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COHERENT BAYESIAN INFERENCE ON BINARY INSPIRAL SIGNALS

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Presented is a Bayesian parameter estimation method for the analysis of interferometric gravitational wave observations of an inspiral of binary compact objects using data recorded simultaneously by a network of several interferometers at different sites. We consider neutron star or black hole inspirals that are modeled to 3.5 post-Newtonian (PN) order in phase and 2.5 PN in amplitude. Inference is facilitated using Markov chain Monte Carlo methods that are adapted in order to efficiently explore the particular parameter space. Nine parameters are estimated, including those associated with the binary system, plus its location on the sky. We explain how this technique will be part of a detection pipeline for compact objects with masses up to 20 $M_\odot$, including binary systems with very large mass ratios.

1 Introduction

A world-wide network of interferometric gravitational wave detectors is now on-line. LIGO has reached its target sensitivity [1], and Virgo is fast approaching theirs [2]. Compact binary systems will certainly produce gravitational waves, and they are likely to be one of the most promising sources. The LIGO Scientific Collaboration (LSC) [3] and Virgo [4,5] each have search pipelines for binary inspiral events, and studies have shown that these pipelines have equivalent detection capabilities, and soon the LSC and Virgo will be conducting collaborative searches [6].

The purpose of a binary inspiral detection pipeline is to find a signal within the data. Once researchers suspect that a signal is present then parameter estimation techniques can be applied in order to produce estimates and summary statistics for the astrophysical parameters. Bayesian Markov chain Monte Carlo (MCMC) methods [7] are well suited for this problem, especially since it is possible to produce accurate predictions for the form of the signal. MCMC parameter estimation techniques have been developed for binary neutron star inspirals, as seen by a single interferometer [8]. In addition, MCMC methods have been developed for the coherent analysis of data from a world-wide network of interferometers [9].

A difficult detection scenario involves finding a signal produced by a binary system where the mass ratio between the two objects is large. In such a case, the signal will likely have its amplitude significantly modulated, and it will be necessary to use higher-order post-Newtonian (PN) approximations. Here we summarize our method for producing parameter estimates associated with a binary inspiral modeled to 3.5 post-Newtonian (PN) order in phase, and 2.5 PN in amplitude [10,12]. We employ new and more advanced MCMC methods, such as evolutionary MCMC [14]. The higher order PN templates will also allow for examination of signals where the amplitude is modulated, as may be the case with rather large ratios between the masses of the compact objects. Finally, we see this MCMC program as part of a larger detection pipeline for signals from binary inspirals with large mass ratios, and individual masses going up to 20 $M_{\odot}$. We imagine, for example, using an existing detection pipeline [5] to generate a reasonable number of triggers; the MCMC would then analyze each of the triggers in detail. Once the MCMC has reached convergence, an estimate for the signal parameters would be produced.

2 Coherent MCMC for Binary Inspiral Signals

We follow a Bayesian approach in order to do inference on the inspiral signal's parameters. Bayesian inference depends on evaluating the parameters' posterior distribution, which is given in terms of the (non-normalized) posterior density, in our case a function of 9 parameters. Typically, one will be interested in figures such as posterior means, confidence bounds, or marginal densities for individual parameters, which require integration of the posterior over the parameter space. This problem is commonly approached using Monte Carlo integration, i.e. by simulating random draws from the posterior distribution, and then approximating the desired integrals by sample statistics (means by averages, etc.). The most popular algorithms for this purpose are Markov chain Monte Carlo (MCMC) samplers that simulate a random walk through parameter space whose stationary distribution is the posterior distribution [15,7]. For our purpose we used a basic Metropolis sampler that we recently upgraded to an *evolutionary MCMC* algorithm [14], a generalization that is motivated by genetic algorithms [16]. This extension offers substantial improvement over the previously employed parallel tempering [9] and yielded a sampler that reliably converged towards the true posterior distribution in the examples discussed below.

Our simulated data consist of simultaneous measurements from several interferometric detectors, superimposed with interferometer-specific Gaussian noise. The signal waveform that was injected into and recovered from the data was implemented using a 3.5 post-Newtonian (PN) approximation for the phase evolution [10,17], and a 2.5 PN model for the amplitude [12]. The 9 parameters determining the responses at different interferometers are: individual masses (m_1, m_2), luminosity distance (d_L), inclination angle (ι), coalescence phase (ϕ_0), coalescence time at geocenter (t_c), declination (δ), right ascension (α) and polarization angle (ψ).

We applied non-informative priors on the 'geometrical' parameters that describe the inspiral event's location and orientation. The coalescence time t_c is assumed to be known in advance up to a certain accuracy from the detection pipeline that would in reality precede such an analysis [5]; here we set the prior to be uniform across ± 5 ms around the time-trigger value. The prior for the masses (m_1, m_2) reflects the distribution of the masses among binary inspirals, which could be based on observational evidence as well as theoretical considerations. For now, we simply defined it as uniform across a range of 1–10 $M_{\odot}$. Assuming that inspirals happen uniformly across space leads to a prior $P(d_L \leq x) \propto x^3$ for the luminosity distance d_L. This is an improper prior, seemingly implying there was an 'infinite' probability for 'infinitely remote' inspiral events. It is also unrealistic, since an inspiral event needs to happen within a certain range in order to be detectable. We incorporated this restriction into the prior specification by considering the *detection probability* of an inspiral event, depending on the signal-to-noise ratio (SNR).

We implemented the MCMC sampler as a basic Metropolis algorithm [15,7] that was then extended to a parallel tempering algorithm. The 'tempering' here works as in a simulated annealing algorithm, and prevents MCMC chains from getting stuck in local modes of the posterior distribution. Parallel tempering is the special case of a Metropolis-coupled MCMC (MCMCMC) algorithm [7], where several tempered MCMC chains, each at different temperatures, are run in parallel, and additional proposals are introduced to 'swap' parameter sets between chains [18,9]. This algorithm can be further refined by implementing elements of genetic algorithms [16]. The set of parallel chains may be thought of as constituting a 'population' whose individuals may be crossed to form 'hybrids' that inherit properties from both 'parental' chains, the result being an evolutionary MCMC algorithm [14]. The 'crossovers' between sets of parameters were implemented as *real crossovers*, in which offsprings are formed by randomly reassembling the parental parameter sets, as well as *snooker crossovers*, in which a new offspring is proposed somewhere on the straight line connecting the two parental points in parameter space [19]. We applied our MCMC routine to a simulated data set, corresponding to an inspiral signal that is received at three interferometers, specifically the two 4-km LIGO detectors at Hanford (LHO) and Livingston (LLO), and the 3-km Virgo interferometer near Pisa (V). The simulated inspiral involved masses of $m_1 = 2\,\mathrm{M}_\odot$ and $m_2 = 5\,\mathrm{M}_\odot$ (chirp mass $m_c = 2.70\,\mathrm{M}_\odot$, mass ratio $\eta = 0.204$), observed from a distance of $d_L = 30\,\mathrm{Mpc}$ at $t_c = 700\,009\,012.345$ GPS seconds. For the synthesized data that we use the noise characteristics were assumed to match the target sensitivities for LIGO and Virgo [20]. The resulting SNRs [9] at the three sites were 8.4 (LHO), 10.9 (LLO), 6.4 (V); the network SNR was 15.2. Figure 1 shows the marginal posterior distributions for several individual parameters in comparison to the true values for the injected signal. While some

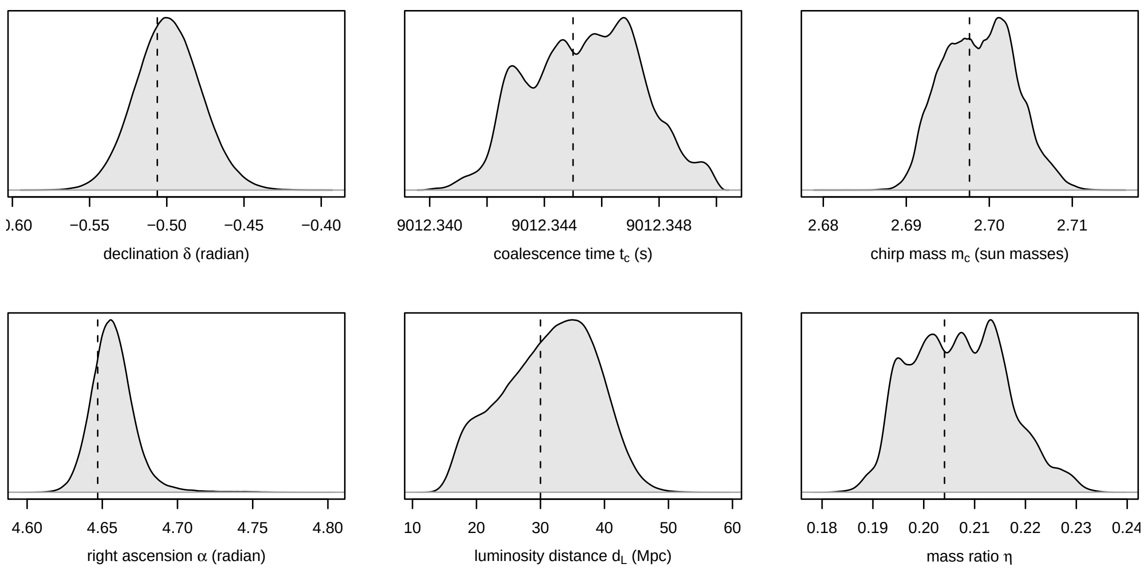

Figure 1: Marginal joint posterior densities for some of the parameters. Dashed lines indicate the true parameter values.

of the distributions appear roughly Gaussian, others are clearly not; some even possess multiple modes. This illustrates some of the strengths of a fully Bayesian approach: no approximations to the posterior's (or likelihood's) shape are made, an irregular posterior surface does not pose a problem, and the assessment of relative importance of multiple modes arises naturally [9].

3 Discussion

We have presented a description of our coherent MCMC code for estimating nine parameters associated with a binary inspiral signal detected by a network of interferometric detectors.

This program uses time-domain inspiral templates that are 3.5 PN in phase and 2.5 PN in amplitude. New MCMC techniques, such as evolutionary MCMC and genetic algorithms, have been implemented in our code. The code can be applied to inspiral signals where the masses of the components can be as large as 20 $M_{\odot}$; inspirals with large mass ratios can also be successfully analyzed. This code is part of a large mass ratio inspiral detection pipeline that we are currently developing; a *loose-net* inspiral detection pipeline (using, for example, lower order PN templates) will generate a reasonable number of triggers, and this MCMC will then be applied to those times where triggers were recorded. The next logical extension of our binary inspiral MCMC work will be to systems with spin; this is currently an area of active research for us.

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References

1. D. Sigg. *Classical and Quantum Gravity* **21**, S409 (2004).
2. F. Acernese et al. *Classical and Quantum Gravity* **23**,S63 (2006).
3. B. Abbott et al. *Physical Review D* **69**, 122001 (2004).
4. F. Marion et al. In *Proceedings of the Rencontres de Moriond 2003*. Editions Frontiáeres, Gif-sur-Yvette, France (2004).
5. P. Amico et al. *Computer Physics Communications* **153**, 179 (2003).
6. F. Beauville et al. Detailed comparison of LIGO and Virgo inspiral pipelines in preparation for a joint search. *gr-qc/0701027* (2007).
7. W. R. Gilks, S. Richardson, and D. J. Spiegelhalter. *Markov chain Monte Carlo in practice*. (Chapman & Hall / CRC, Boca Raton, 1996).
8. C. Röver, R. Meyer, and N. Christensen. *Classical and Quantum Gravity* **23**, 4895 (2006).
9. C. Röver, R. Meyer, and N. Christensen. *Physical Review D* **75**, 062004 (2007).
10. L. Blanchet, G. Faye, B. R. Iyer, and B. Joguet. *Physical Review D* **65**, 061501 (2002). Note the erratum [11].
11. L. Blanchet, G. Faye, B. R. Iyer, and B. Joguet. *Phys. Rev. D* **71**, 129902 2005. (See [10]).
12. K. G. Arun, L. Blanchet, B. R. Iyer, and M. S. S. Qusailah. *Classical and Quantum Gravity* **21**, 3771 (2004). Note the erratum [13].
13. K. G. Arun, L. Blanchet, B. R. Iyer, and M. S. S. Qusailah. *Classical and Quantum Gravity* **22**, 3115 (2005). (See also [12]).
14. F. Liang and H. W. Wong. *Journal of the American Statistical Association* **96**, 653 (2001).
15. A. Gelman, J. B. Carlin, H. Stern, and D. B. Rubin. *Bayesian data analysis*. (Chapman & Hall / CRC, Boca Raton, 1997).
16. D. E. Goldberg. *Genetic algorithms in search, optimization, and machine learning*. (Addison-Wesley, Reading, Mass., 1989).
17. L. Blanchet, T. Damour, G. Esposito-Farèse, and B. R. Iyer. *Physical Review Letters* **93**, 091101 (2004).
18. K. Hukushima and K. Nemoto. *Journal of the Physical Society of Japan* **65**, 1604 (1996).
19. W. R. Gilks, G. O. Roberts, and E. I. George. Adaptive direction sampling. *The Statistician* **43**, 179 (1994).
20. F. Beauville et al. *Classical and Quantum Gravity* **22**, S1149 (2005).