Quantum noise of a Michelson-Sagnac interferometer with translucent mechanical oscillator

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(Dated: October 9, 2009)

Abstract

Quantum fluctuation in the radiation pressure of light excites the motion of a mechanical oscillator. When performing a precise measurement of the position of an oscillator, this effect therefore results in quantum radiation pressure noise. Up to now this effect has not been observed experimentally yet. Recently, extremely thin SiN membranes (about 100 ng) attracted attentions for oscillator in radiation pressure noise measurement. Since the transmittance of this membrane is much lower than unity, the scheme that the membrane in Fabry-Perot cavity has already been proposed [Nature **452**, 72 (2008)]. However, the heat in the membrane and mirrors are a problem because the high incident power is necessary to enhance radiation pressure noise. We propose and theoretically analyze a Michelson-Sagnac interferometer, which includes an extremely light membrane as a common end mirror for the Michelson interferometer part. The mutually independent interferometric techniques of power- and signal-recycling amplify the radiation pressure noise even if the reflectance of the membrane has much lower than unity. This interferometer topology allows that build-ups of the signal from the oscillation of membrane and quantum fluctuation from outside of interferometer (signal-recycling) is separated from build-ups of the laser power (power-recycling). Thus, the incident power can be decreased in order to avoid heating of the membrane and beam splitter due to the absorption of light. We derived formulas for quantum radiation pressure noise and shot noise of the oscillator position measurement and compared them with theoretical models of the thermal noise of a SiN membrane with a fundamental resonant frequency of 75 kHz and an effective mass of 125 ng. We found that quantum radiation pressure noise should be observable with a incident power of 1 W on the interferometer and a membrane temperature of 1 K.

PACS numbers: 04.80.Nn, 95.35.+d, 95.55.Vj, 95.85.Ry

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I. INTRODUCTION

Laser interferometers belong to the most sensitive measurement devices ever built. The currently operated gravitational wave detectors achieve a linear noise spectral density for the differential position measurement of two mirrors as low as 10^{-23} $/\mathrm{Hz}^{1/2}$ [1]. The gravitational wave detectors of the second generation [2–4] are designed to have a ten time better sensitivity. The sensitivity of these interferometers will be limited by quantum radiation pressure noise [5] at low Fourier frequencies and by photon shot noise [5] at high frequencies. While the shot noise limited regime of laser interferometers has fully been investigated, the radiation pressure noise has not been observed yet. The experimental investigation of this quantum measurement regime is interesting in view of future gravitational wave detectors. It is also interesting from the fundamental physics point of view because the observation of the quantum radiation pressure noise corresponds to the realization of a quantum measurement process in which one quantum system (the light) performs a measurement on another quantum system (a mechanical oscillator).

In order to observe the quantum radiation pressure noise, one may want to use a mechanical oscillator with low effective mass, high mechanical quality, and high optical quality (Ref. [6] as review). Recently, commercially available SiN membranes attract a lot of attention [7–9]. The membrane can have an effective mass of the order of 100 ng, a thickness of about 100 nm and a surface area of about 1 mm^2. Its measured mechanical quality value was 10^6 at room temperature and 10^7 at 300 mK at its fundamental resonant frequency (about 100 kHz) [8]. Even though the surface quality of these membranes is high, they can not be used as a mirror as power reflectance is only about 35%. A solution for this problem is to put the membrane between two high reflectance mirrors and establish an optical cavity that is resonant for the laser light [7, 9]. For a sufficiently short cavity, signals below the membrane resonant frequency are also (quasi) resonant and enhanced in the cavity. The disadvantage of this approach is that the signal build-up cannot be adjusted independently from the power build-up. The signal build-up accompanies the power build-up, which leads to heat generation in optical components.

Here, we propose a Michelson-Sagnac interferometer [14–17] in order to access the radiation pressure noise regime of a translucent mechanical oscillator. This oscillator forms the joint end mirror of a Michelson interferometer (see Fig. 1), while the transmitted light estab-

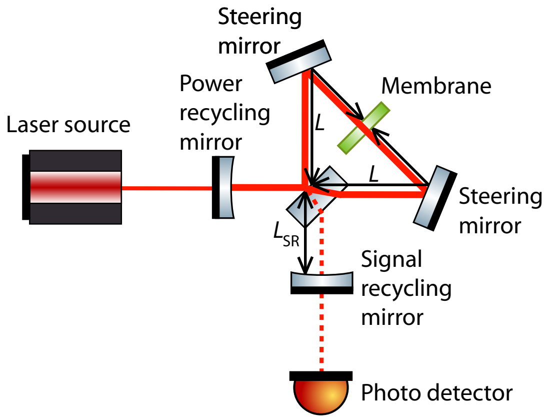

FIG. 1: Schematic view of a Michelson-Sagnac interferometer which consists of the membrane, a beam splitter, and two steering mirrors. The membrane is on the optical path of a Sagnac interferometer. A part of light is reflected by this membrane. This light contributes a signal of a Michelson interferometer. Power- and signal-recycling mirrors are put in input and output ports. In order to simplify the discussion, it is supposed that the length of two optical paths from the beam splitter and membrane is equal. The parameters L and L_{SR} are the arm length of Michelson interferometer and the distance between the beam splitter and signal-recycling mirror, respectively.

lishes a Sagnac interferometer. The Michelson-Sagnac interferometer enables the realization of power- and signal-recycling [10–13] to enhance the radiation pressure noise via additional mirrors at input and output ports of the interferometer even if the oscillator has high transmittance. Since the two recycling cavities are independent from each other, they can be used to create independent build-up factors for the laser power (power-recycling) and the oscillator signal and quantum fluctuation (signal-recycling). Especially, the signal-recycling enables one to use decent light power at the membrane and beam splitter. Comparing the power spectral density of radiation pressure noise, shot noise, and thermal noise, we found that the observation of radiation pressure noise should be possible for an oscillator temperature around 1 K.

II. QUANTUM NOISE OF MICHELSON-SAGNAC INTERFEROMETER

Quantum noise of the Michelson-Sagnac interferometer is different from that of a simple Michelson interferometer. In the case of latter, only the back action of the reflected light must be considered. On the contrary, in the Michelson-Sagnac interferometer, not only the reflected and transmitted light but also interference between them must be taken into account.

A. Photon shot noise of membrane displacement measurement

A Sagnac interferometer with a 50/50 beam splitter reflects all the light back to the input port. The fraction reflected at the membrane also recombines at the beam splitter. The intensity at the output port depends on the position of the membrane. It is assumed that the displacement of the membrane from lock position is smaller than the wavelength of light. The intensity at the output port is given by (using Taylor expansion)

$$
\begin{aligned}
I_{\mathrm{out}} &= \frac{r^2 I_0}{2}\left[1+\cos\left(\Phi_0+\frac{8\pi}{\lambda}x\right)\right] \\
&\sim \frac{r^2 I_0}{2}\left[1+\cos(\Phi_0)-\left(\frac{8\pi}{\lambda}x\right)\sin(\Phi_0)\right],
\end{aligned} \tag{1}
$$

where I_0 is the power at the beam splitter, r is the amplitude reflectance of the membrane, Φ_0 is the phase of the lock point, x is the displacement of the membrane from the lock point, and λ is the wavelength of light. It must be noted that the differential length change of the arms is two times larger than the displacement of the membrane. The one-sided linear spectral density of the shot noise of light power at the output port G_{out} is described as [18]

$$
\sqrt{G_{\mathrm{out}}} = \sqrt{2\hbar\omega_0 I_{\mathrm{out}}} = \sqrt{\frac{4\pi\hbar c I_{\mathrm{out}}}{\lambda}}, \tag{2}
$$

where $\hbar$ is the reduced Planck constant, $\omega_0(=2\pi c/\lambda)$ is the angular frequency of light, and c is the speed of light. The signal-normalized shot noise is then given by

$$
\begin{aligned}
\sqrt{G_{\mathrm{shot}}} &= \sqrt{\frac{4\pi\hbar c I_{\mathrm{out}}}{\lambda}}\left|\left.\frac{\partial I_{\mathrm{out}}}{\partial x}\right|_{x=0}\right|^{-1} = \sqrt{\frac{2\pi\hbar c r^2 I_0(1+\cos\Phi_0)}{\lambda}}\frac{2}{r^2 I_0}\frac{\lambda}{8\pi|\sin\Phi_0|} \\
&= \sqrt{\frac{\hbar c\lambda}{16\pi r^2 I_0}}\frac{1}{|\sin(\Phi_0/2)|}.
\end{aligned} \tag{3}
$$

Hence, for $\Phi_0 = \pi$, which corresponds to the dark fringe; no light at the output port, the signal-to-noise ratio is optimized for a given power

$$
\sqrt{G_{\mathrm{shot}}} = \sqrt{\frac{\hbar c\lambda}{16\pi r^2 I_0}}. \tag{4}
$$

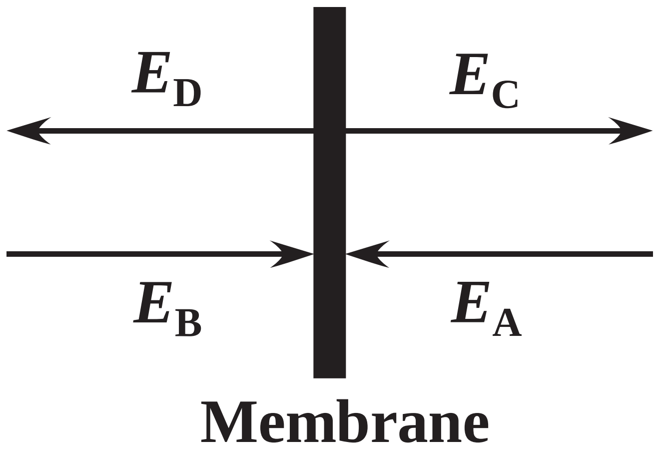

FIG. 2: Input and output scheme of the membrane. Four arrows represent the incident light ($E_{\rm A}, E_{\rm B}$) and outgoing interference of the reflected and transmitted light ($E_{\rm C}, E_{\rm D}$), respectively. These four fields give rise to radiation pressure effects at the membrane. The reflectance (transmittance) on both sides is the same because the membrane is symmetric. This implies that there is a 90 degree phase difference between the reflected and transmitted fields owing to energy conservation.

B. Quantum radiation pressure noise of membrane displacement measurement

The radiation pressure is the product of the light intensity and $1/c$. The intensity is the product of time average of square of light field and $\mathcal{A}c/(4\pi)$ (the value $\mathcal{A}$ is the effective cross section area). Since light is impinging on the membrane from two sides, we have to consider four light fields as shown in Fig. 2. It must be noted that the directions of radiation pressure force caused by $E_{\rm A}$ and $E_{\rm C}$ are opposite to those of $E_{\rm B}$ and $E_{\rm D}$. Therefore, the radiation pressure on the membrane is described as [19]

$$F_{\rm RP} = \frac{1}{c} \times \frac{\mathcal{A}c}{4\pi} \left(\overline{|E_{\rm A}|^2} - \overline{|E_{\rm B}|^2} + \overline{|E_{\rm C}|^2} - \overline{|E_{\rm D}|^2} \right). \tag{5}$$

DC-components of radiation pressure cancel because of the symmetry of the membrane. If the fluctuation of $E_{\rm A}$ equal to that of $E_{\rm B}$ (perfectly positive correlation), radiation pressure forces on both sides of the membrane cancel each other. However, vacuum fluctuations enter the interferometer from the output port. This results in quantum amplitude fluctuation via the interference with the carrier light. In this case, there is perfectly negative correlation between the fluctuations of $E_{\rm A}$ and $E_{\rm B}$ because of the energy conservation at the beam splitter [5]. This is the origin of quantum radiation pressure noise for a Michelson-Sagnac interferometer.

The incident light field amplitudes are [19],

$$E_{\rm A} = \frac{1}{\sqrt{2}}\sqrt{\frac{4\pi\hbar\omega_0}{\mathcal{A}c}}\left[\sqrt{2}D + E_{\rm v1}\right]\cos(\omega_0 t) + \frac{1}{\sqrt{2}}\sqrt{\frac{4\pi\hbar\omega_0}{\mathcal{A}c}}E_{\rm v2}\sin(\omega_0 t), \tag{6}$$

$$E_{\rm B} = \frac{1}{\sqrt{2}}\sqrt{\frac{4\pi\hbar\omega_0}{\mathcal{A}c}}\left[\sqrt{2}D - E_{\rm v1}\right]\cos(\omega_0 t) - \frac{1}{\sqrt{2}}\sqrt{\frac{4\pi\hbar\omega_0}{\mathcal{A}c}}E_{\rm v2}\sin(\omega_0 t). \tag{7}$$

The parameter D is the amplitude for the carrier. The time development of $E_{\rm v1}$ and $E_{\rm v2}$ represents quantum fluctuations in amplitude and phase caused by vacuum from the output port. The relation between D and the incident power I_0 is described as

$$I_0 = 2 \times \frac{\overline{E_{\rm A}}^2}{4\pi}\mathcal{A}c = 2 \times \frac{\overline{E_{\rm B}}^2}{4\pi}\mathcal{A}c = \hbar\omega_0 D^2. \tag{8}$$

The reason why the factor 2 appears in Eq. (8) is that $E_{\rm A}$ and $E_{\rm B}$ represent the light as it is split by the beam splitter.

The complex transmittance of the membrane depends on the reflectance owing to energy conservation ($\overline{E_{\rm A}^2} + \overline{E_{\rm B}^2} = \overline{E_{\rm C}^2} + \overline{E_{\rm D}^2}$). Since the membrane is symmetric, there is a 90 degree phase difference between the reflected and transmitted fields. The outgoing fields are written as

$$\begin{aligned} E_{\rm C} &= \frac{1}{\sqrt{2}}\sqrt{\frac{4\pi\hbar\omega_0}{\mathcal{A}c}}\left[\sqrt{2}tD - tE_{\rm v1} - rE_{\rm v2}\right]\cos(\omega_0 t) \\ &+ \frac{1}{\sqrt{2}}\sqrt{\frac{4\pi\hbar\omega_0}{\mathcal{A}c}}\left[\sqrt{2}rD + rE_{\rm v1} - tE_{\rm v2}\right]\sin(\omega_0 t), \end{aligned} \tag{9}$$

$$\begin{aligned} E_{\rm D} &= \frac{1}{\sqrt{2}}\sqrt{\frac{4\pi\hbar\omega_0}{\mathcal{A}c}}\left[\sqrt{2}tD + tE_{\rm v1} + rE_{\rm v2}\right]\cos(\omega_0 t) \\ &+ \frac{1}{\sqrt{2}}\sqrt{\frac{4\pi\hbar\omega_0}{\mathcal{A}c}}\left[\sqrt{2}rD - rE_{\rm v1} + tE_{\rm v2}\right]\sin(\omega_0 t). \end{aligned} \tag{10}$$

The parameters r and t are amplitude reflectance and transmittance of the membrane.

Substituting Eqs. (6), (7), (9), and (10) in Eq. (5), we obtain the quantum radiation pressure force on the membrane;

$$F_{\rm RP} = \frac{2}{c}\sqrt{2\hbar\omega_0 I_0}r^2 E_{\rm v1} - \frac{2}{c}\sqrt{2\hbar\omega_0 I_0}rtE_{\rm v2}. \tag{11}$$

The functions $E_{\rm v1}$ and $E_{\rm v2}$ are random processes. They are uncorrelated and their one-sided power spectral densities are unity [20]. The linear spectrum of the radiation pressure is described as

$$\sqrt{G_{F_{\rm RP}}} = \sqrt{\left(\frac{2}{c}\sqrt{2\hbar\omega_0 I_0}r^2\right)^2 + \left(\frac{2}{c}\sqrt{2\hbar\omega_0 I_0}rt\right)^2} = \sqrt{\frac{16\pi\hbar r^2 I_0}{c\lambda}}. \tag{12}$$

The motion of the membrane caused by this force is

$$\sqrt{G_{\rm rad}} = H\sqrt{\frac{16\pi\hbar r^2 I_0}{c\lambda}}, \tag{13}$$

$$H = \left|\frac{1}{-m(2\pi f)^2 + m(2\pi f_{\rm mem})^2(1+{\rm i}f/(Qf_{\rm mem}))}\right|. \tag{14}$$

The function H shows the mechanical response of the membrane, defined by its mass ($m_{\rm mem}$), resonant frequency ($f_{\rm mem}$), and Q-value ($Q_{\rm mem}$).

C. Standard quantum limit

The standard quantum limit (SQL) of the Michelson-Sagnac interferometer is written as

$$\sqrt{G_{\rm shot} + G_{\rm rad}} \geq \sqrt{2\sqrt{G_{\rm shot}G_{\rm rad}}} = \sqrt{2\hbar H} = \sqrt{G_{\rm SQL(MS)}}. \tag{15}$$

This is $\sqrt{2}$ times smaller than the SQL of a Michelson interferometer $\sqrt{4\hbar H}$ [18] and one half of that of a Fabry-Perot Michelson interferometer $\sqrt{8\hbar H}$ (e.g. Refs. [19, 21]). The difference between these interferometer topologies is the number of mirrors. Since H implies the response of a mirror, the SQL of an interferometer having n mirrors is

$$\sqrt{G_{\rm SQL}} = \sqrt{2\hbar n H}. \tag{16}$$

If the mechanical responses of the mirrors are different from each other, nH must be replaced by the summation $\sum H_n$.

D. Power- and signal-recycling techniques

If the Michelson-Sagnac interferometer is operated in the dark fringe, its reflectance is almost unity even though the membrane has a low reflectance. This enables to use interferometric techniques to increase quantum radiation pressure noise, namely power- and signal-recycling [10–13]. They are realized via additional mirrors (see Fig. 1) that together with the Michelson-Sagnac interferometer forms cavities for carrier light (power-recycling) and signals and vacuum fluctuation (signal-recycling). In this paper, we consider only tuned signal recycling (signal recycling cavity is tuned to the carrier).

In the case of the power-recycling [10, 11], the recycling cavity enhances the incident (carrier) power by factor $G_{\rm PR}$, which is the power-recycling (energy) gain. The incident power I_0 in formulas of shot noise and radiation pressure noise, Eqs. (4) and (13), is replaced by $G_{\rm PR}I_0$. In the case of the signal-recycling [12, 13], the amplitude of sidebands caused by the membrane motion ($|\partial I_{\rm out}/\partial x|$ in Eq. (3)) and the vacuum fluctuations from the output port are amplified by $\sqrt{G_{\rm SR}}$, which is the signal-recycling (amplitude) gain.

It must be noted that the sideband frequency of the signal and the corresponding vacuum fluctuation should be smaller than the signal-recycling cavity linewidth $f_{\rm SR}$. If this sideband frequency is larger than $f_{\rm SR}$, signal sidebands and vacuum fluctuation are not amplified by cavity. Shot noise increases. On the other hand, radiation pressure noise decreases. This cut-off frequency $f_{\rm SR}$ depends on the summation of the distance between beam splitter and signal-recycling mirror $L_{\rm SR}$ and the arm length L (distance between the beam splitter and membrane) defined in Fig. 1. Formulas of shot noise and radiation pressure noise with power- and signal-recycling are written as [22, 23]

$$\sqrt{G_{\rm shot}} = \sqrt{\frac{\hbar c\lambda}{16\pi G_{\rm PR}G_{\rm SR}r^2 I_0}}\sqrt{1+\left(\frac{f}{f_{\rm SR}}\right)^2}, \tag{17}$$

$$\sqrt{G_{\rm rad}} = H\sqrt{\frac{16\pi\hbar G_{\rm PR}G_{\rm SR}r^2 I_0}{c\lambda}}\frac{1}{\sqrt{1+\left(\frac{f}{f_{\rm SR}}\right)^2}}, \tag{18}$$

$$f_{\rm SR} = \frac{c(1-r_{\rm SR})}{4\pi(L_{\rm SR}+L)}. \tag{19}$$

If the reflectance of the signal recycling mirror is almost unity, the recycling gains are

$$G_{\rm PR} = \frac{1+r_{\rm PR}}{1-r_{\rm PR}}, \tag{20}$$

$$G_{\rm SR} = \frac{1+r_{\rm SR}}{1-r_{\rm SR}}, \tag{21}$$

$$\tag{22}$$

where $r_{\rm PR}$ and $r_{\rm SR}$ are amplitude reflectance of power- and signal-recycling mirrors, respectively.

III. SPECIFICATIONS TO MEASURE RADIATION PRESSURE NOISE

In this section, the design of Michelson-Sagnac interferometer to observe radiation pressure noise is discussed. We compared two different schemes to reach a quantum radiation pressure noise dominated regime, a Michelson-Sagnac interferometer with and without

TABLE I: Specifications of Michelson-Sagnac interferometer.

Light wavelength (λ)	1064 nm
Arm length of Michelson interferometer (L)	0.6 m
Length between the signal-recycling mirror and beam splitter ($L_{\rm SR}$)	3 cm
Amplitude reflectance of signal-recycling mirror ($r_{\rm SR}$)	0.998
Signal-recycling (amplitude) gain ($\sqrt{G_{\rm SR}}$)	32
Power at beam splitter (I_0)(with/without signal recycling)	1 W/1 kW
Power reflectance of membrane (r^2)	0.35
Resonant frequency of membrane ($f_{\rm mem}$)	75 kHz
Effective mass of membrane ($m_{\rm mem}$)	125 ng
Q-value of membrane ($Q_{\rm mem}$)	10^7
Temperature of membrane ($T_{\rm mem}$)	1 K

signal-recycling and found that signal-recycling technique is promising. For the parameters given in Table I (Ref. [8] and our measurement) the predictions for quantum noise and thermal noise are shown in Fig. 3. For the case without signal recycling, a power of 1 kW is needed, which means that power recycling becomes necessary. If signal recycling is used, this value can be decreased to 1 W. Both types show the same quantum noise below the membrane resonant frequency of 75 kHz. In this frequency region, radiation pressure noise is at least two times larger than shot noise. Signal recycling has the potential to decrease the thermal load in the optics without decreasing the radiation pressure noise. For frequencies higher than the cut-off frequency of the signal recycling cavity (set to $f_{\rm SR} = 76$ kHz), shot noise increases. If the signal-recycling gain increases, $f_{\rm SR}$ becomes lower. Equations (18), (21), and (19) show that radiation pressure noise is independent of the signal-recycling gain above the cut-off frequency. Thus, the radiation pressure noise near 100 kHz does not increase effectively even though the signal recycling gain increases.

Figure 3 also includes linear spectrums of thermal noise. We considered two different kinds of dissipation, namely viscous and structural damping [24]. The thermal noise formulas are

$$G_{\rm thermal} = |H(f)|^2 \frac{4k_{\rm B}T_{\rm mem}m_{\rm mem}(2\pi f_{\rm mem})}{Q_{\rm mem}} \text{ (viscous damping)}, \tag{23}$$

$$G_{\rm thermal} = |H(f)|^2 \frac{4k_{\rm B}T_{\rm mem}m_{\rm mem}(2\pi f_{\rm mem})^2}{Q_{\rm mem}(2\pi f)} \text{ (structural damping)}, \tag{24}$$

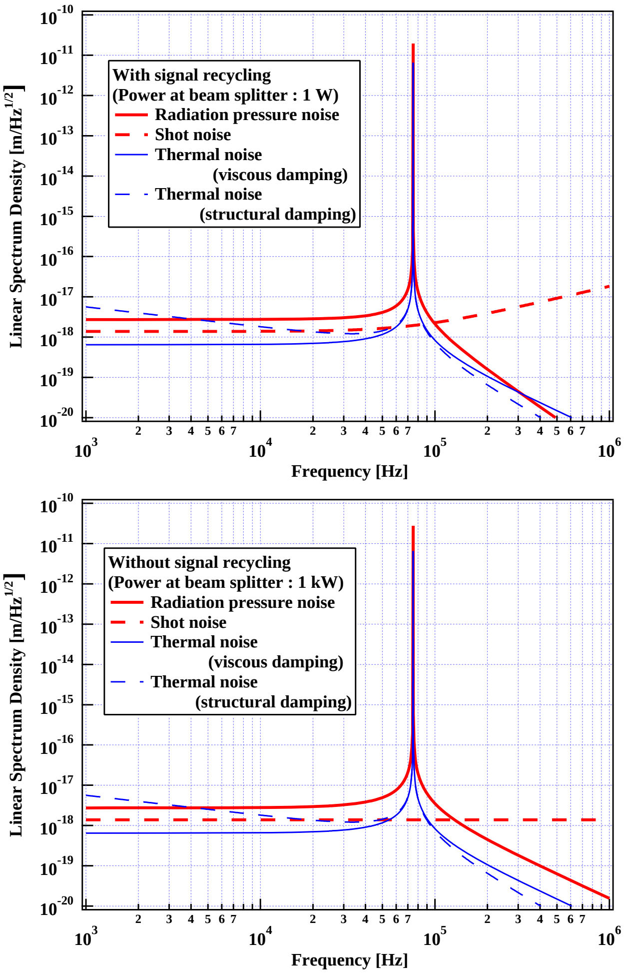

FIG. 3: Goal sensitivity of the Michelson-Sagnac interferometer to measure radiation pressure noise. The graphs on the left and right hand sides show the sensitivity with and without signal-recycling, respectively. Thick solid and dashed lines (red in online) are the radiation pressure noise and shot noise, respectively. Thin solid and dashed lines (blue in online) are thermal noise in the cases of the viscous damping and structural damping[24].

where k_{B} and T_{mem} are the Boltzmann constant and temperature of the membrane. For Fig. 3 it is assumed that a temperature of 1 K because thermal noise decreases below radiation pressure noise by a factor of three around the membrane resonant frequency. At room temperature thermal noise becomes 50 times larger because the quality factor of the membrane is 10 times smaller (Ref. [8] and our room temperature measurement). In this case, the power at the beam splitter must be 3 kW even if signal-recycling is adopted. Since higher laser power leads to thermal problems caused by absorption, a cryogenic cooling of the membrane seems necessary.

IV. SUMMARY AND CONCLUSION

We have analyzed the quantum noise of a Michelson-Sagnac interferometer containing a light translucent mechanical oscillator in order to measure quantum radiation pressure noise. This interferometer topology is compatible with power- and signal-recycling techniques to increase the radiation pressure noise even if the oscillator transmits most of the incident laser light. We have presented noise formulas for shot noise and radiation pressure noise for a position measurement of a translucent membrane. The expressions differ from those of a usual Michelson interferometer because of the interference between the beams reflected and transmitted by the membrane. We have found that the radiation pressure noise is twice as large as the shot noise below the membrane resonant frequency for a laser power of 1 W at the beam splitter and a signal-recycling amplitude gain of 32. If the membrane temperature is 1 K, the calculated thermal noise of the oscillator fundamental mode is below the radiation pressure noise. A higher signal-recycling gain reduces the laser power required for pushing the radiation pressure noise above the shot-noise. This might turn out to be important in order to reduce the heating of the membrane and beam splitter through absorbed laser power.

Acknowledgments

This work is supported by the Deutsche Forschungsgemeinschaft and is part of Sonderforschungsbereich 407. We are grateful to Yanbei Chen for useful comments.

[1] S.E. Whitcomb, Class. Quantum Grav. **25**, 114013 (2008).

[2] P. Fritschel, in *Proceedings of the SPIE meeting Gravitational-Wave Detection (4856-39), Waikoloa, Hawaii, 2002*, edited by P. Saulson and M. Cruise (International Society for Optical Engineering, WA, 2002), p. 282.

[3] F. Acernese *et al.*, J. Opt. A - Pure Appl. Opt. **10**, 064009 (2008).

[4] K. Kuroda *et al.*, Prog. Theor. Phys. Suppl. **163**, 54 (2006).

[5] C.M. Caves, Phys. Rev. Lett. **45**, 75 (1980).

[6] T.J. Kippenberg and K.J. Vahala, Science **321**, 1172 (2008).

[7] J.D. Thomson *et al.*, Nature **452**, 72 (2008).

[8] B.M. Zwickl *et al.*, Appl. Phys. Lett. **92**, 103125 (2008).

[9] A.M. Jayich *et al.*, New J. of Phys. **10**, 095008 (2008).

[10] R.W.P. Drever *et al.*, *"Quantum Optics, Experimental Gravitation, and Measurement Theory"*, ed. P. Meystre and M.O. Scully (Plenum Press, New York, 1983).

[11] R.W.P. Drever, *"Gravitational Radiation"*, ed. N. Deruelle and T. Piran (North-Holland, Amsterdam, 1983).

[12] J.Y. Vinet, B. Meers, C.N. Man, and A. Brillet, Phys. Rev. D **38**, 433 (1988).

[13] B.J. Meers, Phys. Rev. D **38** 2317 (1988).

[14] Y. Chen and P. Purdue, Phys. Rev. D **66** 122004 (2002).

[15] Y. Chen, Phys. Rev.D **67** 122004 (2003).

[16] S. L. Danilishin, Phys. Rev. D **69** 102003 (2004).

[17] Y. Chen, S. L. Danilishin, F. Ya. Khalili, and H. Mueller-Ebhardt, arXiv:0910.0319, submitted to General Relativity and Gravitation.

[18] W.A. Edelstein, J. Hough, J.R. Pugh, and W. Martin, J. Phys. E: Sci. Instrum. **11** 710 (1978).

[19] H.J. Kimble, Yuri Levin, A.B. Matsko, K.S. Thorne, and S.P. Vyatchanin, Phys. Rev. D **65**, 022002 (2001).

[20] V.B. Braginsky *et al.*, Phys. Rev. D **67**, 082001 (2003).

[21] V.B. Braginsky *et al.*, Phys. Rev. D **67**, 082001 (2003).

[22] P. Fritschel, D. Shoemaker, and R. Weiss, Appl. Opt. **31** 1412 (1992).

[23] K. Strain and B.J. Meers, Phys. Rev. Lett. **66**, 1391 (1991).

[24] P.R. Saulson, Phys. Rev. D **42**, 2437 (1990).