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Note on Choice of Lowest Stage Wire Radius for Quad Reaction Chain

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## 1 Introduction

The purpose of this document is to capture the final design choice for the wire radius for the final stage of the reaction chains for the ETM and ITM suspensions.

## 2 Background

The choice of wire radii for Advanced LIGO suspensions has in general been made by using a nominal value for tensile strength for music wire of 2000 MPa and using a safety factor of  $\sim 3$ . We note that the minimum tensile strength as per ASTM A-228M-07 is  $\sim 2000$  MPa for wire of diameter  $\sim 1.5$  mm, radius 0.75 mm, and gradually increases for smaller diameters (see T0900502). The range of wire radii for a quad is from  $\sim 0.5$  mm at the top to  $\sim 0.23$  mm at the bottom and for these values the minimum tensile strength is of order 2400 to 2600 MPa, so we have in fact a safety factor closer to 4.

The quadruple pendulums are all assembled firstly with metal masses and wires at every stage. Thus we have to specify a wire radius for the bottom stage of the main chain, though this gets replaced eventually by the silica fibres when the monolithic suspension is installed. This wire is chosen using the criteria above (stress of  $\sim 1/3$  of 2000 MPa) applied to a 40 kg mass and the resulting value is 0.457 mm diameter, 0.2285 mm radius, ref D060516-v2. However the bottom mass of the reaction chain is lighter, 26 kg in the case of the end reaction mass for the ETM, and 20 kg for the newly proposed thinner compensator plate in the reaction chain of the ITM. If one follows the same criteria for choosing those wires, the wires for suspending those masses would be thinner than for the main chain, and indeed different for the two types of reaction chain. We had originally proposed to use thinner wire for the reaction chain – see T060283-02. However for simplicity of assembly we are now proposing that the same wire diameter is used for all chains at the final stage of the quad suspensions, regardless of the final mass. We review the implications of this decision in section 4.

Note that choice of wire diameters at upper stages is not affected by any difference in the lowest mass since we keep the overall mass the same by increasing the penultimate mass as necessary.

## 3 Parameters varied for this study.

We have looked at the effect of different wire diameters using the latest published quad MATLAB model from Mark Barton, ref T1000263-v1, which is the current baseline model of the quad suspension main chain. To use this model for looking at the reaction chain we require to change several of the parameters

- 1) Material (Young's modulus) and radius of the bottom wire.
- 2) Masses and moments of inertia for the penultimate and bottom mass.
- 3) The effective  $d$  values at the ends of the lowest wire, to simulate keeping the physical  $d$  values (i.e. positions of attachment point for the break off prisms or clamps) the same. The effective  $d$  values will change with wire diameter since they include the wire flexure lengths, and the code is currently written so that if a wire is changed, the physical  $d$  is changed to keep the effective  $d$  constant. Thus if we want to keep the *physical*  $d$ s constant (corresponding to the production as-built values) we need to change the effective  $d$ s in the code.

The appendix (section 6) lists the various parameter sets used in this investigation.

Note that the parameters are only an approximation to the production values. For example we have modeled the penultimate reaction masses as right circular cylinders and in reality they are more complicated. Also no wedges are included for the ERM or CP. But the values are close enough to draw conclusions about the changes which occur when different radii of wire are used at the final stage.

## 4 Observations regarding effect of wire radius

The major effects seen when the wire radius of the final stage in the reaction chain is changed is that:

- i) With increasing wire radius, the highest two pitch modes are raised and the pitch isolation is slightly decreased (by less than factor of two at 10 Hz for the wire radii considered). See figure 1.
- ii) With increasing wire radius, the highest vertical and roll modes are increased, see figures 2 and 3.

For the ERM with  $r = 0.2285$  mm (default value), highest vertical mode = 18.3 Hz and highest roll mode = 26.1 Hz

For the ERM with  $r = 0.175$  mm (thinner wire), highest vertical mode = 14.4 Hz and highest roll mode = 20.5 Hz

For the thinner CP with  $r = 0.2285$  mm (default value), highest vertical mode = 19.8 Hz and highest roll mode = 28.2 Hz

Other modes and transfer functions are essentially unchanged.

## 5 Conclusions

The isolation required in the reaction chain of the quad is modest compared to the main chain – see for example T060043-00-K. Thus the small decrease in pitch isolation when using thicker wire is not significant. We have no requirement on the frequencies of the highest vertical and roll modes for the reaction chains. These peaks will only weakly couple into the main chain and will have much lower  $Q$ s than the main chain which contains silica fibres. We can see no adverse effect in their being at slightly higher frequencies due to the use of thicker wire. The frequencies are similar to those from other wire suspensions (HLTS and HSTS) and are all below 30 Hz.

The advantage of using the same wire,  $r = 0.2285$  mm, is that it simplifies the assembly process for the quadruple suspensions. Having only one set of four wire sizes for the quad for both main and reaction chains makes keeping track of which wire goes where a simpler process. In addition this choice of wire is what has been assumed in the design of parts by the RAL group. **Thus in conclusion we propose to use the same lowest wire radius, namely  $r = 0.2285$  mm (diameter = 0.457 mm) for all quad suspension chains regardless of the mass of the bottom stage.**

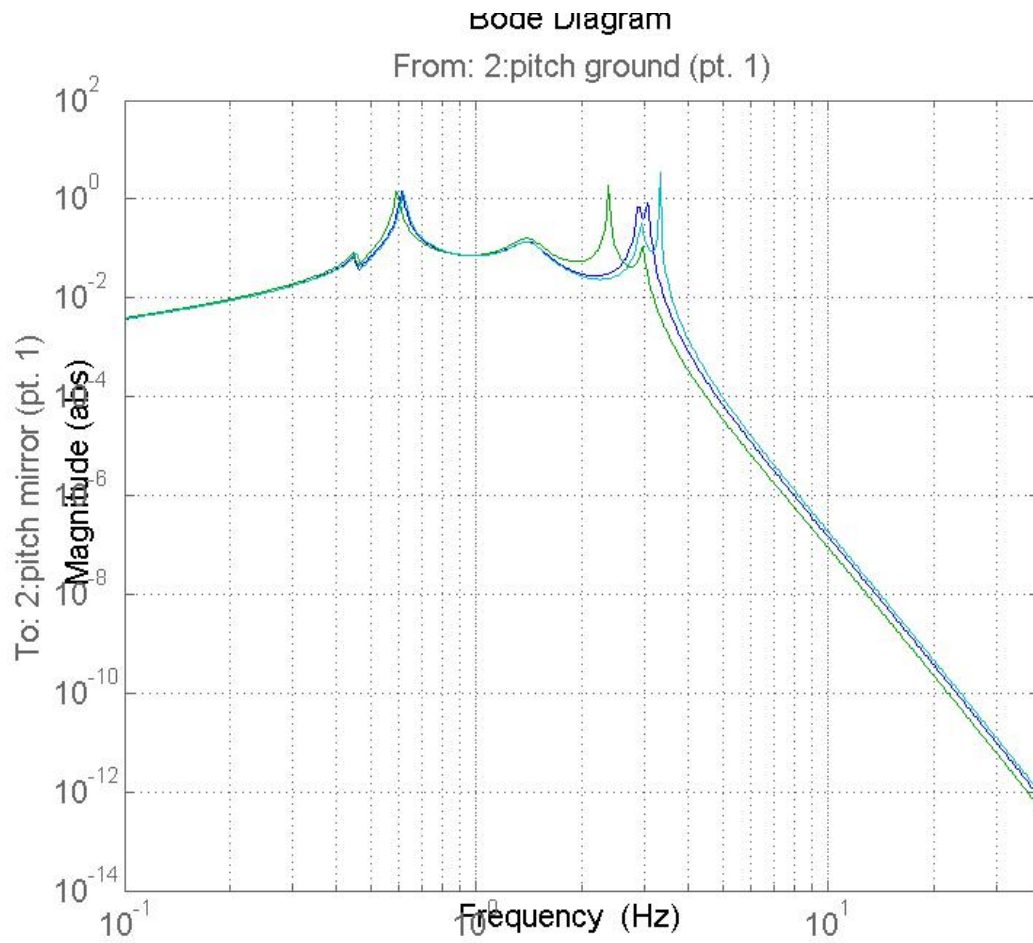


Figure 1. Pitch transfer functions for ERM chain (green and blue) and for thin CP chain (turquoise).  
Green :  $r_3 = 0.2285$  mm. Blue:  $r_3 = 0.175$  mm. Turquoise:  $r_3 = 0.2285$  mm.

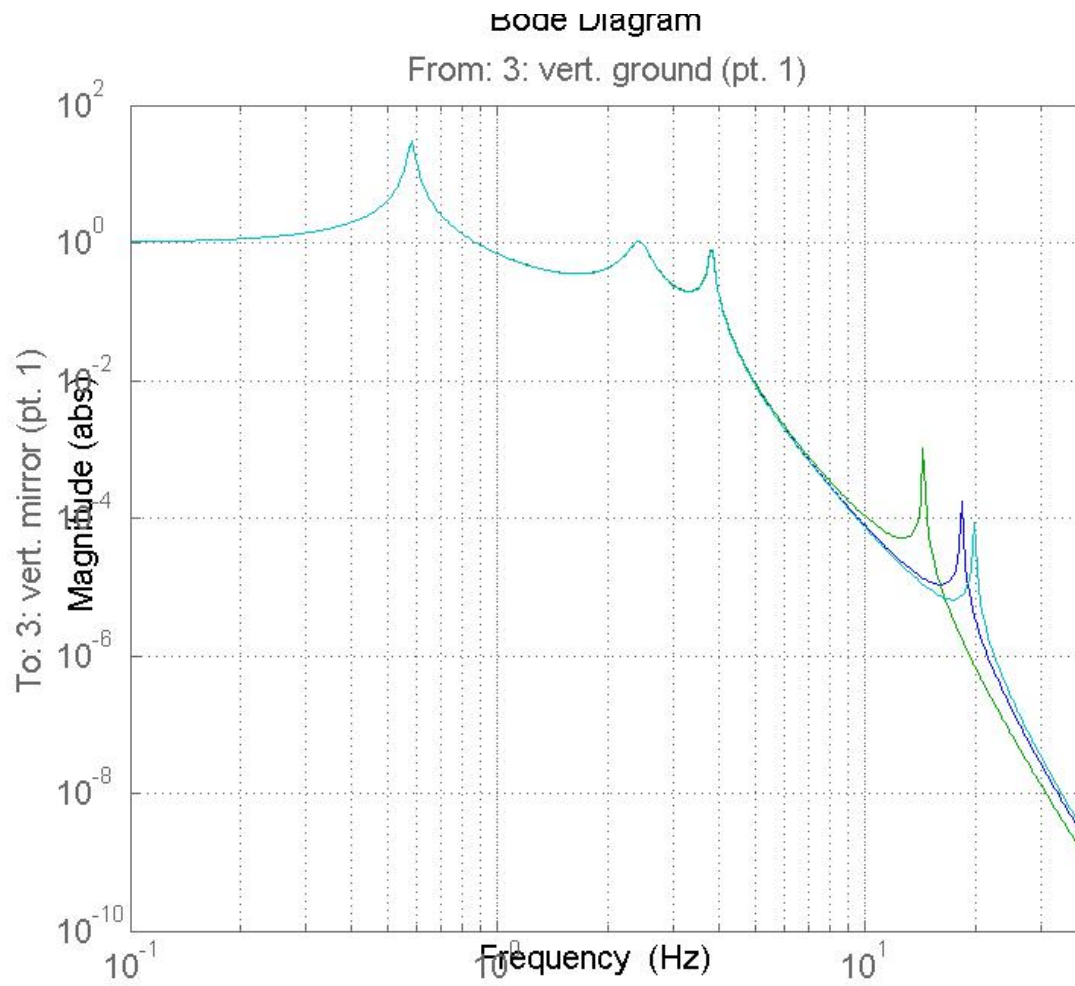


Figure 2. Vertical transfer functions for ERM chain (green and blue) and for thin CP chain (turquoise).  
Green :  $r3 = 0.2285$  mm. Blue:  $r3 = 0.175$  mm. Turquoise:  $r3 = 0.2285$  mm.

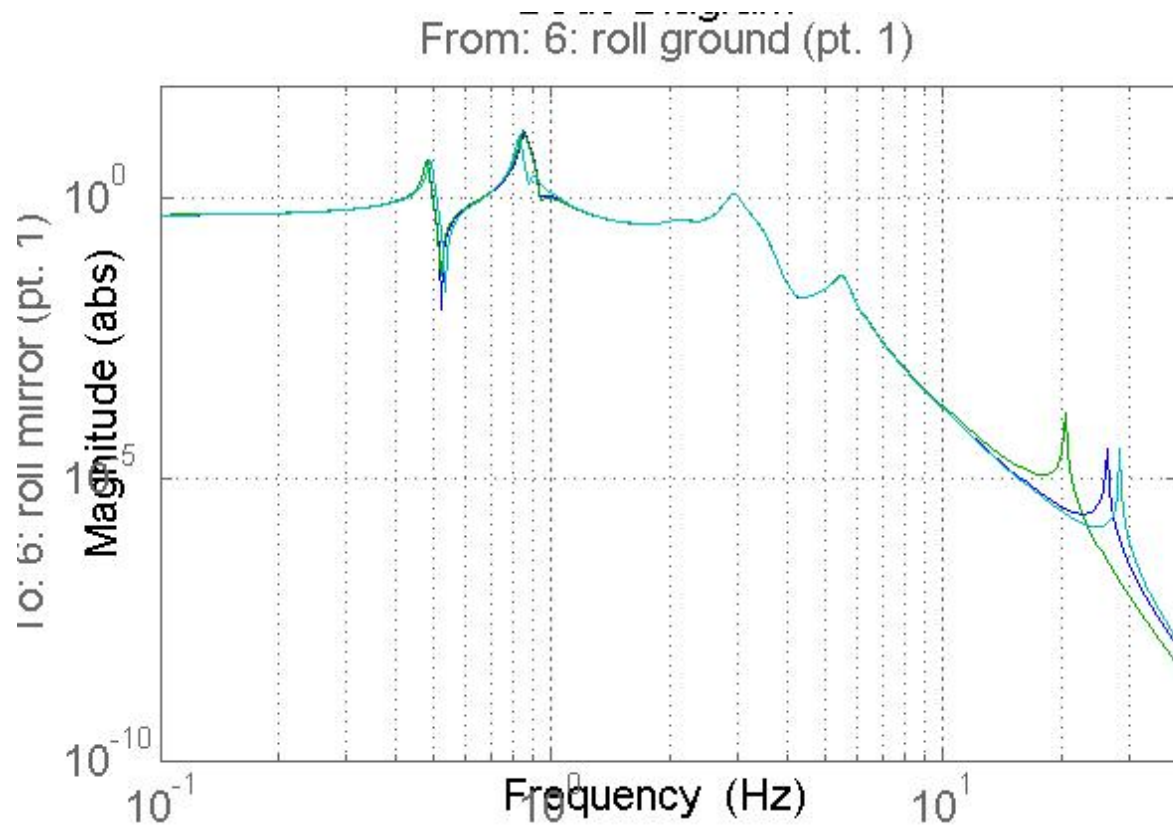


Figure 3. Roll transfer functions for ERM chain (green and blue) and for thin CP chain (turquoise).  
Green :  $r3 = 0.2285$  mm. Blue:  $r3 = 0.175$  mm. Turquoise:  $r3 = 0.2285$  mm.

## 6 Appendix

Default parameters in T1000263 (set for noise prototype with silica fibres in final stage)

pend =

```
stage2: 1
ribbon: 0
  g: 9.8100
  nx: 0.1300
  ny: 0.5000
  nz: 0.0840
denn: 4000
  mn: 22.1100
Inx: 0.4558
Iny: 0.0712
Inz: 0.4547
  ux: 0.1300
  uy: 0.5000
  uz: 0.0840
den1: 4000
  m1: 21.0110
I1x: 0.5174
I1y: 0.0598
I1z: 0.5205
  ix: 0.2000
  ir: 0.1700
  if: 0.0950
den2: 2200
  m2: 39.5696
I2x: 0.5666
I2y: 0.4204
```

I2z: 0.4101  
tx: 0.2000  
tr: 0.1700  
tf: 0.0950  
den3: 2200  
m3: 39.5696  
I3x: 0.5666  
I3y: 0.4204  
I3z: 0.4101  
nwn: 2  
nw1: 4  
nw2: 4  
nw3: 4  
bd: 0  
rn: 5.2000e-004  
r1: 3.5000e-004  
r2: 3.1000e-004  
r3: 4.0000e-004  
r3n: 4.0000e-004  
r3m: 2.0000e-004  
nl: 0.0150  
Yn: 2.1200e+011  
Y1: 2.1200e+011  
Y2: 2.1200e+011  
Y3: 7.2000e+010  
twistlength: 0  
d3tr: 1.0000e-003  
d4tr: 1.0000e-003  
sn: 0  
su: 0.0030  
si: 0.0030  
sl: 0.0150  
nn0: 0.2500



nn1: 0.0900  
n0: 0.2000  
n1: 0.0600  
n2: 0.1400  
n3: 0.1762  
n4: 0.1712  
n5: 0.1712  
ln: 0.4486  
l1: 0.3090  
l2: 0.3327  
l3: 0.6077  
dm: -0.0031  
dn: 0.0032  
d0: -0.0017  
d1: 0.0031  
d2: 0.0072  
d3: -0.0029  
d4: -0.0029  
mn3: 122.2602  
m13: 100.1502  
m23: 79.1392  
tlnspec: 0.4160  
tl1spec: 0.2770  
tl2spec: 0.3410  
tl3spec: 0.6020  
kxn: 100000  
kx1: 100000  
kx2: 80000  
dmspec: 1.0000e-003  
dnspec: 1.0000e-003  
d0spec: 1.0000e-003  
d1spec: 1.0000e-003  
d2spec: 0.0100

d3spec: 1.0000e-003  
d4spec: 1.0000e-003  
kcn: 1.6111e+003  
kc1: 1.8114e+003  
kc2: 2.6347e+003  
kw3n: 4.8255e+006  
kw3m: 3.1322e+004  
kw3: 3.0921e+004  
tln: 0.4160  
tl1: 0.2770  
tl2: 0.3410  
tl3: 0.6020  
Mn1: 5.7425e-014  
M11: 1.1786e-014  
M21: 7.2533e-015  
M31: 2.0106e-014  
M32: 2.0106e-014  
flexn: 0.0041  
flex1: 0.0027  
flex2: 0.0028  
flex3: 0.0039  
ufcn: 1.9213  
ufc1: 2.0899  
ufc2: 1.8366  
kwn: 4.0148e+005  
kw1: 5.2808e+005  
kw2: 3.8475e+005  
l\_suspoint\_to\_centreofoptic: 1.6360  
l\_suspoint\_to\_bottomofoptic: 1.8060  
flex3tr: 0.0039  
longpitch1: [0.4332 0.5193 0.9870 1.2331]  
longpitch2: [1.5674 2.0072 2.9454 3.4189]  
yaw: [0.5981 1.3452 2.4083 3.0462]

transroll1: [0.4625 0.8615 1.0442 2.1240]

transroll2: [2.6925 3.3254 5.0994 12.8743]

vertical: [0.5815 2.3386 3.7604 9.0024]

Note that sum of test mass (m3) and penultimate mass (m2) = 2 x 39.57 = 79.14 kg

### A.1 End Reaction Mass

ERM mass (m3) = 25.97 kg (340 mm diameter by 130 mm thick silica)

Thus penultimate mass (m2) = 79.14 – 26.0 = 53.14 kg

m2 has been approximated below by a cylinder of same dimensions as the ERM and fictitious density, as given below.

ix: 0.1300

ir: 0.1700

den2: 4502

m2: 53.1369

I2x: 0.7678

I2y: 0.4587

I2z: 0.4587

tx: 0.1300

tr: 0.1700

den3: 2200

m3: 25.9665

I3x: 0.3752

I3y: 0.2242

I3z: 0.2242

Complete baseline parameter set is as below. Note that d3 and d4 are -1.0 mm, which are physical break-off values as noted by Joe Odell in email to NAR 28 May 2010, and taken from D060338-v1. These values are held constant in subsequent analyses.

r3 is set at 0.2285 mm, default value.

pend =

stage2: 1  
ribbon: 0  
    g: 9.8100  
    nx: 0.1300  
    ny: 0.5000  
    nz: 0.0840  
denn: 4000  
    mn: 22.1100  
Inx: 0.4558  
Iny: 0.0712  
Inz: 0.4547  
    ux: 0.1300  
    uy: 0.5000  
    uz: 0.0840  
den1: 4000  
    m1: 21.0110  
I1x: 0.5174  
I1y: 0.0598  
I1z: 0.5205  
    ix: 0.1300  
    ir: 0.1700  
    if: 0.0950  
den2: 4502  
    m2: 53.1369  
I2x: 0.7678  
I2y: 0.4587  
I2z: 0.4587  
    tx: 0.1300  
    tr: 0.1700  
    tf: 0.0950  
den3: 2200  
    m3: 25.9665  
I3x: 0.3752

I3y: 0.2242  
I3z: 0.2242  
nwn: 2  
nw1: 4  
nw2: 4  
nw3: 4  
bd: 0  
rn: 5.2000e-004  
r1: 3.5000e-004  
r2: 3.1000e-004  
r3: 2.2850e-004  
r3n: 2.2850e-004  
r3m: 2.2850e-004  
nl: 0.0150  
Yn: 2.1200e+011  
Y1: 2.1200e+011  
Y2: 2.1200e+011  
Y3: 2.1200e+011  
twistlength: 0  
d3tr: 1.0000e-003  
d4tr: 1.0000e-003  
sn: 0  
su: 0.0030  
si: 0.0030  
sl: 0.0150  
nn0: 0.2500  
nn1: 0.0900  
n0: 0.2000  
n1: 0.0600  
n2: 0.1400  
n3: 0.1762  
n4: 0.1712  
n5: 0.1712

ln: 0.4486  
l1: 0.3090  
l2: 0.3327  
l3: 0.6041  
dm: -0.0031  
dn: 0.0032  
d0: -0.0017  
d1: 0.0031  
d2: 0.0072  
d3: -0.0010  
d4: -0.0010  
mn3: 122.2245  
m13: 100.1145  
m23: 79.1035  
tlnspec: 0.4160  
tl1spec: 0.2770  
tl2spec: 0.3410  
tl3spec: 0.6020  
kxn: 100000  
kx1: 100000  
kx2: 80000  
dmspec: 1.0000e-003  
dnspec: 1.0000e-003  
d0spec: 1.0000e-003  
d1spec: 1.0000e-003  
d2spec: 0.0100  
d3spec: 0.0016  
d4spec: 0.0016  
kcn: 1.6111e+003  
kc1: 1.8114e+003  
kc2: 2.6347e+003  
kw3n: 4.6366e+006  
kw3m: 1.2115e+005

kw3: 1.1514e+005

tlm: 0.4160

tl1: 0.2770

tl2: 0.3410

tl3: 0.6020

Mn1: 5.7425e-014

M11: 1.1786e-014

M21: 7.2533e-015

M31: 2.1411e-015

M32: 2.1411e-015

flexn: 0.0041

flex1: 0.0027

flex2: 0.0028

flex3: 0.0027

ufcn: 1.9213

ufc1: 2.0899

ufc2: 1.5849

kwn: 4.0147e+005

kw1: 5.2808e+005

kw2: 3.8475e+005

l\_suspoint\_to\_centreofoptic: 1.6360

l\_suspoint\_to\_bottomofoptic: 1.8060

flex3tr: 0.0027

longpitch1: [0.4505 0.6128 0.8590 1.3716]

longpitch2: [1.9256 2.8707 3.0690 3.4091]

yaw: [0.6532 1.3450 2.3237 2.9966]

transroll1: [0.4804 0.8508 0.9096 2.0566]

transroll2: [2.7038 3.3251 5.0981 26.0771]

vertical: [0.5825 2.3494 3.7765 18.3095]

Parameter set for thinner wire (as highlighted)

pend =

stage2: 1  
ribbon: 0  
  g: 9.8100  
  nx: 0.1300  
  ny: 0.5000  
  nz: 0.0840  
denn: 4000  
  mn: 22.1100  
  Inx: 0.4558  
  Iny: 0.0712  
  Inz: 0.4547  
  ux: 0.1300  
  uy: 0.5000  
  uz: 0.0840  
den1: 4000  
  m1: 21.0110  
  I1x: 0.5174  
  I1y: 0.0598  
  I1z: 0.5205  
  ix: 0.1300  
  ir: 0.1700  
  if: 0.0950  
den2: 4502  
  m2: 53.1369  
  I2x: 0.7678  
  I2y: 0.4587  
  I2z: 0.4587  
  tx: 0.1300  
  tr: 0.1700  
  tf: 0.0950  
den3: 2200  
  m3: 25.9665



I3x: 0.3752  
I3y: 0.2242  
I3z: 0.2242  
nwn: 2  
nw1: 4  
nw2: 4  
nw3: 4  
bd: 0  
rn: 5.2000e-004  
r1: 3.5000e-004  
r2: 3.1000e-004  
r3: 1.7500e-004  
r3n: 0.0022  
r3m: 1.7500e-004  
nl: 0.0150  
Yn: 2.1200e+011  
Y1: 2.1200e+011  
Y2: 2.1200e+011  
Y3: 2.1200e+011  
twistlength: 0  
d3tr: 1.0000e-003  
d4tr: 1.0000e-003  
sn: 0  
su: 0.0030  
si: 0.0030  
sl: 0.0150  
nn0: 0.2500  
nn1: 0.0900  
n0: 0.2000  
n1: 0.0600  
n2: 0.1400  
n3: 0.1762  
n4: 0.1712

n5: 0.1712  
ln: 0.4486  
l1: 0.3090  
l2: 0.3327  
l3: 0.6040  
dm: -0.0031  
dn: 0.0032  
d0: -0.0017  
d1: 0.0031  
d2: 0.0072  
d3: -0.0010  
d4: -0.0010  
mn3: 122.2245  
m13: 100.1145  
m23: 79.1035  
tlnspec: 0.4160  
tl1spec: 0.2770  
tl2spec: 0.3410  
tl3spec: 0.6020  
kxn: 100000  
kx1: 100000  
kx2: 80000  
dmspec: 1.0000e-003  
dnspec: 1.0000e-003  
d0spec: 1.0000e-003  
d1spec: 1.0000e-003  
d2spec: 0.0100  
d3spec: 5.6000e-004  
d4spec: 5.6000e-004  
kcn: 1.6111e+003  
kc1: 1.8114e+003  
kc2: 2.6347e+003  
kw3n: 4.2009e+008

kw3m: 7.1067e+004  
kw3: 7.1043e+004  
tln: 0.4160  
tl1: 0.2770  
tl2: 0.3410  
tl3: 0.6020  
Mn1: 5.7425e-014  
M11: 1.1786e-014  
M21: 7.2533e-015  
M31: 7.3662e-016  
M32: 7.3662e-016  
flexn: 0.0041  
flex1: 0.0027  
flex2: 0.0028  
flex3: 0.0016  
ufcn: 1.9213  
ufc1: 2.0899  
ufc2: 1.5849  
kwn: 4.0147e+005  
kw1: 5.2808e+005  
kw2: 3.8475e+005  
l\_suspoint\_to\_centreofptic: 1.6360  
l\_suspoint\_to\_bottomofptic: 1.8060  
flex3tr: 0.0016  
longpitch1: [0.4504 0.5930 0.8582 1.3646]  
longpitch2: [1.9244 2.3692 2.9517 3.4088]  
yaw: [0.6531 1.3431 2.3233 2.9965]  
transroll1: [0.4803 0.8448 0.9095 2.0566]  
transroll2: [2.7021 3.3241 5.0981 20.5003]  
vertical: [0.5825 2.3485 3.7753 14.3941]

## A.2 Compensator plate (thin version)

CP mass (m3 below) = 20.0 kg (340 mm diameter by 100 mm thick silica)

Thus penultimate mass (m2 below) =  $79.14 - 20.0 = 59.14$  kg

m2 has been approximated below by a cylinder of same dimensions as the ERM (and the original CP) and fictitious density, as given below.

ix: 0.1300  
ir: 0.1700  
den2: 5011  
m2: 59.1447  
I2x: 0.8546  
I2y: 0.5106  
I2z: 0.5106  
tx: 0.1000  
tr: 0.1700  
den3: 2200  
m3: 19.9742  
I3x: 0.2886  
I3y: 0.1610  
I3z: 0.1610

Complete parameter set is as below. Note that d3 and d4 are -1.0 mm, which are physical break-off values as noted by Joe Odell in email to NAR 28 May 2010, and taken from D060338-v1. These values are held constant.

r3 is set at 0.2285 mm, default value.

pend =

stage2: 1  
ribbon: 0  
g: 9.8100  
nx: 0.1300  
ny: 0.5000  
nz: 0.0840  
denn: 4000  
mn: 22.1100  
Inx: 0.4558

Iny: 0.0712  
Inz: 0.4547  
ux: 0.1300  
uy: 0.5000  
uz: 0.0840  
den1: 4000  
m1: 21.0110  
I1x: 0.5174  
I1y: 0.0598  
I1z: 0.5205  
ix: 0.1300  
ir: 0.1700  
if: 0.0950  
den2: 5011  
m2: 59.1447  
I2x: 0.8546  
I2y: 0.5106  
I2z: 0.5106  
tx: 0.1000  
tr: 0.1700  
tf: 0.0950  
den3: 2200  
m3: 19.9742  
I3x: 0.2886  
I3y: 0.1610  
I3z: 0.1610  
nwn: 2  
nw1: 4  
nw2: 4  
nw3: 4  
bd: 0  
rn: 5.2000e-004  
r1: 3.5000e-004

r2: 3.1000e-004

r3: 2.2850e-004

r3n: 2.2850e-004

r3m: 2.2850e-004

nl: 0.0150

Yn: 2.1200e+011

Y1: 2.1200e+011

Y2: 2.1200e+011

Y3: 2.1200e+011

twistlength: 0

d3tr: 1.0000e-003

d4tr: 1.0000e-003

sn: 0

su: 0.0030

si: 0.0030

sl: 0.0150

nn0: 0.2500

nn1: 0.0900

n0: 0.2000

n1: 0.0600

n2: 0.1400

n3: 0.1762

n4: 0.1712

n5: 0.1712

ln: 0.4486

l1: 0.3090

l2: 0.3327

l3: 0.6041

dm: -0.0031

dn: 0.0032

d0: -0.0017

d1: 0.0031

d2: 0.0072

d3: -0.0010  
d4: -0.0010  
mn3: 122.2399  
m13: 100.1299  
m23: 79.1189  
tlnspec: 0.4160  
tl1spec: 0.2770  
tl2spec: 0.3410  
tl3spec: 0.6020  
kxn: 100000  
kx1: 100000  
kx2: 80000  
dmspec: 1.0000e-003  
dnspec: 1.0000e-003  
d0spec: 1.0000e-003  
d1spec: 1.0000e-003  
d2spec: 0.0100  
d3spec: 0.0020  
d4spec: 0.0020  
kcn: 1.6111e+003  
kc1: 1.8114e+003  
kc2: 2.6347e+003  
kw3n: 4.6366e+006  
kw3m: 1.2115e+005  
kw3: 1.1513e+005  
tln: 0.4160  
tl1: 0.2770  
tl2: 0.3410  
tl3: 0.6020  
Mn1: 5.7425e-014  
M11: 1.1786e-014  
M21: 7.2533e-015  
M31: 2.1411e-015

M32: 2.1411e-015  
flexn: 0.0041  
flex1: 0.0027  
flex2: 0.0028  
flex3: 0.0030  
ufcn: 1.9213  
ufc1: 2.0899  
ufc2: 1.5023  
kwn: 4.0147e+005  
kw1: 5.2808e+005  
kw2: 3.8475e+005  
l\_suspoint\_to\_centreofoptic: 1.6360  
l\_suspoint\_to\_bottomofoptic: 1.8060  
flex3tr: 0.0030  
longpitch1: [0.4589 0.6188 0.8090 1.3760]  
longpitch2: [1.9046 2.9211 3.3219 3.4078]  
yaw: [0.6655 1.3437 2.2629 2.9686]  
transroll1: [0.4906 0.8249 0.8796 2.0383]  
transroll2: [2.7045 3.3232 5.0981 28.1695]  
vertical: [0.5825 2.3499 3.7772 19.7787]