LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

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Effects of small size anomalies in a FP cavity

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1 Introduction

This note discusses the effect of small size defects on the HR surface in a FP cavity, analyzed using SIS. In this revision, (a) the loss round trip loss and (b) resonating field shapes are calculated when, in a small area on ETM, (1) the loss is not uniform or (2) the surface has a bump.

2 Optical system

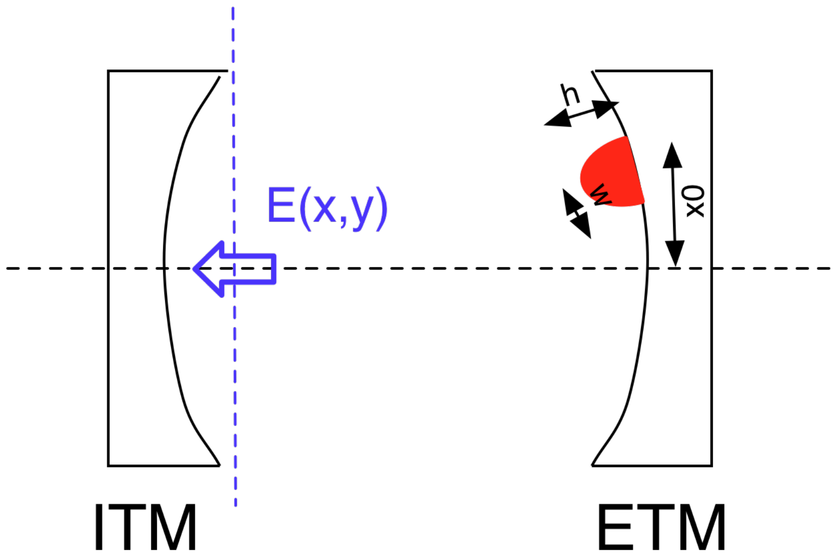

Figure 1. FP cavity and small anomaly

The optical system studied is the advanced LIGO arm with an anomaly placed on ETM. The shape of the anomaly is of Gaussian shape and is placed at (x=x0, y=0) and the anomaly is 0 out of 2w, i.e.,

$$\begin{aligned} d(x,y) &= h \cdot \exp(-\frac{(x-x_0)^2+y^2}{w^2}) : \sqrt{(x-x_0)^2+y^2} < 2w \\ &= 0 : \sqrt{(x-x_0)^2+y^2} > 2w \end{aligned} \quad (1)$$

A TEM00 mode field, which matches with the clean FP cavity, i.e., without the anomaly, is injected to the cavity and the stationary field is calculated.

The round trip loss without the anomaly is 0.6ppm, due to the finite apertures of ITM and ETM.

Field shapes shown below are the one on ITM surface coming from ETM, shown by an arrow in Fig.1. The effect is very small and fields called “deviation” is defined as

$$\delta(x,y) \equiv \frac{E(x,y)}{E(0,0)} - \frac{E_0(x,y)}{E_0(0,0)} \quad (2)$$

where $E_0(x,y)$ is the stationary state field without the anomaly and E(x,y) is the one with the anomaly. E_0 is a almost clean TEM00 mode, and δ is the structure added by the anomaly, which is normalized by the amplitude at the center.

3 Loss anomaly

The effect of loss due to a point defect is calculated using a localized loss with the Gaussian shape. When the loss is of the Gaussian shape, the total loss on one bounce is calculated to be

$$Loss \approx P_{cav}(\frac{w_a}{w_{beam}})^2 2h \cdot \exp(-\frac{2x_0^2}{w_a^2}) \quad (3)$$

when the size of the anomaly, w_a, is much smaller than the beam size, w_{beam}.

FFT-based simulation cannot simulate effects caused by point structures, which is smaller than the FFT grid size, which is a fraction of mm at the smallest.

In order to see how good the approximation is, the round trip loss was calculated using three sets of (w_a, h) which give same loss values: (w_a, h) = (2mm, 0.0025), (1mm, 0.01), (0.5mm, 0.04). For all cases, x_0 = 2cm, and w_{beam} = 6.2cm. The losses calculated using these values were 4.8ppm, 4.7ppm and 4.8ppm, which is comparable to the analytic point loss using Eq.(3), 4.2ppm.

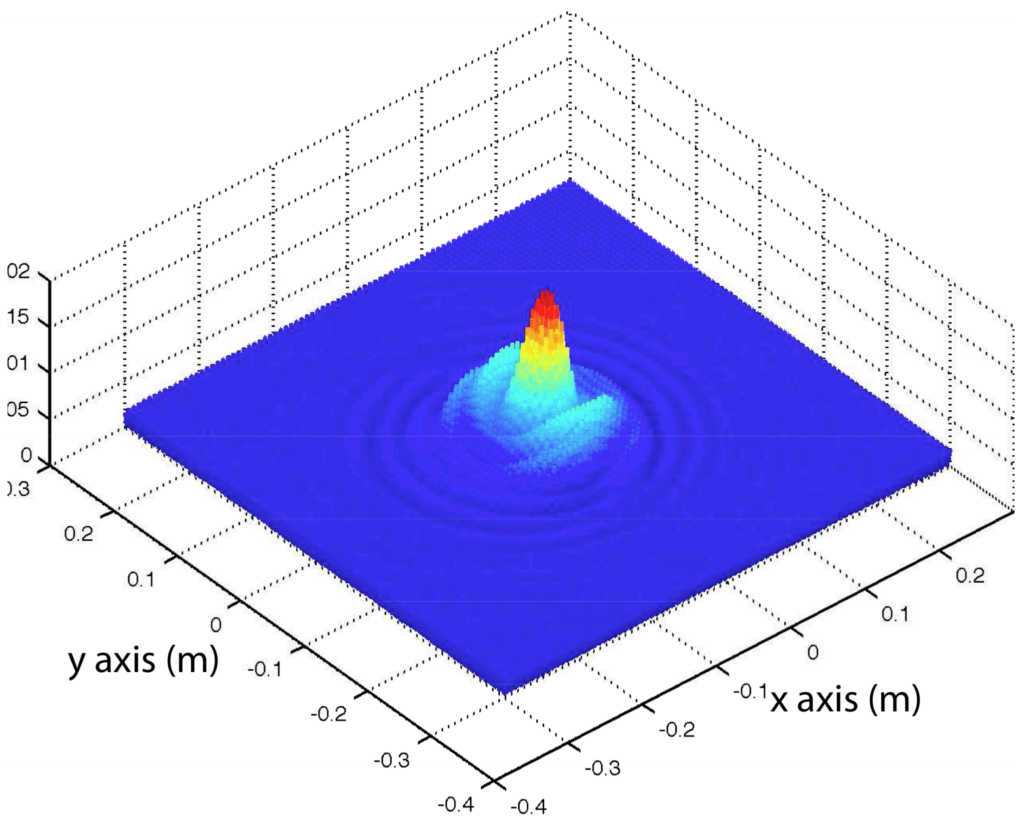

Figure 2. Deviation with loss anomaly

Fig.2 is the deviation field when a loss anomaly with (w_a, h) = (2mm, 0.0025) is placed at x_0=2cm. As is seen from the figure, the effect of a localized anomaly affects the entire beam surface. The same effect is observed when there is a localized bump, which is discussed in the following section.

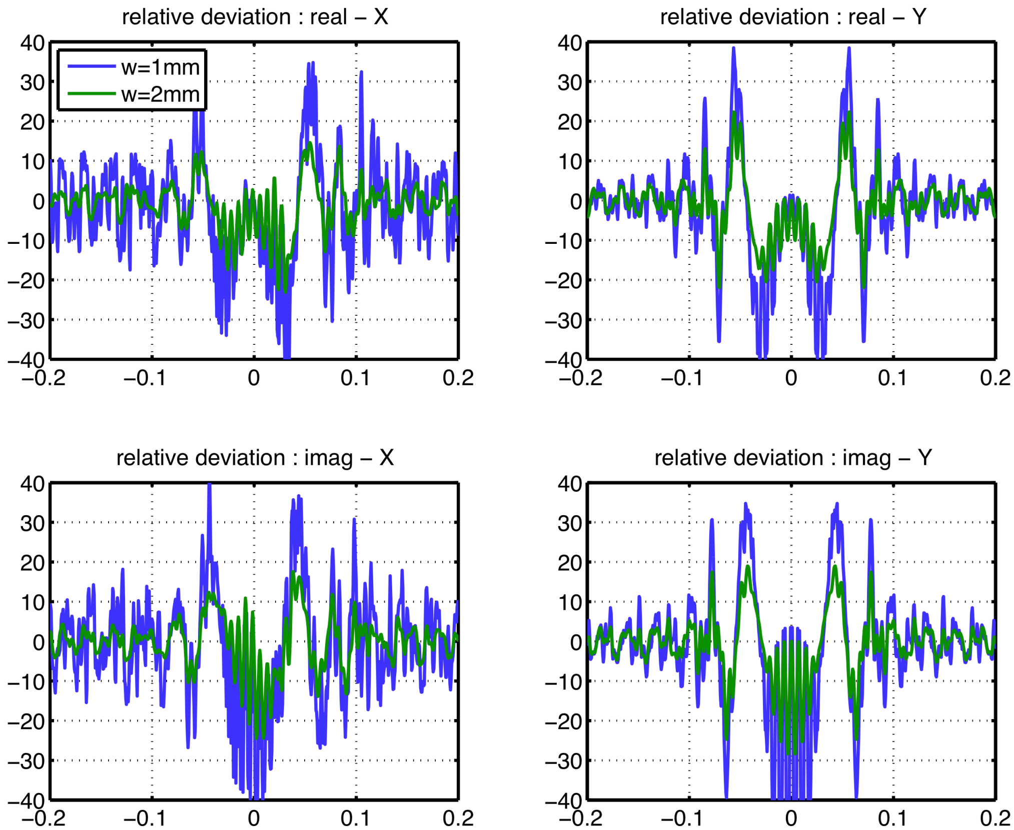

Figure 3. Deviations of real and imaginary component (units in ppm)

Fig.3 shows the real and imaginary components of the deviations. E.g., the top-left plot is real part of $\delta(x,0)$. The spatial wavelengths of the structure is determined by the location of the anomaly, i.e., 2cm in the case, both in x and y directions.

The point scattering loss measured at Caltech lab using a small size laser (200 μm) was mostly less than 100ppm. On ETM, this point scattering loss will scatter out $< (200\ \mu m/6.2cm)^2$ x 100ppm ~ 10^{-9}. This is over 1000 times smaller than the example case used to create plots in Fig.3. So the deviation by anomalous loss will change the field amplitude only by 10^{-7}.

It is necessary to understand when there are many point scatterings how they affect collectively.

4 Shape anomaly

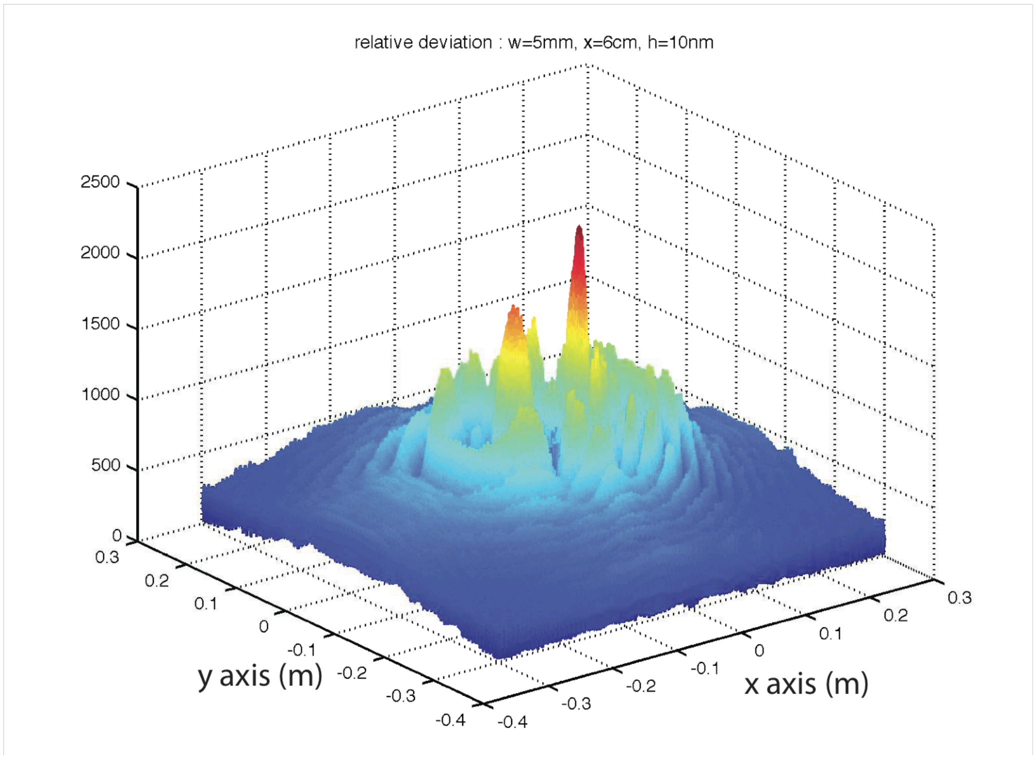

Figure 4. Deviation with shape anomaly

Fig.4 is the deviation when a Gaussian bump with height of 10nm, width of 5mm at location (x=6cm, y=0). As is observed above, the bump affects the entire beam, not localized around the bump.

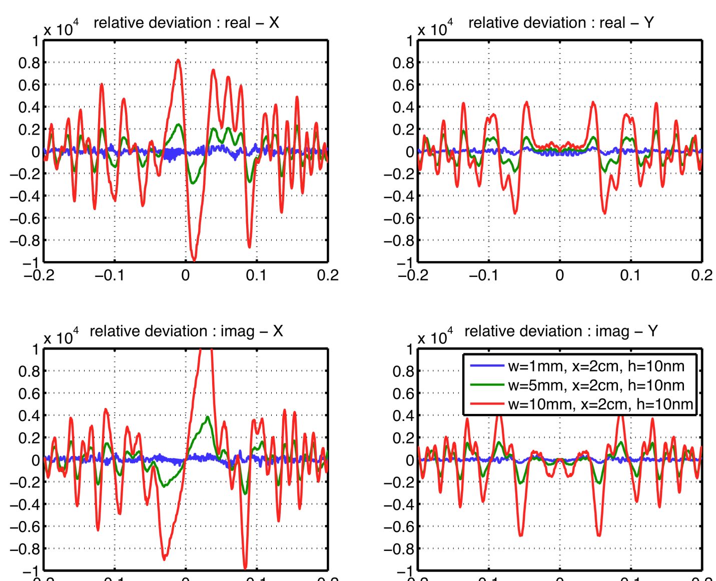

Figure 5. Deviations with different widths

Fig.5 shows the real and imaginary part of deviations when the dumps are placed at 2cm away from the center. Three lines show deviations with different sizes, 1mm, 5mm and 10mm, with same height of 10nm. For each width, the round trip loss is 3.5ppm, 68ppm and 210ppm. The round trip loss is roughly proportional to the square of the height, and if the height is 1nm, the loss values are ~1/100 of these values. These loss values and deviation magnitudes are much larger than the case of the effect of the loss anomaly.

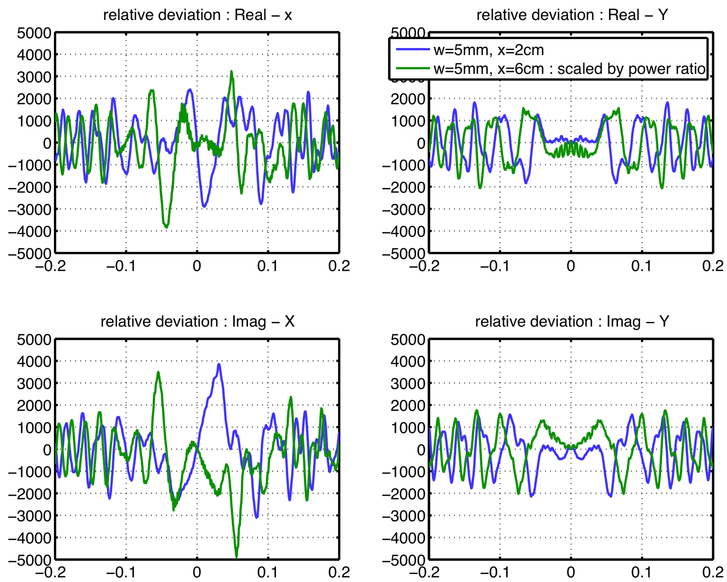

Figure 6. Deviations with different bump locations

Fig. 6 shows the deviations when a bump (width 5mm, height of 10nm) is placed at 2cm and at 6cm. The blue line (x_0=2cm) shows shorter spatial wavelength structure than the green line (x_0=6cm).

5 Thermal deformation in a small region

Muzammil calculated thermal surface shape deformation by the absorption in a small region. Small beams hearted at (x=2cm,y=0cm) and the total absorption is 0.425W or 0.5ppm at the full aLIGO arm power. The beam size chosen are 0.5mm, 1mm and 2mm.

Fig. 7 and Fig.8 shows the surface deformation when the absorption is 1/100 of 0.5ppm. Fig.7 is the 3D plot for the beam size is 1mm, and Fig.8 compares the shapes for the beam size of 1mm and 2mm. For a larger beam size, the surface deformation is also broader.

Fig.9 shows the field distortion defined in Eq.(2) when 1mm beam is absorbed at 0.01 x 0.5ppm rate. As were discussed in previous sections, the point absorption distorted the field in the entire are, not just a limited region.

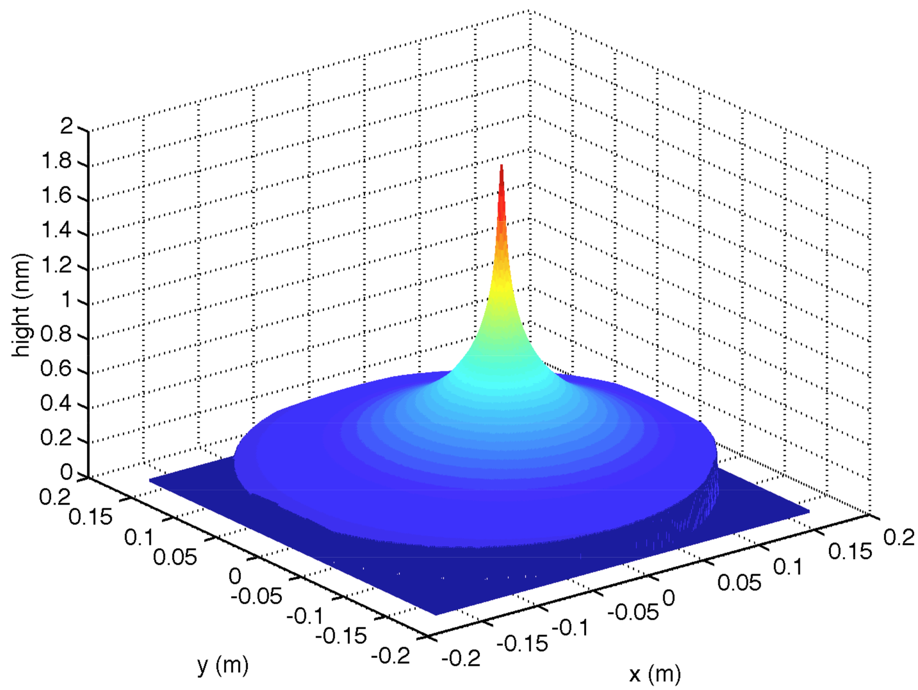

Figure 7 Surface deformation in 3D (beam size 1mm)

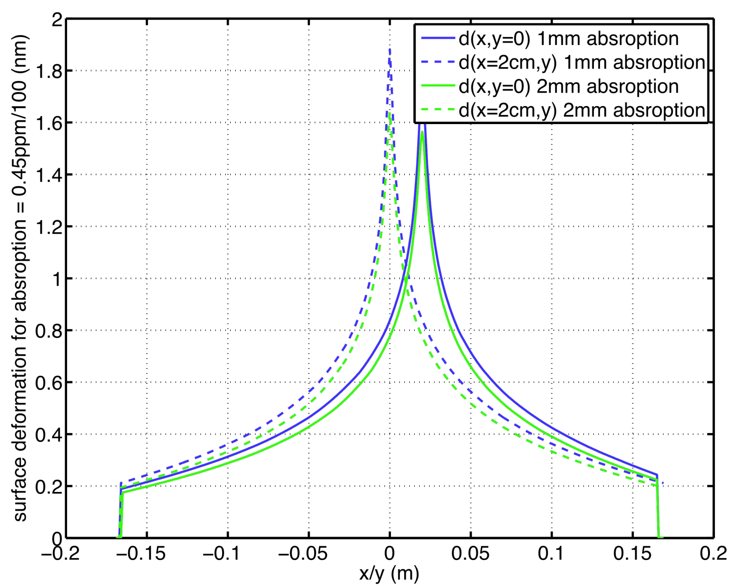

Figure 8 Surface deformation in 2D

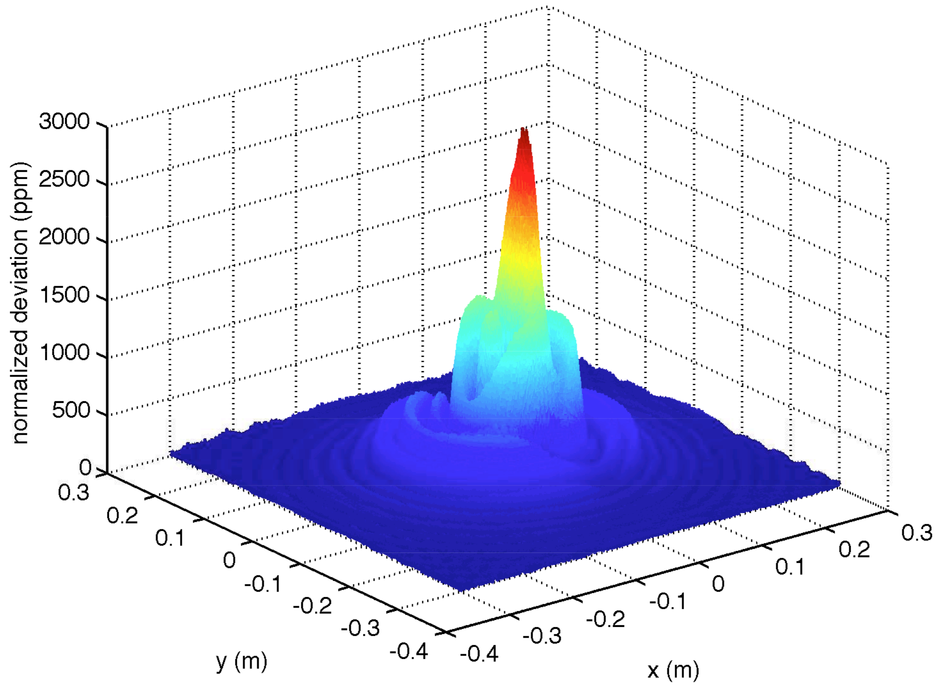

Figure 9 Field on ITM with 1mm heating, 1/100x0.5ppm absorption

Fig.10 shows the relation between the absorption loss vs round trip loss. As is seen from Fig.8, the surface distortion is in a region of a several mm with a height of a few nm when the absorption is 1/100 of 0.5ppm. This induces round trip loss of a few ppm. This is consistent with the loss calculated in the previous section using a Gaussian shape.

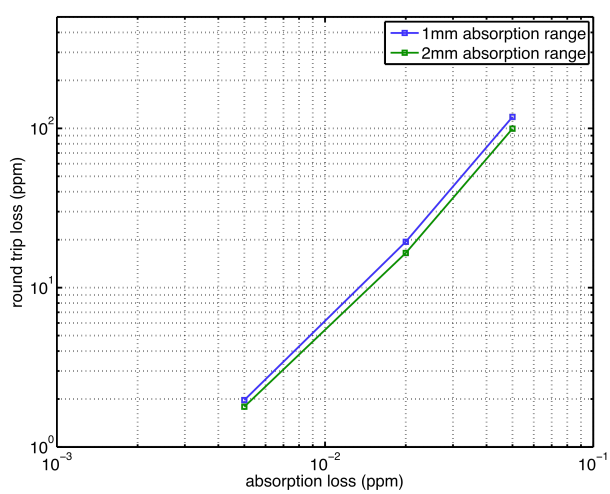

Figure 10 Absorption vs round trip loss

When this calculation was done, the transmittance and reflectance of ETM was changed in the small region to reflect the local loss due to absorption. But the round trip loss is almost unaffected by the change of the optical quantities (R and T), but is determined by the surface shape. This is also consistent with the generic calculation in Sec. 3.

6 Point scattering using Huygen's Integral

6.1 Near field using Huygen's integral

The Huyhen's integral of the field propagation from z_0 to z is expressed as follows.

$$\begin{aligned} &E(x,y,z) \equiv \frac{i}{\lambda}\iint dx_0\, dy_0 E_0(x_0,y_0,z_0)\frac{\exp(-ik\rho)}{\rho}\cos\theta \\ &\Delta x = x - x_0, \Delta y = y - y_0, L = z - z_0, k = 2\pi/\lambda \\ &\rho = \sqrt{\Delta x^2 + \Delta y^2 + L^2}, \cos\theta = \frac{L}{\rho} \end{aligned} \tag{4}$$

When a Gaussian field is reflected by a point defect, the reflected field is written as follows:

$$\begin{aligned} &E_0(x_0,y_0,z_0) = TEM00(z_0)\cdot\exp(2ik\delta(x_0,y_0)) \\ &TEM00(z_0) = \sqrt{\frac{2}{\pi}}\frac{1}{w_0}\exp(-\frac{x_0^2+y_0^2}{w_0^2}) \end{aligned} \tag{5}$$

For simplicity, the waist position is on the reflection surface and δ is the point defect.

When inserting Eq.(5) to Eq.(4), the reflected field can be written as follows keeping only the first order of δ:

$$E(x,y,z) = F_0(x,y,z) + dF(x,y,z) \tag{6}$$

$$F_0(x,y,z) = \frac{i}{\lambda}\iint dx_0\, dy_0 TEM00(z_0)\frac{\exp(-ik\rho)}{\rho}\cos\theta \tag{7}$$

$$\begin{aligned} dF(x,y,z) &= \sqrt{\frac{2}{\pi}}\frac{1}{w_0}\frac{i}{\lambda}\iint dx_0\, dy_0 (\exp(2ik\delta)-1)\exp(-\frac{x_0^2+y_0^2}{w_0^2})\frac{\exp(-ik\rho)}{\rho}\cos\theta \\ &\approx \sqrt{\frac{2}{\pi}}\frac{1}{w_0}\frac{i}{\lambda}\iint dx_0\, dy_0\, 2ik\delta(x_0,y_0)\exp(-\frac{x_0^2+y_0^2}{w_0^2})\frac{\exp(-ik\rho)}{\rho}\cos\theta \end{aligned} \tag{8}$$

F_0 is the unperturbed components and dF is the perturbed component by the point defect. The integral goes over the point defect where δ is non-zero in a region O(1μm). For simplify, the defect is placed at the center of the reflection point.

The field is calculated in a plane near the reflection surface with the following approximation.

$$\begin{aligned} &r \equiv \sqrt{x^2+y^2} \gg L \gg point\ source\ size \\ &\rho \approx \sqrt{r^2+L^2} \end{aligned} \tag{9}$$

With this approximation, dF can be calculated as follows:

$$
\begin{aligned}
dF(x,y,z) &\approx \sqrt{\frac{2}{\pi}}\frac{1}{w_0}\frac{i}{\lambda}2ik\frac{\exp(-ik\rho)}{\rho}\frac{L}{\rho}\iint dx_0\, dy_0 \delta(x_0,y_0) \\
&= -\sqrt{\frac{2}{\pi}}\frac{1}{w_0}\frac{4\pi}{\lambda^2}\frac{\exp(-ik\rho)}{\rho}\frac{L}{\rho}a^2 h \\
&= -\sqrt{\frac{2}{\pi}}\frac{a}{w_0}\frac{a}{\lambda}\frac{h}{\lambda}4\pi\frac{\exp(-ik\rho)}{\rho}\frac{L}{\rho} \\
a^2 h &= \iint dx_0\, dy_0 \delta(x_0,y_0)
\end{aligned}
\tag{9}
$$

With this approximation, the effect of the point source is to generate a wave from a point source independent of the shape of the point defect. The shape affects the magnitude of the perturbed field. The integral of the defect is expressed by a^2h. The only physical quantity is this product, but it is convenient to express this way so that typical size and height of the defect can be used for the estimation.

The perturbed field can go out of the target mirror and this is the loss. For the overestimation, the total power of the perturbed field is calculated as the estimation of the loss. The above expression is not correct near the source, i.e., r ~ 1 μm, but so long as the field is smooth, the integral in this small region will not be large. Integration is carried out over the full surface.

$$
\begin{aligned}
po\mathrm{int}\,Loss &= \iint dS\left|dF(x,y,z)\right|^2 \\
&= \left(\sqrt{\frac{2}{\pi}}\frac{a}{w_0}\frac{a}{\lambda}\frac{h}{\lambda}4\pi\right)^2\iint dx\, dy\frac{L^2}{(r^2+L^2)^2} \\
&= 32\pi^2\left(\frac{a}{w_0}\right)^2\left(\frac{a}{\lambda}\right)^2\left(\frac{h}{\lambda}\right)^2
\end{aligned}
\tag{10}
$$

Using values, a=2μ, w_0=6cm, λ=1.064μ, and h=20nm, this point loss estimation comes out to be 4.4×10^{-4} ppm.

6.2 Fresnel approximation

With the Fresnel approximation, the Huyhen's integral becomes as follows.

$$
\begin{aligned}
E(x,y,z) &= \exp(-ikL)\cdot E_t(x,y,z) \\
E_t(x,y,z) &\equiv \frac{i}{L\cdot\lambda}\iint dx_0\, dy_0 E_0(x_0,y_0,z_0)\exp\left(-ik\frac{\Delta x^2+\Delta y^2}{2L}\right)
\end{aligned}
\tag{11}
$$

With this approximation, F_0 and dF become as follows:

$$
\begin{aligned}
F_0(x,y,z) &= \sqrt{\frac{2}{\pi}}\frac{1}{w_0}\frac{i}{L\cdot\lambda}\iint dx_0\, dy_0 \exp\left(-ik\frac{\Delta x^2+\Delta y^2}{2L}\right)\exp\left(-\frac{x_0^2+y_0^2}{w^2}\right) \\
&= \sqrt{\frac{2}{\pi}}\frac{1}{w(z)}\exp\left(-\frac{x^2+y^2}{w(z)^2}\right)\exp\left(i\eta(z)-i\frac{r^2}{2R(z)}\right) \\
&= TEM00(z)
\end{aligned}
\tag{7}
$$

$$dF(x,y,z)=\sqrt{\frac{2}{\pi}}\frac{1}{w_0}\frac{i}{L\cdot\lambda}\iint dx_0\,dy_0(\exp(2ik\delta)-1)\exp(-ik\frac{\Delta x^2+\Delta y^2}{2L})\exp(-\frac{x_0^2+y_0^2}{w_0^2})$$
$$\approx\sqrt{\frac{2}{\pi}}\frac{1}{w_0}\frac{i}{L\cdot\lambda}\iint dx_0\,dy_0 2ik\delta(x_0,y_0)\exp(-ik\frac{\Delta x^2+\Delta y^2}{2L})\exp(-\frac{x_0^2+y_0^2}{w_0^2}) \quad (8)$$

For the typical propagation distance (~km) and the beam size (~several cm), the expression of dF can be simplified as follows:

$$dF(x,y,z)=\sqrt{\frac{2}{\pi}}\frac{1}{w_0}\frac{i}{L\cdot\lambda}\exp(-ik\frac{x^2+y^2}{2L})2ik\iint dx_0\,dy_0\delta(x_0,y_0)$$
$$=-\sqrt{\frac{2}{\pi}}\frac{1}{w_0}\frac{4\pi}{L\cdot\lambda^2}\exp(-ik\frac{x^2+y^2}{2L})a^2h \quad (9)$$
$$=-\sqrt{\frac{2}{\pi}}\frac{a}{w_0}\frac{a}{\lambda}\frac{h}{\lambda}\frac{4\pi}{L}\exp(-ik\frac{x^2+y^2}{2L})$$

With this approximation, the effect of the point source is to generate a spherical wave independent of the shape of the point defect. The shape affects the magnitude of the perturbed field as was the case when the hear field was calculated.

The point loss is calculated to be:

$$po\,\text{int}\,Loss=\iint dS|dF(x,y,z)|^2$$
$$=2\pi L^2(\sqrt{\frac{2}{\pi}}\frac{a}{w_0}\frac{a}{\lambda}\frac{h}{\lambda}\frac{4\pi}{L})^2 \quad (10)$$
$$=64\pi^2(\frac{a}{w_0})^2(\frac{a}{\lambda})^2(\frac{h}{\lambda})^2$$

This is factor 2 larger than the hear field calculation, 8.8 x10^{-4} ppm.

The Fresnel calculation is valid in the forward region and the amplitude at the large angle can be in accurate. But the two calculation give similar results, and the results will be correct with in a factor, not order of magnitude error.

6.3 Total loss

The loss is proportional to the power hitting the point defect, and PL is the value estimated using the power at the origin. For N defects per mm^2 region, the total loss is estimated to be the following using the point loss (PL) = 4.4 x 10^{-4} ppm from the calculation in Sec.6.1.

$$totalLoss=\iint dx\,dyPL\cdot\exp(-\frac{2(x^2+y^2)}{w_0^2})\times N(1/m^2)$$
$$=PL\cdot\frac{\pi}{2}w_0^2N(1/m^2) \quad (11)$$
$$=2.5\times N(1/mm^2)ppm$$

7 Summary

Effects caused by localized anomalies in a FP cavity are studied using SIS. Localized anomalies can disturb the entire field. The effect of shape anomaly seems to be more problematic than the effect of the loss anomaly.

The surface deformation due to local heating was calculated using the surface deformation map by Muzzamil. If the point absorption rate is more than 1% of the nominal absorption rate, the round trip loss may become too large to affect the arm performance.

8 References

1. H. Yamamoto, "SIS (Stationary Interferometer Simulation) manual", LIGO-T070039