in elevator frame
    - stuff @ sides accel inward bc lines to ctr of $\oplus$ converge
- This relative acceleration is measurable manifestation of gravity: **tidal force**

Spacetime Geometry

- Recall in special relativity, speed of light c same for all inertial observers
- Given pair of events, different observers measure different Δx, Δy, Δz & even Δt, but all agree on

$$(\Delta s)^2 = -c^2(\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$$



- If $(\Delta s)^2 = 0$, have lightlike or null-sep events
- If $(\Delta s)^2 > 0$, have spacelike-separated events
- If $(\Delta s)^2 < 0$, have timelike-separated events

Notational Simplifications

- Work in units where $c = 1$ (defines what we mean by measuring time in meters and distance in (light-)seconds)
- Four-vector $\{x^\alpha\} = \{x^0, x^1, x^2, x^3\} = \{t, x, y, z\}$
- Einstein summation convention: implied sum over repeated indices so for example $g_{\alpha\beta} V^\alpha V^\beta$ means $\sum_{\alpha=0}^{3} \sum_{\beta=0}^{3} g_{\alpha\beta} V^\alpha V^\beta$ & $g_{ij} V^i V^j$ means $\sum_{i=1}^{3} \sum_{j=1}^{3} g_{ij} V^i V^j$
- So $(\Delta s)^2 = \eta_{\alpha\beta} \Delta x^\alpha \Delta x^\beta$ where $\{\eta_{\alpha\beta}\} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

General Relativity in a Nutshell

- In GR, talk about infinitesimal separations $\Delta \rightarrow d$
- Geometry described by

$$(ds)^2 = g_{\alpha\beta} dx^\alpha dx^\beta$$

  $g_{\alpha\beta}(\{x^\gamma\})$ in general is not the flat Minkowski metric $\eta_{\alpha\beta}$
- You can always choose coördinates so that **at one point** $g_{\alpha\beta} = 0$ & $\frac{\partial g_{\alpha\beta}}{\partial x^\gamma} = 0$ (equivalence principle)
- Cannot get rid of $\frac{\partial^2 g_{\alpha\beta}}{\partial x^\gamma \partial x^\delta}$, even at a point (tidal effects)
- Einstein's equations describe how $\frac{\partial^2 g_{\alpha\beta}}{\partial x^\gamma \partial x^\delta}$ determined by density of matter and energy

Outline

Gravitational Wave as Metric Perturbation

- Full GR complicated (choice of coörds, global struct, etc)
- Far from source, much simpler:
  - $\approx$ a plane wave
  - GW $h_{\alpha\beta}$ is a small perturbation on top of flat metric $\eta_{\alpha\beta}$
    $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$
  - Can choose coörds to leave only two polarization states;
    E.g. Plane wave propagating in z direction

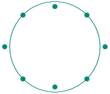
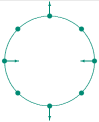
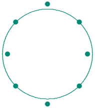
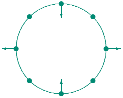
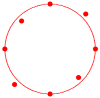
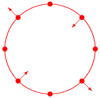
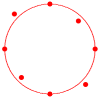
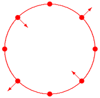
$$\{h_{\alpha\beta}\} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{i2\pi f(z/c-t)}$$

h_+ and $h_\times$ are amplitudes of “plus” and “cross” pol states.

$$\overleftrightarrow{h} = \left[h_+ \overleftrightarrow{e}_+ + h_\times \overleftrightarrow{e}_\times\right] e^{i2\pi f(\hat{k}\cdot\vec{r}/c-t)}$$

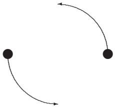
Effects of Gravitational Wave

Fluctuating geom changes distances btwn particles in free-fall:

Plus (+) Polarization				
Cross (×) Polarization				

Gravitational Wave Generation

- Generated by moving/oscillating mass distribution
- Classic example: orbiting binary system



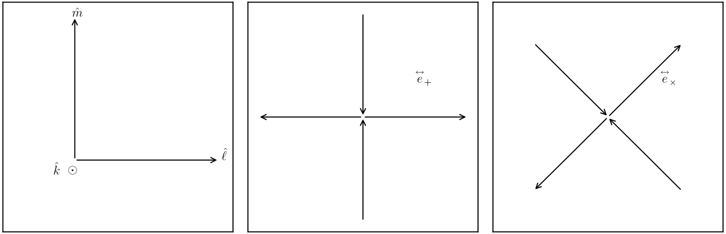
(e.g., Binary Pulsar 1913+16
– Observed energy loss agrees w/GW prediction)

The Polarization Basis

- wave propagating along $\hat{k}$;
  construct $\overleftrightarrow{e}_{+,\times}$ from $\perp$ unit vectors $\hat{\ell}$ & $\hat{m}$:

$$\overleftrightarrow{e}_{+} = \hat{\ell}\otimes\hat{\ell} - \hat{m}\otimes\hat{m} \qquad \overleftrightarrow{e}_{\times} = \hat{\ell}\otimes\hat{m} + \hat{m}\otimes\hat{\ell}$$

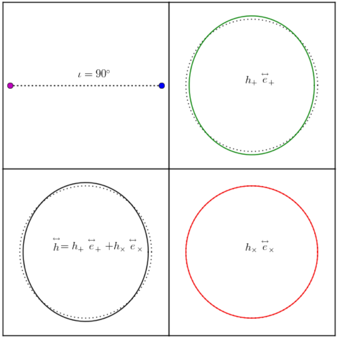
- arbitrary choice of $\hat{\ell}$ within plane $\perp$ $\hat{k}$ (fixes $\hat{m} = \hat{k}\times\hat{\ell}$)
  Free to choose polarization basis convenient to situation

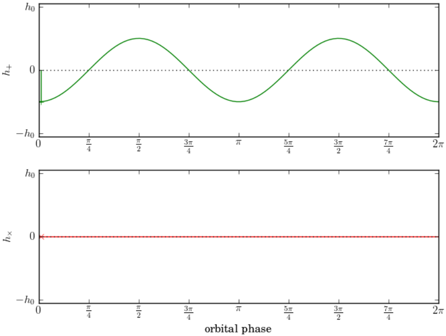


Example: Linear polarization

- Consider binary system seen edge on:
  masses seen going back & forth in one direction; call that $\hat{\ell}$
- In that pol basis, $h_\times = 0$ and only h_+ **linear polarization**

$$h_+ = A \cos \Phi(t) \qquad h_\times = 0$$

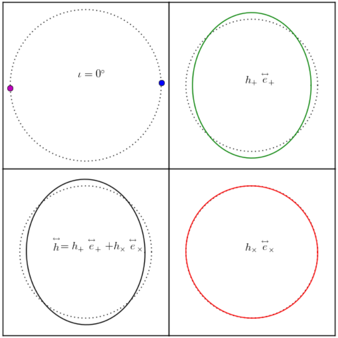


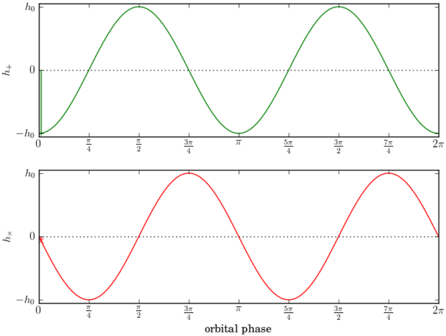


Example: Circular polarization

- Consider binary seen face on: masses seen going in circle
- In any pol basis, h_+ & $h_\times$ have same amp; out of phase **circular polarization**

$$h_+ = A\cos\Phi(t) \qquad h_\times = A\sin\Phi(t)$$

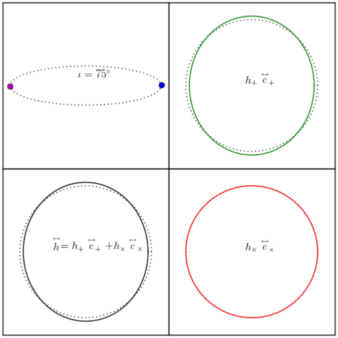


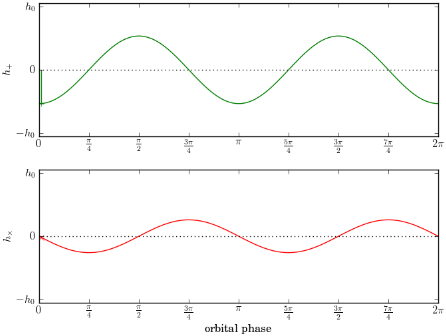


Example: Elliptical polarization

- General case: binary system seen at an angle: masses seen going around an ellipse; long axis of that ellipse picks preferred direction $\hat{\ell}$ for pol basis
- In that pol basis, h_+ & $h_\times$ out of phase; h_+ has greater amp **elliptical polarization** $[|A_+| > |A_\times|]$

$$h_+ = A_+ \cos\Phi(t) \qquad h_\times = A_\times \sin\Phi(t)$$





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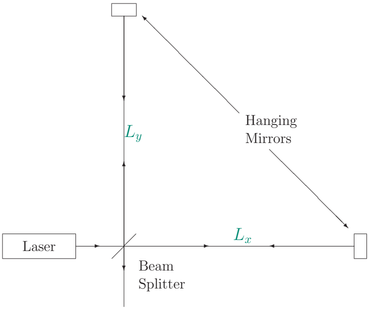
Measuring GWs w/Laser Interferometry

Interferometry: Measure GW-induced distance changes



- Measure small change in

$$L_x - L_y = \sqrt{g_{11} L_0^2} - \sqrt{g_{22} L_0^2}$$
$$= \sqrt{(1+h_{11}) L_0^2} - \sqrt{(1+h_{22}) L_0^2}$$
$$\approx L_0 \frac{h_{11} - h_{22}}{2} \sim L_0 h_+$$

- More gen,

$$(L_1 - L_2)/L_0 = \overleftrightarrow{h} : \overleftrightarrow{d}$$

with "response tensor"

$$\overleftrightarrow{d} = \frac{\hat{n}_1 \otimes \hat{n}_1 - \hat{n}_2 \otimes \hat{n}_2}{2}$$

(also when $\hat{n}_1$ & $\hat{n}_2$ not $\perp$)

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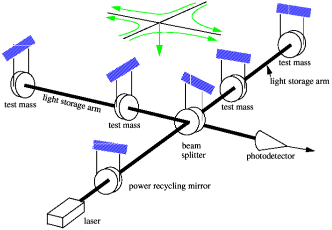

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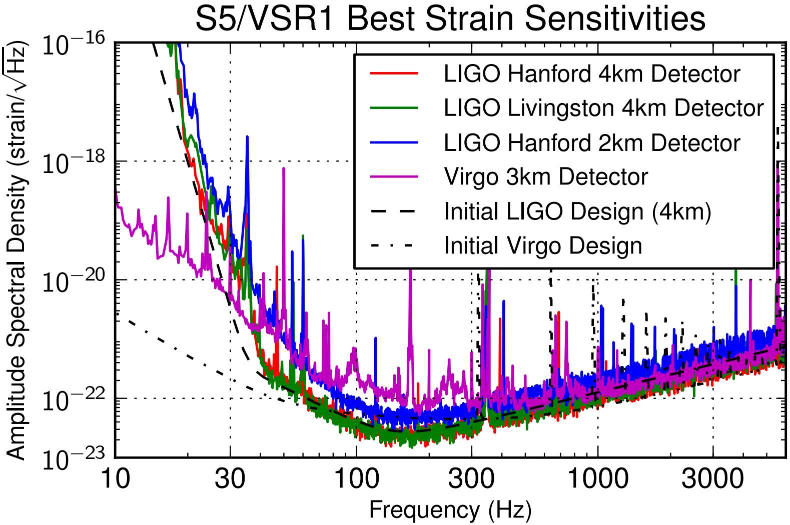
Rogues’ Gallery of Ground-Based Interferometers

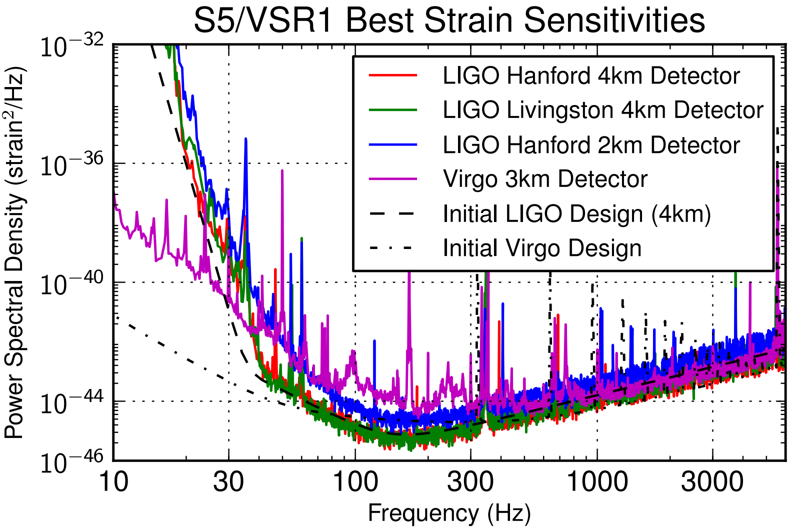
LIGO Hanford (Wash.)

LIGO Livingston (La.)

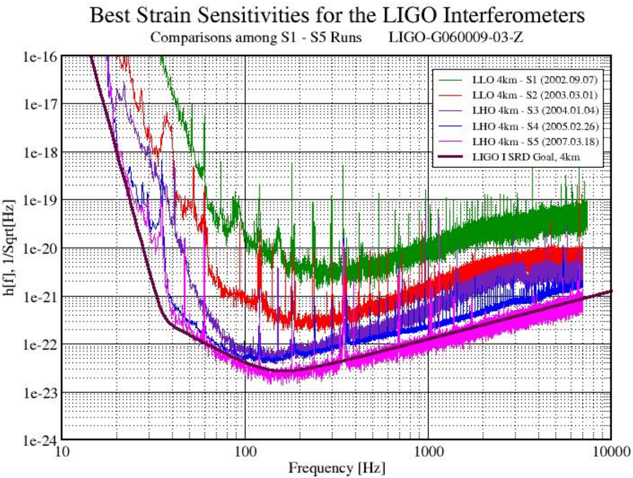
GEO-600 (Germany)

Virgo (Italy)


S5/VSR1 Best Strain Sensitivities
Amplitude Spectral Density (strain/√Hz)
10^{-16}
10^{-18}
10^{-20}
10^{-22}
10^{-23}
10
30
100
300
1000
3000
Frequency (Hz)
LIGO Hanford 4km Detector
LIGO Livingston 4km Detector
LIGO Hanford 2km Detector
Virgo 3km Detector
Initial LIGO Design (4km)
Initial Virgo Design


S5/VSR1 Best Strain Sensitivities
Power Spectral Density (strain²/Hz)
10⁻³²
10⁻³⁶
10⁻⁴⁰
10⁻⁴⁴
10⁻⁴⁶
10
30
100
300
1000
3000
Frequency (Hz)
LIGO Hanford 4km Detector
LIGO Livingston 4km Detector
LIGO Hanford 2km Detector
Virgo 3km Detector
Initial LIGO Design (4km)
Initial Virgo Design

Evolution of LIGO Sensitivity S1-S5



GW Observatory Network

- LSC detectors conducting science runs since 2002
  - LIGO Hanford (4km H1 & 2km H2)
  - LIGO Livingston (4km L1)
  - GEO-600 (600m G1)
- Virgo (3km V1) started science runs in 2007
- Recent long runs:
  - LIGO/GEO S5: Nov 2005-Sep 2007: LIGO @ design sens
  - Virgo VSR1: May-Sep 2007: Begin joint LSC-Virgo analysis
- Current/Ongoing joint runs:
  - LIGO (H1 & L1) S6: Jul 2009-Oct 2010
  - Virgo VSR2 Jul 2009-Jan 2010 & VSR3 about to start
- LIGO & Virgo will go offline in 2010/2011
  to begin upgrade to Advanced Detectors

Outline

Classification of GW Signals

In LIGO/Virgo band (10s-1000s of Hz),
natural division of sources:

	modelled	unmodelled
long	Periodic Sources (e.g., Rotating Neutron Star)	Stochastic Background (Cosmological or Astrophysical)
short	Binary Coalescence (Black Holes and/or Neutron Stars)	Bursts (Supernova, messy merger, etc.)

Summary

- Relativistic causality implies gravitational waves
- General Relativity describes gravity as geometry
- Far from source, GWs are plane waves w/2 pol states
- GW detectors measure fluctuations in distances