LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

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Effects of small size anomalies in a FP cavity

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1 Introduction

This note discusses the effect of small size defects on the HR surface in a FP cavity, analyzed using SIS. In this revision, (a) the loss round trip loss and (b) resonating field shapes are calculated when, in a small area on ETM, (1) the loss is not uniform or (2) the surface has a bump.

2 Optical system

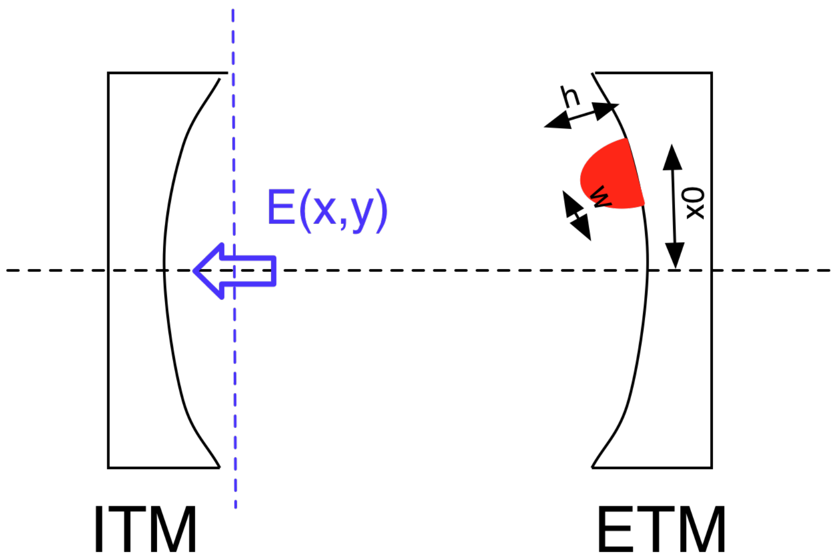

Figure 1. FP cavity and small anomaly

The optical system studied is the advanced LIGO arm with an anomaly placed on ETM. The shape of the anomaly is of Gaussian shape and is placed at (x=x0, y=0) and the anomaly is 0 out of 2w, i.e.,

$$
\begin{aligned}
d(x,y) &= h \cdot \exp(-\frac{(x-x_0)^2+y^2}{w^2}) : \sqrt{(x-x_0)^2+y^2} < 2w \\
&= 0 : \sqrt{(x-x_0)^2+y^2} > 2w
\end{aligned} \tag{1}
$$

A TEM00 mode field, which matches with the clean FP cavity, i.e., without the anomaly, is injected to the cavity and the stationary field is calculated.

The round trip loss without the anomaly is 0.6ppm, due to the finite apertures of ITM and ETM.

Field shapes shown below are the one on ITM surface coming from ETM, shown by an arrow in Fig.1. The effect is very small and fields called "deviation" is defined as

$$
\delta(x,y) \equiv \frac{E(x,y)}{E(0,0)} - \frac{E_0(x,y)}{E_0(0,0)} \tag{2}
$$

where $E_0(x,y)$ is the stationary state field without the anomaly and E(x,y) is the one with the anomaly. E_0 is a almost clean TEM00 mode, and δ is the structure added by the anomaly, which is normalized by the amplitude at the center.

3 Loss anomaly

The effect of loss due to a point defect is calculated using a localized loss with the Gaussian shape. When the loss is of the Gaussian shape, the total loss on one bounce is calculated to be

$$Loss \approx P_{cav}\left(\frac{w_a}{w_{beam}}\right)^2 2h \cdot \exp\left(-\frac{2x_0^2}{w_a^2}\right) \quad (3)$$

when the size of the anomaly, w_a, is much smaller than the beam size, w_{beam}.

FFT-based simulation cannot simulate effects caused by point structures, which is smaller than the FFT grid size, which is a fraction of mm at the smallest.

In order to see how good the approximation is, the round trip loss was calculated using three sets of (w_a, h) which give same loss values: (w_a, h) = (2mm, 0.0025), (1mm, 0.01), (0.5mm, 0.04). For all cases, x_0 = 2cm, and w_{beam} = 6.2cm. The losses calculated using these values were 4.8ppm, 4.7ppm and 4.8ppm, which is comparable to the analytic point loss using Eq.(3), 4.2ppm.

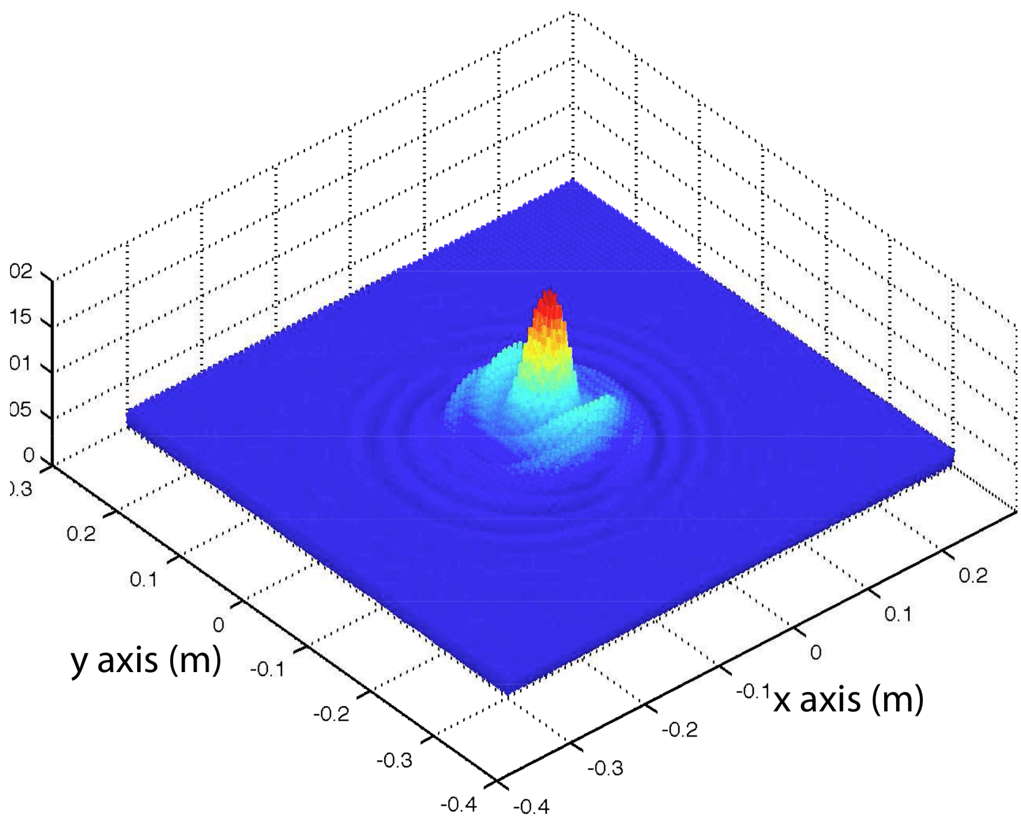

Figure 2. Deviation with loss anomaly

Fig.2 is the deviation field when a loss anomaly with (w_a, h) = (2mm, 0.0025) is placed at x_0=2cm. As is seen from the figure, the effect of a localized anomaly affects the entire beam surface. The same effect is observed when there is a localized bump, which is discussed in the following section.

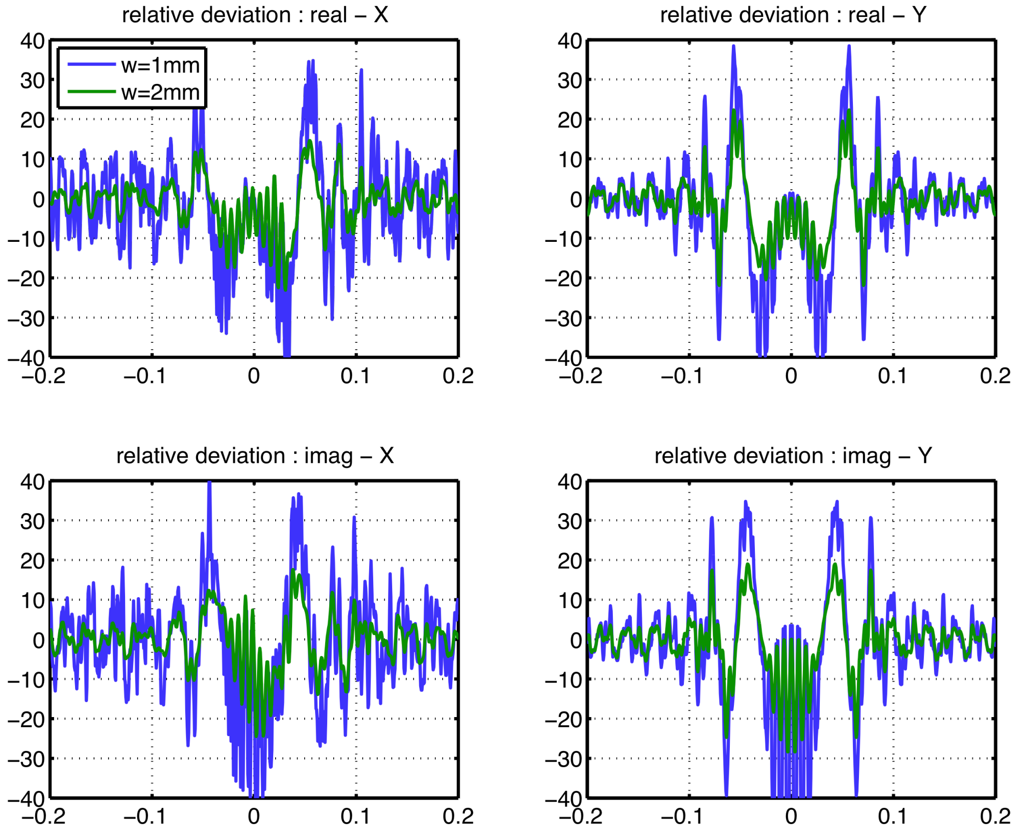

Figure 3. Deviations of real and imaginary component (units in ppm)

Fig.3 shows the real and imaginary components of the deviations. E.g., the top-left plot is real part of δ(x,0). The spatial wavelengths of the structure is determined by the location of the anomaly, i.e., 2cm in the case, both in x and y directions.

The point scattering loss measured at Caltech lab using a small size laser (200 μm) was mostly less than 100ppm. On ETM, this point scattering loss will scatter out < $(200\ \mu m/6.2cm)^2$ x 100ppm ~ 10^{-9}. This is over 1000 times smaller than the example case used to create plots in Fig.3. So the deviation by anomalous loss will change the field amplitude only by 10^{-7}.

It is necessary to understand when there are many point scatterings how they affect collectively.

4 Shape anomaly

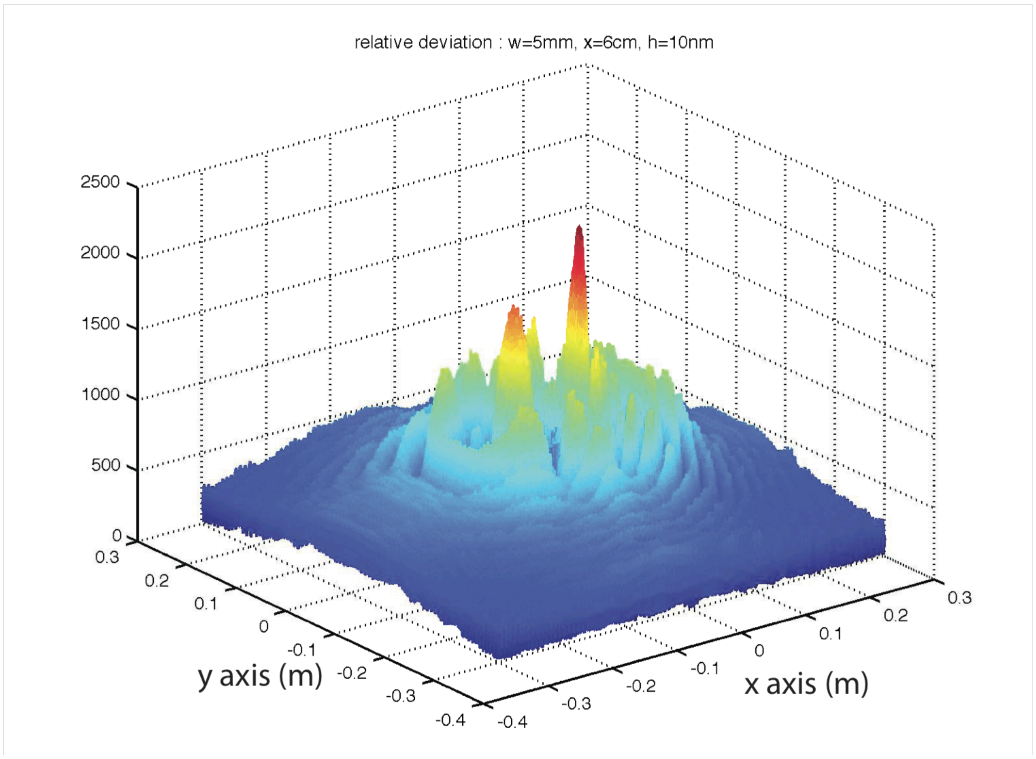

Figure 4. Deviation with shape anomaly

Fig.4 is the deviation when a Gaussian bump with height of 10nm, width of 5mm at location (x=6cm, y=0). As is observed above, the bump affects the entire beam, not localized around the bump.

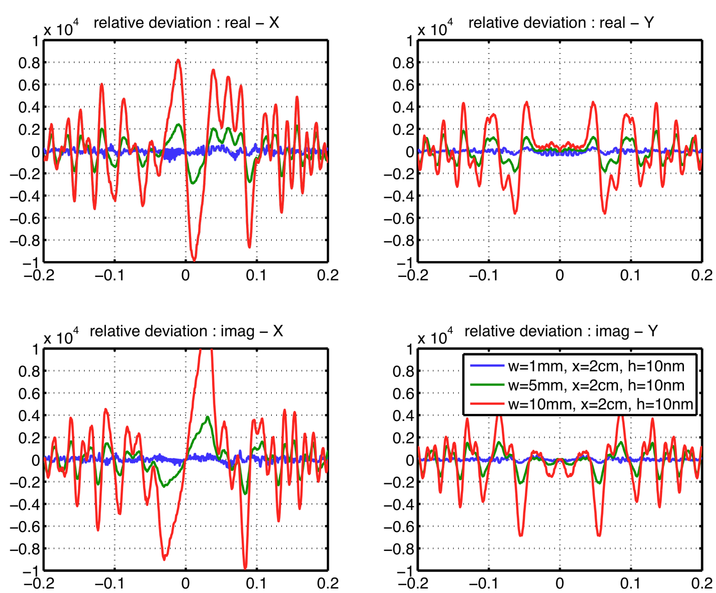

Figure 5. Deviations with different widths

Fig.5 shows the real and imaginary part of deviations when the dumps are placed at 2cm away from the center. Three lines show deviations with different sizes, 1mm, 5mm and 10mm, with same height of 10nm. For each width, the round trip loss is 3.5ppm, 68ppm and 210ppm. The round trip loss is roughly proportional to the square of the height, and if the height is 1nm, the loss values are ~1/100 of these values. These loss values and deviation magnitudes are much larger than the case of the effect of the loss anomaly.

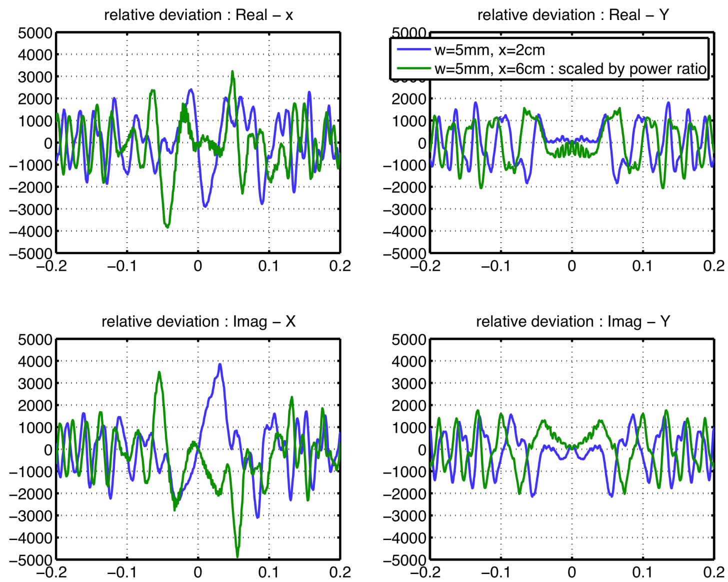

Figure 6. Deviations with different bump locations

Fig. 6 shows the deviations when a bump (width 5mm, height of 10nm) is placed at 2cm and at 6cm. The blue line (x_0=2cm) shows shorter spatial wavelength structure than the green line (x_0=6cm).

5 Thermal deformation in a small region

Muzammil calculated thermal surface shape deformation by the absorption in a small region. Small beams hearted at (x=2cm,y=0cm) and the total absorption is 0.425W or 0.5ppm at the full aLIGO arm power. The beam size chosen are 0.5mm, 1mm and 2mm.

Fig. 7 and Fig.8 shows the surface deformation when the absorption is 1/100 of 0.5ppm. Fig.7 is the 3D plot for the beam size is 1mm, and Fig.8 compares the shapes for the beam size of 1mm and 2mm. For a larger beam size, the surface deformation is also broader.

Fig.9 shows the field distortion defined in Eq.(2) when 1mm beam is absorbed at 0.01 x 0.5ppm rate. As were discussed in previous sections, the point absorption distorted the field in the entire are, not just a limited region.

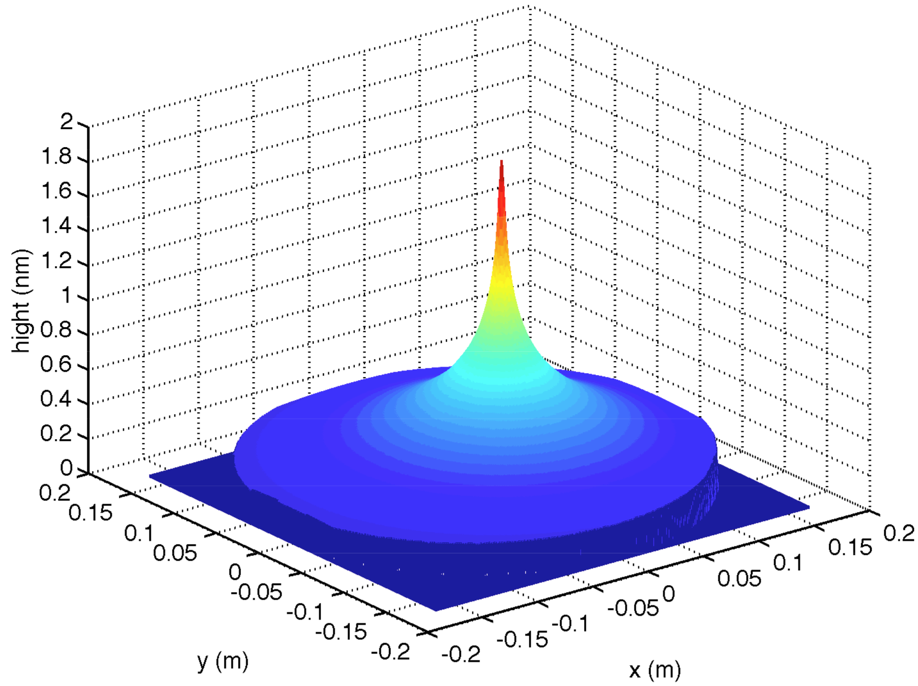

Figure 7 Surface deformation in 3D (beam size 1mm)

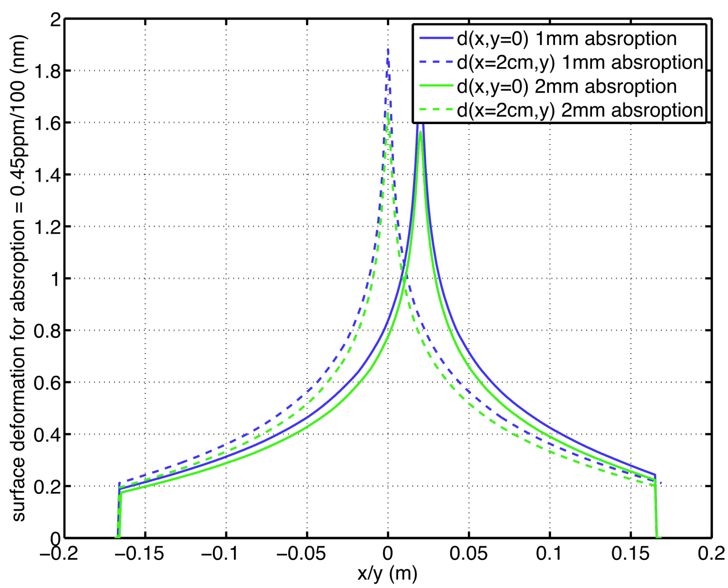

Figure 8 Surface deformation in 2D

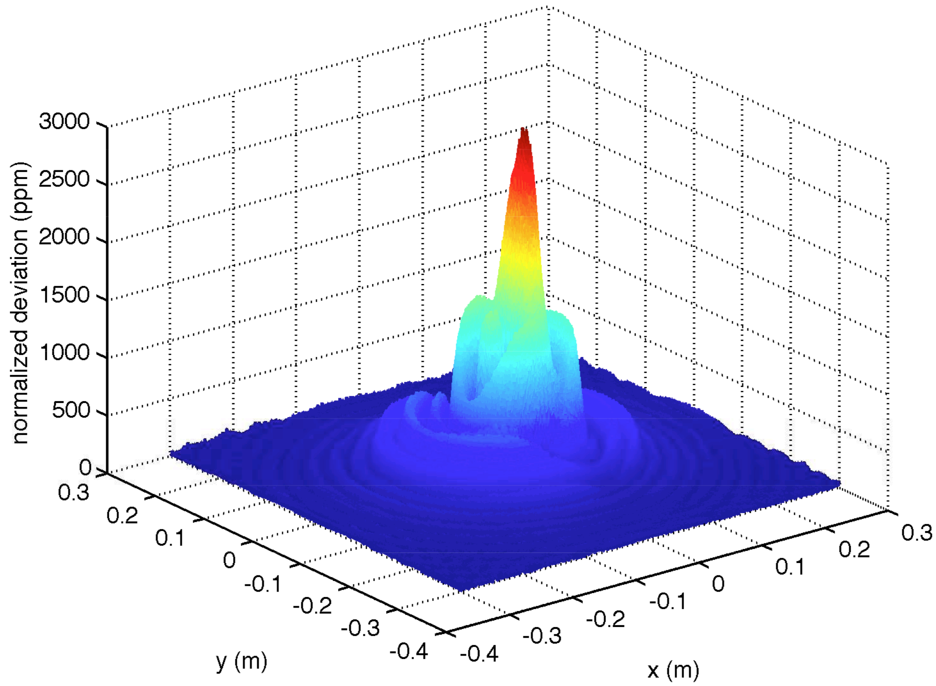

Figure 9 Field on ITM with 1mm heating, 1/100x0.5ppm absorption

Fig.10 shows the relation between the absorption loss vs round trip loss. As is seen from Fig.8, the surface distortion is in a region of a several mm with a height of a few nm when the absorption is 1/100 of 0.5ppm. This induces round trip loss of a few ppm. This is consistent with the loss calculated in the previous section using a Gaussian shape.

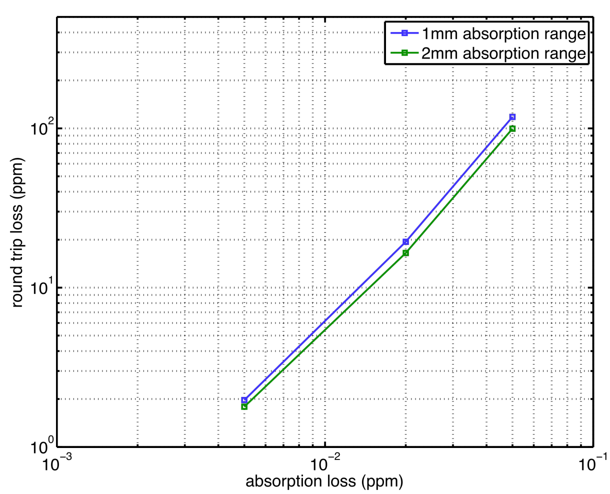

Figure 10 Absorption vs round trip loss

When this calculation was done, the transmittance and reflectance of ETM was changed in the small region to reflect the local loss due to absorption. But the round trip loss is almost unaffected by the change of the optical quantities (R and T), but is determined by the surface shape. This is also consistent with the generic calculation in Sec. 3.

6 Point scattering using Huygen's Integral

[The content of this section is a rewrite of Jean-Yves Vinet's note.]

The Huyhen's integral of the field propagation from z_0 to z is expressed as follows.

$$E(x,y,z) \equiv \frac{i}{\lambda}\iint dx_0\, dy_0 E_0(x_0,y_0,z_0)\frac{\exp(-ik\rho)}{\rho}\cos\theta$$
$$\Delta x = x - x_0, \Delta y = y - y_0, L = z - z_0, k = 2\pi/\lambda \tag{4}$$
$$\rho = \sqrt{\Delta x^2 + \Delta y^2 + L^2}, \cos\theta = \frac{L}{\rho}$$

When a Gaussian field is reflected by a point defect, the reflected field is written as follows:

$$E_0(x_0,y_0,z_0) = TEM00(z_0)\cdot\exp(2ikf(x_0,y_0))$$
$$TEM00(z_0) = \sqrt{\frac{2}{\pi}}\frac{1}{w_0}\exp(-\frac{x_0^2+y_0^2}{w_0^2}) \tag{5}$$

For simplicity, the waist position is on the reflection surface and f is the point defect.

When inserting Eq.(5) to Eq.(4), the reflected field can be written as follows keeping only the first order of f:

$$E(x,y,z) = F_0(x,y,z) + dF(x,y,z) \tag{6}$$

$$F_0(x,y,z) = \frac{i}{\lambda}\iint dx_0\, dy_0 TEM00(z_0)\frac{\exp(-ik\rho)}{\rho}\cos\theta \tag{7}$$

$$dF(x,y,z) = \sqrt{\frac{2}{\pi}}\frac{1}{w_0}\frac{i}{\lambda}\iint dx_0\, dy_0(\exp(2ikf)-1)\exp(-\frac{x_0^2+y_0^2}{w_0^2})\frac{\exp(-ik\rho)}{\rho}\cos\theta$$
$$\approx \sqrt{\frac{2}{\pi}}\frac{1}{w_0}\frac{i}{\lambda}\iint dx_0\, dy_0\, 2ikf(x_0,y_0)\exp(-\frac{x_0^2+y_0^2}{w_0^2})\frac{\exp(-ik\rho)}{\rho}\cos\theta \tag{8}$$

F_0 is the unperturbed components and dF is the perturbed component by the point defect. The integral goes over the point defect where f is non-zero in a region O(1μm). For simplify, the defect is placed at the center of the reflection point.

With the Fresnel approximation, the Huyhen's integral becomes as follows.

$$E(x,y,z) = \exp(-ikL)\cdot E_t(x,y,z)$$
$$E_t(x,y,z) \equiv \frac{i}{L\cdot\lambda}\iint dx_0\, dy_0 E_0(x_0,y_0,z_0)\exp(-ik\frac{\Delta x^2+\Delta y^2}{2L}) \tag{9}$$

With this approximation, F_0 and dF become as follows:

$$F_0(x,y,z) = \sqrt{\frac{2}{\pi}}\frac{1}{w_0}\frac{i}{L\cdot\lambda}\iint dx_0\,dy_0\exp(-ik\frac{\Delta x^2+\Delta y^2}{2L})\exp(-\frac{x_0^2+y_0^2}{w^2})$$
$$= \sqrt{\frac{2}{\pi}}\frac{1}{w(z)}\exp(-\frac{x^2+y^2}{w(z)^2})\exp(i\eta(z)-i\frac{r^2}{2R(z)}) \quad (10)$$
$$= TEM00(z)$$

$$dF(x,y,z) = \sqrt{\frac{2}{\pi}}\frac{1}{w_0}\frac{i}{L\cdot\lambda}\iint dx_0\,dy_0(\exp(2ikf)-1)\exp(-ik\frac{\Delta x^2+\Delta y^2}{2L})\exp(-\frac{x_0^2+y_0^2}{w_0^2})$$
$$\approx \sqrt{\frac{2}{\pi}}\frac{1}{w_0}\frac{i}{L\cdot\lambda}\iint dx_0\,dy_0\,2ikf(x_0,y_0)\exp(-ik\frac{\Delta x^2+\Delta y^2}{2L})\exp(-\frac{x_0^2+y_0^2}{w_0^2}) \quad (11)$$

For the typical propagation distance (~km) and the beam size (~several cm), the expression of dF can be simplified as follows, which is the Fraunhofer approximation.

$$dF(x,y,z) = \sqrt{\frac{2}{\pi}}\frac{1}{w_0}\frac{i}{L\cdot\lambda}\exp(-ik\frac{x^2+y^2}{2L})2ik\iint dx_0\,dy_0 f(x_0,y_0)\exp(ik\frac{x\cdot x_0+y\cdot y_0}{L}) \quad (12)$$

The point loss is calculated by integrating the power of this perturbed field:

$$point\,Loss = \iint dx\,dy\left|dF(x,y,z)\right|^2$$
$$= \frac{32\pi}{w_0^2L^2\lambda^4}\iint dx\,dy\iint dx_1\,dy_1\iint dx_2\,dy_2 f(x_1,y_1)f(x_2,y_2)\times$$
$$\exp(i2\pi\frac{x(x_1-x_2)+y(y_1-y_2)}{L\lambda})$$
$$= \frac{32\pi}{w_0^2L^2\lambda^4}\iint dx_1\,dy_1\iint dx_2\,dy_2 f(x_1,y_1)f(x_2,y_2)(L\lambda)^2\delta(x_1-x_2)\delta(y_1-y_2) \quad (13)$$
$$= \frac{32\pi}{w_0^2\lambda^2}\iint dx_0\,dy_0 f(x_0,y_0)^2$$
$$= 32\pi(\frac{a}{w_0})^2(\frac{h}{\lambda})^2$$

which comes out to be 3.9 $\times 10^{-5}$ ppm with a = 2μm, w_0=6cm, h=20nm and λ=1.064μm.

The loss is proportional to the power hitting the point defect, and PL is the value estimated using the power at the origin. For N defects per mm^2 region, the total loss is estimated to be the following.

$$\begin{aligned} totalLoss &= \iint dx\,dy PL \cdot \exp(-\frac{2(x^2+y^2)}{w_0^2}) \times N(1/m^2) \\ &= PL \cdot \frac{\pi}{2} w_0^2 N(1/m^2) \\ &= 16\pi^2 \left(\frac{h}{\lambda}\right)^2 a^2 N(1/m^2) \\ &= (\frac{4\pi h}{\lambda})^2 a^2 N(1/m^2) \\ &= 0.22 \times N(1/mm^2) ppm \end{aligned} \tag{17}$$

7 Scattering loss using near field calculation

When a field is reflected by a surface with surface aberration f(x,y), the reflected field can be approximated by

$$E_{ref} = E_{ref}^0 \exp(i\omega t - ikz) \exp(2ikf(x,y)) \tag{18}$$

where $E_{ref}^0 \exp(i\omega t - ikz)$ is the reflected field without the aberration. When the aberration is small, i.e, kf <<1, this can be expanded keeping up to the second order of kf:

$$\begin{aligned} E_{ref} &= E_{ref}^0 \exp(i\omega t - ikz)(1 + i2kf - 2(kf)^2) \\ &= E_{ref}^0 \exp(i\omega t - ikz)(1 - 2(kf)^2) + E_{ref}^0 \exp(i\omega t - ikz) i2kf \end{aligned} \tag{19}$$

The first term is the amplitude modulation and the second term, dF, is the phase modulation. The power of the second term is

$$\begin{aligned} dP &= \iint dx\,dy \left|E_{ref}^0\right|^2 4k^2 f^2 \\ &\approx P_{ref}^0 4k^2 \iint dx\,dy f^2 \\ &= P_{ref}^0 4k^2 \sigma^2 S \\ &= P_{ref}^0 \left(\frac{4\pi\sigma}{\lambda}\right)^2 S \end{aligned} \tag{20}$$

where the variation of the field is assumed to be small compared to the variation of f, S is the area of the integration and σ^2 is defined as follows:

$$\sigma^2 = \frac{\iint dx\,dy f^2}{S} \tag{21}$$

The mean value of f is assumed to be 0, which corresponds to a displacement of the reflection plane.

When you use Eq.(5) for the power (i.e., $2/\pi w^2$), σ=h and $S=a^2$, this is the same result as Eq.(13), the result using a far field calculation based on Fraunhofer approximation.

When the aberration f is expressed using Fourier expansion as

$$f(x,y)=\sum_{nx,ny} a_{nx,ny}\sin(n_x\omega_x x+n_y\omega_y y+\varphi_{nx,ny}) \tag{22}$$

the phase modulation term, dF, can be written as follows:

$$\begin{aligned} dF &= E^0_{ref}\exp(i\omega t-ikz)i2kf \\ &= E^0_{ref}k\sum_{nx,ny} a_{nx,ny}(\exp(i\Phi^+_{nxny})-\exp(i\Phi^-_{nxny})) \\ \Phi^0 &= \omega t-kz \\ \Phi^\pm_{nxny} &\equiv \Phi^0\pm(n_x\omega_x x+n_y\omega_y y+\varphi_{nx,ny}) \end{aligned} \tag{23}$$

The time evolution of the field wave front can be trace by requiring the phase to be constant. The phase of the amplitude modulated component is Φ^0, and the field is moving along the z direction. The phase modulated part has two phases, $\Phi^\pm$, and each component of the field is going with an opening angle of

$$\sqrt{\omega_x^2+\omega_y^2}\,/\,k \sim n\cdot\lambda\,/\,a \tag{24}$$

where λ is the wavelength of the field, a is a typical spatial wavelength characterize the structure an n is a number larger than 1.

After a propagation of distance L, the scattered light will be away from the beam axis by

$$L\cdot\lambda\,/\,a \tag{25}$$

For an aberration whose size is less than 1mm, the phase modulated components miss the target mass more than L/1000 m, much larger than the test mass size. So it will be good assumption that the energy of the phase modulated term is the loss due to the aberration.

8 Summary

Effects caused by localized anomalies in a FP cavity are studied using SIS. Localized anomalies can disturb the entire field. The effect of shape anomaly seems to be more problematic than the effect of the loss anomaly.

The surface deformation due to local heating was calculated using the surface deformation map by Muzzamil. If the point absorption rate is more than 1% of the nominal absorption rate, the round trip loss may become too large to affect the arm performance.

9 References

1. H. Yamamoto, “SIS (Stationary Interferometer Simulation) manual”, LIGO-T070039
2. Jean-Yves Vinet, “Losses caused by a point-like surface defect”, April 2, 2010, private communication.