The importance of the ”magnetic” components of gravitational waves in the response functions of interferometers

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Abstract

With an enlighting treatment Baskaran and Grishchuk have recently shown the presence and importance of the so-called “magnetic” components of gravitational waves (GWs), which have to be taken into account in the context of the total response functions of interferometers for GWs propagating from arbitrary directions. In this paper more detailed angular and frequency dependences of the response functions for the magnetic components are given in the approximation of wavelength much larger than the linear dimensions of the interferometer, with a specific application to the parameters of the LIGO and Virgo interferometers. Our results agree with the work of Baskaran and Grishchuk in which it has been shown that the identification of “electric” and “magnetic” contributions is unambiguous in the long-wavelenght approximation. At the end of this paper the angular and frequency dependences of the total response functions of the LIGO and Virgo interferometers are given. In the high-frequency regime the division on “electric” and “magnetic” components becomes ambiguous, thus the full theory of gravitational waves has to be used. Our results are consistent with the ones of Baskaran and Grishchuk in this case too.

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1 Introduction

The design and construction of a number of sensitive detectors for GWs is underway today. There are some laser interferometers like the VIRGO detector, being built in Cascina, near Pisa by a joint Italian-French collaboration [1, 2], the GEO 600 detector, being built in Hanover, Germany by a joint Anglo-Germany collaboration [3, 4], the two LIGO detectors, being built in the United States (one in Hanford, Washington and the other in Livingston, Louisiana) by a joint Caltech-Mit collaboration [5, 6], and the TAMA 300 detector, being built near Tokyo, Japan [7, 8]. There are many bar detectors currently in operation too, and several interferometers and bars are in a phase of planning and proposal stages.

The results of these detectors will have a fundamental impact on astrophysics and gravitation physics. There will be lots of experimental data to be analyzed, and theorists will be forced to interact with lots of experiments and data analysts to extract the physics from the data stream.

Detectors for GWs will also be important to confirm or ruling out the physical consistency of General Relativity or of any other theory of gravitation [9, 10, 11, 12]. This is because, in the context of Extended Theories of Gravity, some differences from General Relativity and the others theories can be seen starting by the linearized theory of gravity [9, 10, 12].

With an enlighting treatment, recently, Baskaran and Grishchuk have shown the presence and importance of the so-called "magnetic" components of GWs, which have to be taken into account in the context of the total response functions (angular patterns) of interferometers for GWs propagating from arbitrary directions [13]. In this paper more detailed angular and frequency dependences of the response functions for the magnetic components are given in the approximation of wavelength much larger than the linear dimensions of the interferometer, with a specific application to the parameters of the LIGO and Virgo interferometers. Our results agree with the work of [13] in which it has been shown that the identification of "electric" and "magnetic" contributions is unambiguous in the long-wavelenght approximation. At the end of this paper the angular and frequency dependences of the total response functions of the LIGO and Virgo interferometers are given. In the high-frequency regime the division on "electric" and "magnetic" components become ambiguous, thus the full theory of gravitational waves has to be used [13]. Our results are consistent with the ones of [13] in this case too.

2 Analysis in the frame of the local observer

In a laboratory enviroment on earth, the coordinate system in which the space-time is locally flat is typically used [12, 13, 15, 16, 17] and the distance between any two points is given simply by the difference in their coordinates in the sense of Newtonian physics. In this frame, called the frame of the local observer, GWs manifest themself by exerting tidal forces on the masses (the mirror and

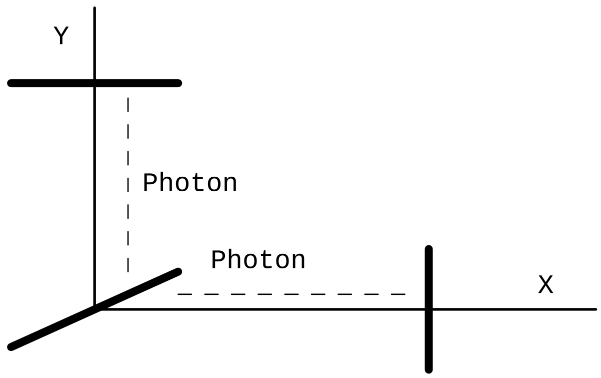

Figure 1: photons can be launched from the beam-splitter to be bounced back by the mirror

the beam-splitter in the case of an interferometer, see figure 1).

A detailed analysis of the frame of the local observer is given in ref. [15], sect. 13.6. Here we remember only the more important features of this frame:

the time coordinate x_0 is the proper time of the observer O;

spatial axes are centered in O;

in the special case of zero acceleration and zero rotation the spatial coordinates x_j are the proper distances along the axes and the frame of the local observer reduces to a local Lorentz frame: in this case the line element reads

$$ds^2 = -(dx^0)^2 + \delta_{ij} dx^i dx^j + O(|x^j|^2) dx^\alpha dx^\beta; \tag{1}$$

the effect of GWs on test masses is described by the equation for geodesic deviation in this frame

$$\ddot{x}^i = -\widetilde{R}^i_{0k0} x^k, \tag{2}$$

where we have called $\widetilde{R}^i_{0k0}$ the linearized Riemann tensor [15].

Recently Baskaran and Grishchuk have shown the presence and importance of the so-called magnetic component of GWs and have computed the detector pattern in the low frequencies approximation [13]. Actually a more detailed angular and frequency dependences of the response functions for the magnetic components can be given in the same approximation (i.e. wavelength much larger than the linear dimensions of the interferometer), with a specific application to the parameters of the LIGO and Virgo interferometers.

Before starting with the analysis of the response functions of interferometers, a brief review of Section 3 of [13] is necessary to understand the importance of the "magnetic" components of GWs. In this paper we use different notations with respect the ones used in [13]. We work with $G = 1$, $c = 1$ and $\hbar = 1$ and we call $h_+(t_{tt} + z_{tt})$ and $h_\times(t_{tt} + z_{tt})$ the weak perturbations due to the $+$ and the $\times$ polarizations of the GW which are expressed in terms of syncrony coordinates $t_{tt}, x_{tt}, y_{tt}, z_{tt}$ in the transverse-traceless (TT) gauge. In this way the most general GW propagating in the z_{tt} direction can be written in terms of plane monochromatic waves [15, 16, 17, 18]

$$\begin{aligned} h_{\mu\nu}(t_{tt}+z_{tt}) &= h_+(t_{tt}+z_{tt})e_{\mu\nu}^{(+)} + h_\times(t_{tt}+z_{tt})e_{\mu\nu}^{(\times)} = \\ &= h_{+0}\exp i\omega(t_{tt}+z_{tt})e_{\mu\nu}^{(+)} + h_{\times 0}\exp i\omega(t_{tt}+z_{tt})e_{\mu\nu}^{(\times)}, \end{aligned} \tag{3}$$

and the correspondent line element will be

$$ds^2 = dt_{tt}^2 - dz_{tt}^2 - (1+h_+)dx_{tt}^2 - (1-h_+)dy_{tt}^2 - 2h_\times dx_{tt}dx_{tt}. \tag{4}$$

The wordlines $x_{tt}, y_{tt}, z_{tt} = const$ are timelike geodesics which represent the histories of free test masses [15, 17]. The coordinate transformation $x^\alpha = x^\alpha(x_{tt}^\beta)$ from the TT coordinates to the frame of the local observer is [13, 19]

$$\begin{aligned} t &= t_{tt} + \tfrac{1}{4}(x_{tt}^2 - y_{tt}^2)\dot{h}_+ - \tfrac{1}{2}x_{tt}y_{tt}\dot{h}_\times \\ x &= x_{tt} + \tfrac{1}{2}x_{tt}h_+ - \tfrac{1}{2}y_{tt}h_\times + \tfrac{1}{2}x_{tt}z_{tt}\dot{h}_+ - \tfrac{1}{2}y_{tt}z_{tt}\dot{h}_\times \\ y &= y_{tt} + \tfrac{1}{2}y_{tt}h_+ - \tfrac{1}{2}x_{tt}h_\times + \tfrac{1}{2}y_{tt}z_{tt}\dot{h}_+ - \tfrac{1}{2}x_{tt}z_{tt}\dot{h}_\times \\ z &= z_{tt} - \tfrac{1}{4}(x_{tt}^2 - y_{tt}^2)\dot{h}_+ + \tfrac{1}{2}x_{tt}y_{tt}\dot{h}_\times. \end{aligned} \tag{5}$$

In eqs. (5) it is $\dot{h}_+ \equiv \frac{\partial h_+}{\partial t}$ and $\dot{h}_\times \equiv \frac{\partial h_\times}{\partial t}$. The coefficients of this transformation (components of the metric and its first time derivative) are taken along the central wordline of the local observer [13, 14, 19]. We emphasize that, in refs. [13, 19] it has been shown that the linear and quadratics terms, as powers of x_{tt}^α, are unambiguously determined by the conditions of the frame of the local observer while the cubic and higher-order corrections are not determined by these condictions, thus, at high-frequenies, the expansion in terms of higher-order corrections breaks down [13, 14].

Considering a free mass riding on a timelike geodesic ($x = l_1$, $y = l_2$, $z = l_3$) [13] eqs. (5) define the motion of this mass with respect the introduced frame of the local observer. It is

$$\begin{aligned} x(t) &= l_1 + \tfrac{1}{2}[l_1h_+(t) - l_2h_\times(t)] + \tfrac{1}{2}l_1l_3\dot{h}_+(t) + \tfrac{1}{2}l_2l_3\dot{h}_\times(t) \\ y(t) &= l_2 - \tfrac{1}{2}[l_2h_+(t) + l_1h_\times(t)] - \tfrac{1}{2}l_2l_3\dot{h}_+(t) + \tfrac{1}{2}l_1l_3\dot{h}_\times(t) \\ z(t) &= l_3 - \tfrac{1}{4[}(l_1^2 - l_2^2)\dot{h}_+(t) + 2l_1l_2\dot{h}_\times(t), \end{aligned} \tag{6}$$

which are exactly eqs. (13) of [13] rewritten using our notation. In absence of GWs the position of the mass is (l_1, l_2, l_3). The effect of the GW is to drive the mass to have oscillations. Thus, in general, from eqs. (6) all three components of motion are present [13].

Neglecting the terms with $\dot{h}_+$ and $\dot{h}_\times$ in eqs. (6) the "traditional" equations for the mass motion are obteined [15, 17, 18]

$$\begin{array}{c} x(t) = l_1 + \frac{1}{2}[l_1 h_+(t) - l_2 h_\times(t)] \\ \\ y(t) = l_2 - \frac{1}{2}[l_2 h_+(t) + l_1 h_\times(t)] \\ \\ z(t) = l_3. \end{array} \tag{7}$$

Cleary, this is the analogue of the electric component of motion in electrodinamics [13], while equations

$$\begin{array}{c} x(t) = l_1 + \frac{1}{2}l_1 l_3 \dot{h}_+(t) + \frac{1}{2}l_2 l_3 \dot{h}_\times(t) \\ \\ y(t) = l_2 - \frac{1}{2}l_2 l_3 \dot{h}_+(t) + \frac{1}{2}l_1 l_3 \dot{h}_\times(t) \\ \\ z(t) = l_3 - \frac{1}{4[}(l_1^2 - l_2^2)\dot{h}_+(t) + 2l_1 l_2 \dot{h}_\times(t), \end{array} \tag{8}$$

are the analogue of the magnetic component of motion. One could think that the presence of these magnetic components is a "frame artefact" due to the transformation (5), but it has to be emphasized that in Section 4 of [13] eqs. (6) have been obteined directly by the geodesic deviation equation too, thus the magnetic components have a really physical significance. The fundamental point of [13] is that the magnetic component becomes important when the frequency of the wave increases, like it is shown in Section 3 of [13], but only in the low-frequencies regime. This can be understood directly from eqs. (6). In fact, using eqs. (3) and eqs. (5), eqs. (6) become

$$\begin{array}{c} x(t) = l_1 + \frac{1}{2}[l_1 h_+(t) - l_2 h_\times(t)] + \frac{1}{2}l_1 l_3 \omega h_+(t) + \frac{1}{2}l_2 l_3 \omega h_\times(t) \\ \\ y(t) = l_2 - \frac{1}{2}[l_2 h_+(t) + l_1 h_\times(t)] - \frac{1}{2}l_2 l_3 \omega h_+(t) + \frac{1}{2}l_1 l_3 \omega h_\times(t) \\ \\ z(t) = l_3 - \frac{1}{4[}(l_1^2 - l_2^2)\omega h_+(t) + 2l_1 l_2 \omega h_\times(t). \end{array} \tag{9}$$

This also means that the terms with $\dot{h}_+$ and $\dot{h}_\times$ in eqs. (6) can be neglectet only when the wavelenght goes to infinity [13] while at high-frequencies, the expansion in terms of $\omega l_i l_j$ corrections, with $i = 1, 2, 3$, breaks down [13, 14].

Now let us compute the total response functions of interferometers for the magnetic components.

Equations (6), that represent the coordinates of the mirror of the interferometer in presence of a GW in the frame of the local observer, can be rewritten for the pure magnetic component of the + polarization as

$$\begin{array}{c} x(t) = l_1 + \frac{1}{2}l_1 l_3 \dot{h}_+(t) \\ \\ y(t) = l_2 - \frac{1}{2}l_2 l_3 \dot{h}_+(t) \\ \\ z(t) = l_3 - \frac{1}{4}(l_1^2 - l_2^2)\dot{h}_+(t), \end{array} \tag{10}$$

where l_1, l_2 *and* l_3 are the umperturbed coordinates of the mirror.

To compute the responce functions for an arbitrary incoming direction of the GW we have to remember that the arms of our interferometer are in the $\overrightarrow{u}$ and $\overrightarrow{v}$ directions, while the x, y, z frame is the frame of the local observer (i.e. the observer is assumed located in the position of the beam splitter) in phase with the frame of the propagating GW. Then we have to make a spatial rotation of our coordinate system:

$$\begin{aligned} u &= -x\cos\theta\cos\phi + y\sin\phi + z\sin\theta\cos\phi \\ v &= -x\cos\theta\sin\phi - y\cos\phi + z\sin\theta\sin\phi \\ w &= x\sin\theta + z\cos\theta, \end{aligned} \tag{11}$$

or, in terms of the x, y, z frame:

$$\begin{aligned} x &= -u\cos\theta\cos\phi - v\cos\theta\sin\phi + w\sin\theta \\ y &= u\sin\phi - v\cos\phi \\ z &= u\sin\theta\cos\phi + v\sin\theta\sin\phi + w\cos\theta. \end{aligned} \tag{12}$$

In this way the GW is propagating from an arbitrary direction $\overrightarrow{r}$ to the interferometer (see figure 2). Because the mirror of eqs. (10) is situated in the u direction, using eqs. (10), (11) and (12) the u coordinate of the mirror is given by

$$u = L + \frac{1}{4}L^2 A\dot{h}_+(t). \tag{13}$$

where we have defined

$$A \equiv \sin\theta\cos\phi(\cos^2\theta\cos^2\phi - \sin^2\phi), \tag{14}$$

and $L = \sqrt{l_1^2 + l_2^2 + l_3^2}$ is the lenght of the arms of the interferometer.

The computation for the v arm is parallel to the one above. Using eqs. (10), (11) and (12) the coordinate of the mirror in the v arm is:

$$v = L + \frac{1}{4}L^2 B\dot{h}_+(t), \tag{15}$$

where we have defined

$$B \equiv \sin\theta\sin\phi(\cos^2\theta\cos^2\phi - \sin^2\phi). \tag{16}$$

3 The response function of an interferometer for the magnetic contribution of the + polarization

Equations (13) and (15) represent the distance of the two mirrors of the interferometer from the beam splitter in presence of the GW (i.e. only the contribution

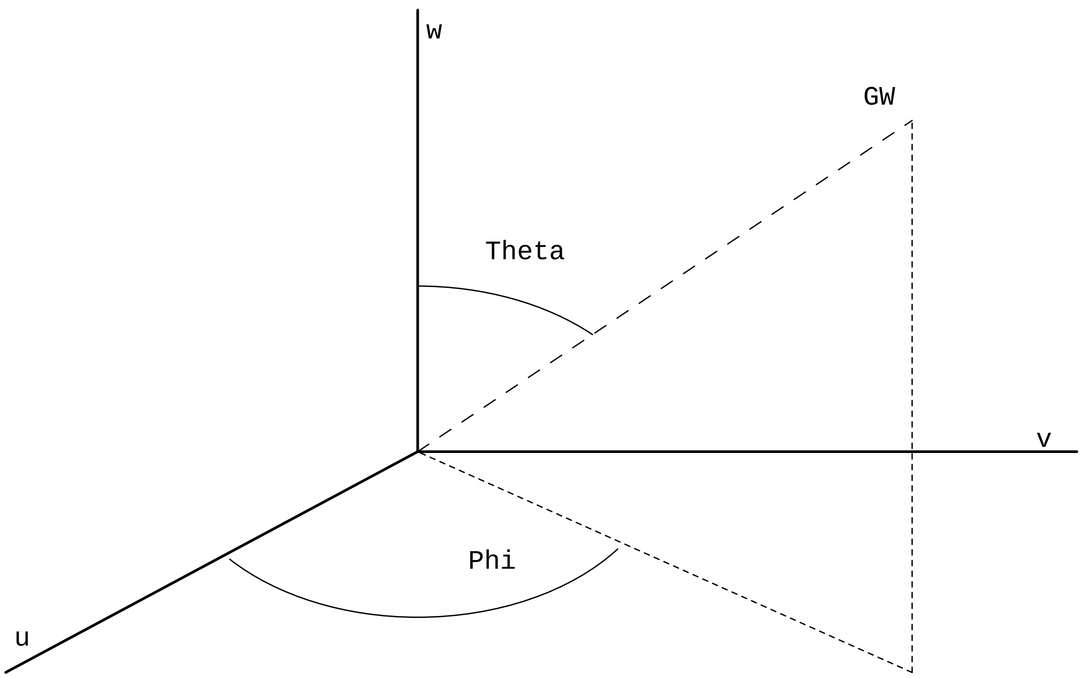

Figure 2: a GW propagating from an arbitrary direction

of the magnetic component of the + polarization of the GW is taken into account). They represent particular cases of the more general form given in eq. (33) of [13].

A "signal" can also be defined in the time domain (i.e. $T = L$ in our notation):

$$\frac{\delta T(t)}{T} \equiv \frac{u-v}{L} = \frac{1}{4}L(A-B)\dot{h}_+(t). \tag{17}$$

The quantity (17) can be computed in the frequency domain using the Fourier transform of h_+ defined by

$$\tilde{h}_+(\omega) = \int_{-\infty}^{\infty} dt h_+(t)\exp(i\omega t), \tag{18}$$

obtaining

$$\frac{\tilde{\delta}T(\omega)}{T} = H^+_{magn}(\omega)\tilde{h}_+(\omega),$$

where the function

$$\begin{array}{c} H^+_{magn}(\omega) = -\frac{1}{8}i\omega L(A-B) = \\ \\ = -\frac{1}{4}i\omega L\sin\theta[(\cos^2\theta + \sin 2\phi\frac{1+\cos^2\theta}{2})](\cos\phi - \sin\phi) \end{array} \tag{19}$$

is the total response function of the interferometer for the magnetic component of the + polarization that is in perfect agreement with the result of Baskaran and Grishchuk (eqs. 46 and 49 of [13]). In the above computation the derivation theorem of the Fourier transform has been used.

We emphasize that in our work the x, y, z frame is the frame of the local observer in phase with respect the propagating GW, while in [13] the two frames are not in phase (i.e. in our work the third angle is put equal to zero, this is not a restriction as it is known in literature, see for example [12]).

In figures 3 and 4 the absolute value of the response functions (19) of the Virgo ($L = 3$Km) and LIGO ($L = 4$Km) interferometers to the magnetic component of the + polarization of GWs for $\theta = \frac{\pi}{4}$ and $\phi = \frac{\pi}{3}$ are respectively shown in the low-frequency range $10Hz \leq 100Hz$. This value grows with frequencies. In figures 5 and 6 the angular dependence of the response function (19) of the Virgo and LIGO interferometers to the magnetic component of the + polarization of GWs for $f = 100Hz$ are shown.

4 Analysis for the × polarization

The analysis can be generalized for the magnetic component of the × polarization too. In this case, equations (6) can be rewritten for the pure magnetic component of the × polarization as

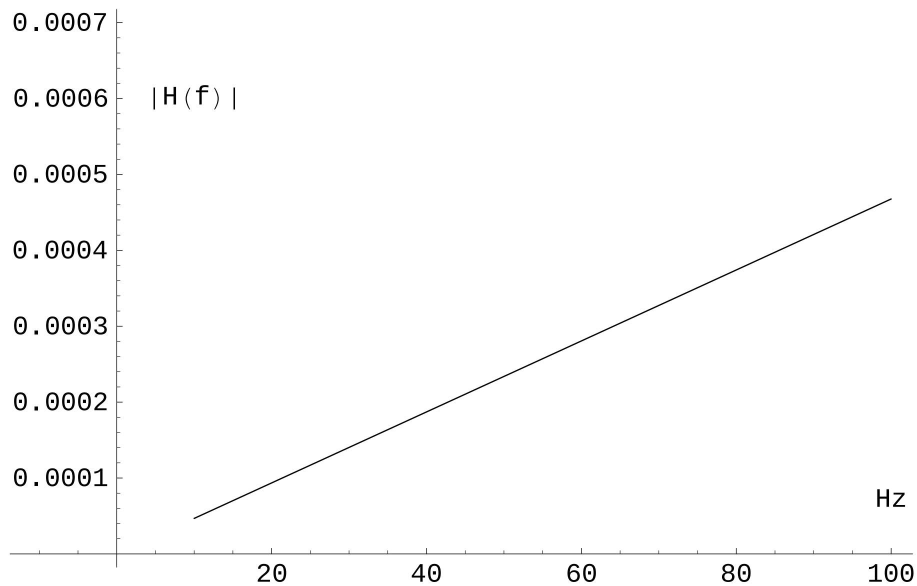

Figure 3: the absolute value of the total response function of the Virgo interferometer to the magnetic component of the + polarization of GWs for $\theta = \frac{\pi}{4}$ and $\phi = \frac{\pi}{3}$ in the low-frequency range $10Hz \leq 100Hz$

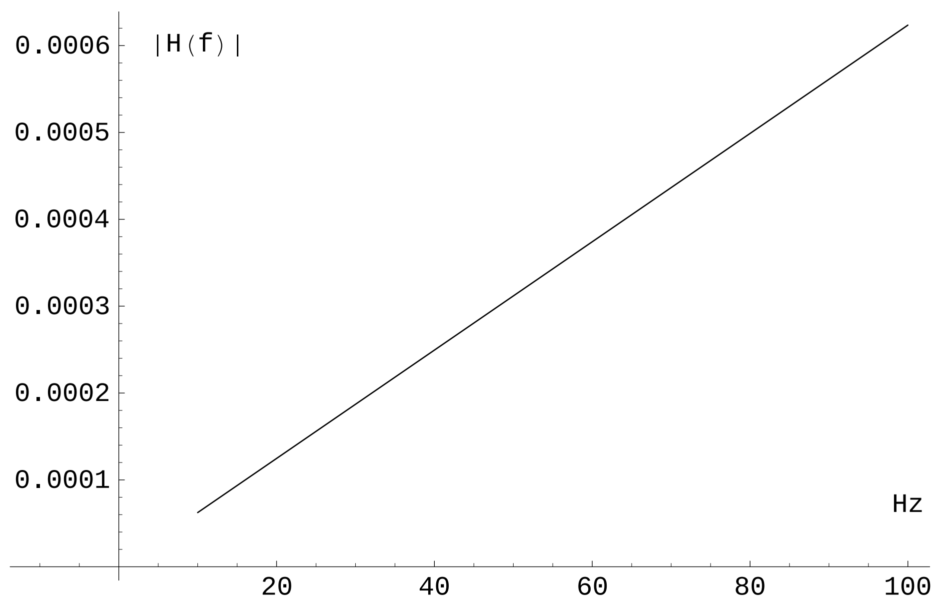

Figure 4: the absolute value of the total response function of the LIGO interferometer to the magnetic component of the + polarization of GWs for $\theta = \frac{\pi}{4}$ and $\phi = \frac{\pi}{3}$ in the low- frequency range $10Hz \leq 100Hz$

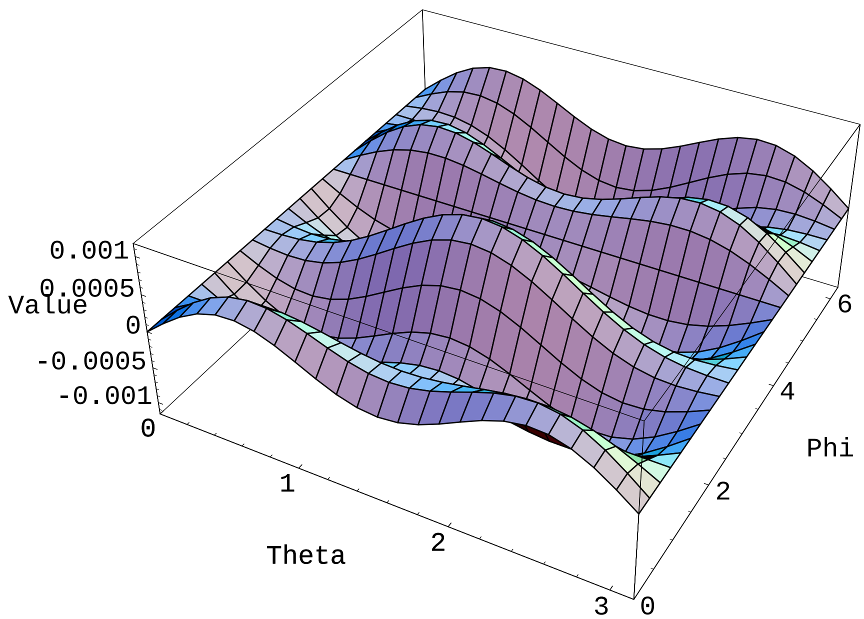

Figure 5: the angular dependence of the response function of the Virgo interferometer to the magnetic component of the + polarization of GWs for $f = 100Hz$

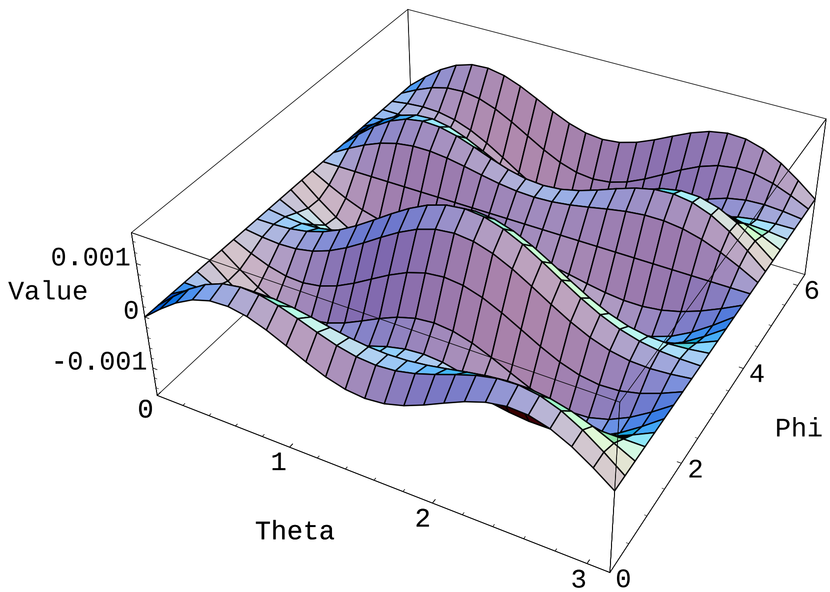

Figure 6: the angular dependence of the response function of the LIGO interferometer to the magnetic component of the + polarization of GWs for $f = 100Hz$

$$x(t+z) = l_1 + \tfrac{1}{2} l_2 l_3 \dot{h}_\times (t+z)$$

$$y(t+z) = l_2 + \tfrac{1}{2} l_1 l_3 \dot{h}_\times (t+z) \tag{20}$$

$$z(t+z) = l_3 - \tfrac{1}{2} l_1 l_2 \dot{h}_\times (t+z).$$

Using eqs. (20), (11) and (12) the u coordinate of the mirror situated in the u arm of the interferometer is given by

$$u = L + \frac{1}{4} L^2 C \dot{h}_\times (t). \tag{21}$$

where we have defined

$$C \equiv -2\cos\theta\cos^2\phi\sin\theta\sin\phi, \tag{22}$$

while the v coordinate of the mirror situated in the v arm of the interferometer is given by

$$v = L + \frac{1}{4} L^2 D \dot{h}_\times (t). \tag{23}$$

where it is

$$D \equiv 2\cos\theta\cos\phi\sin\theta\sin^2\phi. \tag{24}$$

Thus, with an analysis parallel to the one of previous Sections, it is possible to show that the response function of the interferometer for the magnetic component of the $\times$ polarization of GWs is

$$\begin{aligned} H^{\times}_{magn}(\omega) &= -i\omega T(C-D) = \\ &= -i\omega L \sin 2\phi(\cos\phi + \sin\phi)\cos\theta \end{aligned} \tag{25}$$

that is in perfect agreement with the result of Baskaran and Grishchuk (eqs. 46 and 50 of [13]). In figure 7 and 8 the absolute value of the total response functions (25) of the Virgo and LIGO interferometers to the magnetic component of the $\times$ polarization of GWs for $\theta = \frac{\pi}{4}$ and $\phi = \frac{\pi}{3}$ are respectively shown in the low- frequency range $10Hz \leq 100Hz$. This value grows with frequencies in analogy with the case seen in previous Section for the magnetic component of the $+$ polarization. In figure 9 and 10 the angular dependence of the total response function (25) of the Virgo and LIGO interferometers to the magnetic components of the $\times$ polarization of GWs for $f = 100Hz$ are shown.

5 The total response function of interferometers in the full theory of gravitational waves

The low-frequencies approximation, that has been used in previous Sections to show that the "magnetic" and "electric" contributes to the response functions

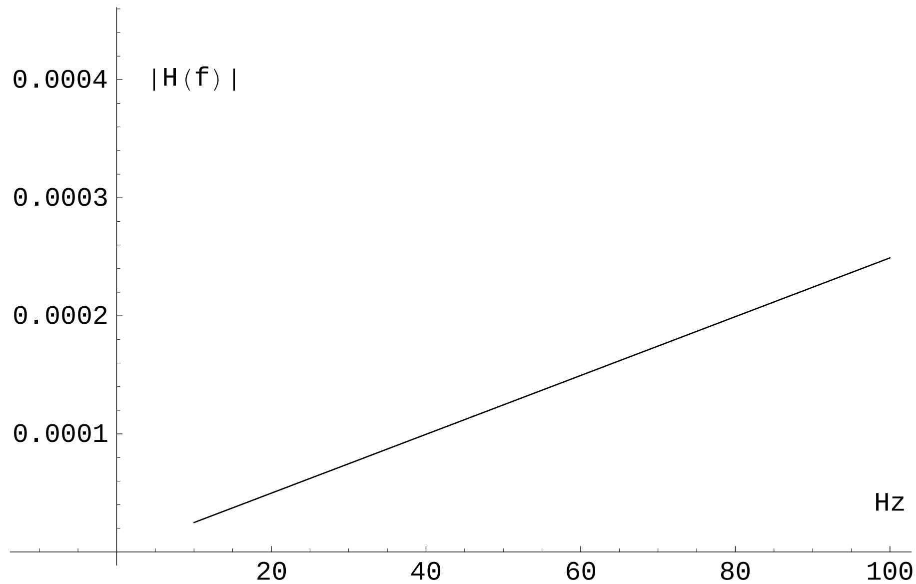

Figure 7: the absolute value of the total response function of the Virgo interferometer to the magnetic component of the $\times$ polarization of GWs for $\theta = \frac{\pi}{4}$ and $\phi = \frac{\pi}{3}$ in the low- frequency range $10Hz \leq 100Hz$

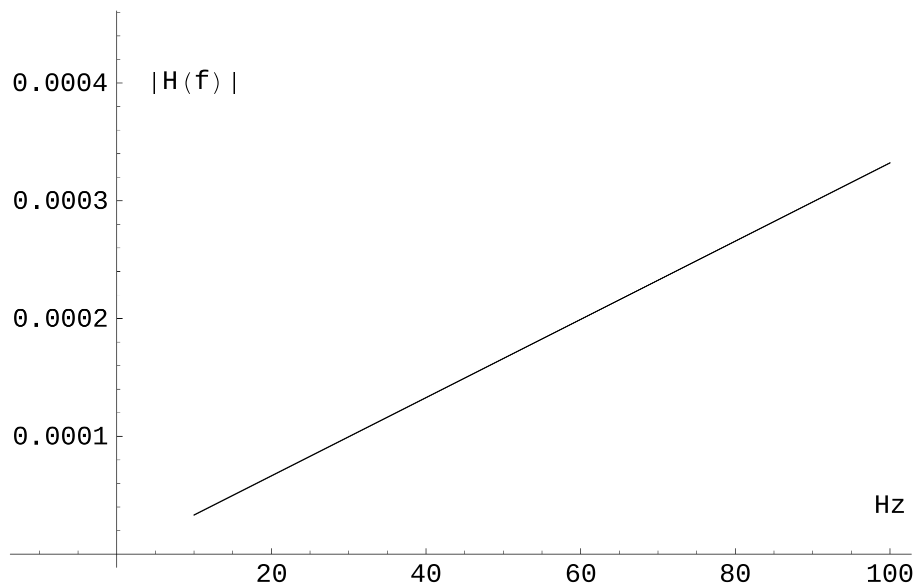

Figure 8: the absolute value of the total response function of the LIGO interferometer to the magnetic component of the $\times$ polarization of GWs for $\theta = \frac{\pi}{4}$ and $\phi = \frac{\pi}{3}$ in the low- frequency range $10Hz \leq 100Hz$

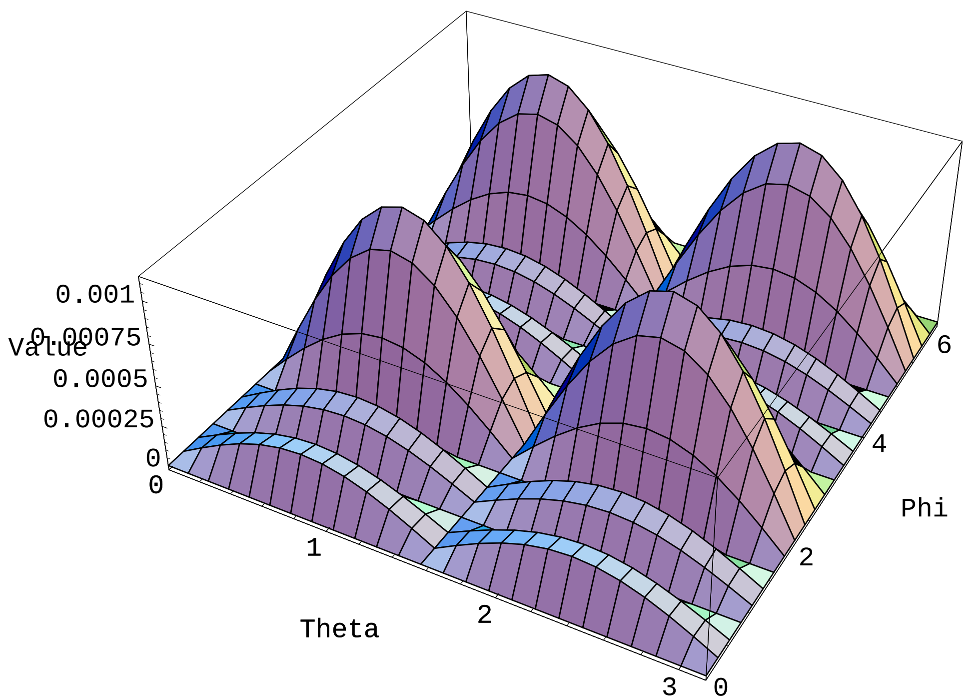

Figure 9: the angular dependence of the total response function of the Virgo interferometer to the magnetic component of the $\times$ polarization of GWs for $f = 100Hz$

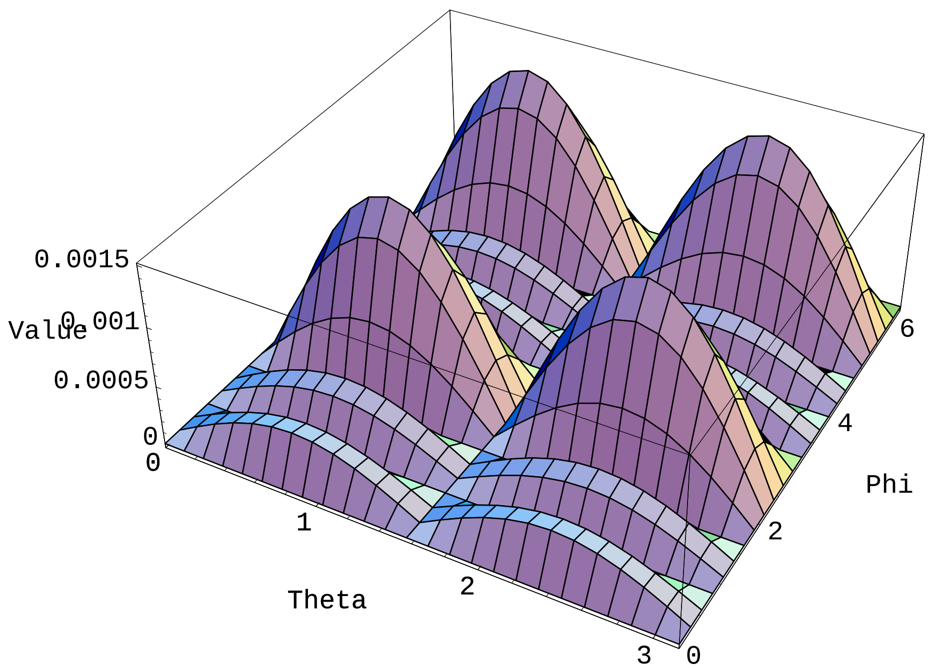

Figure 10: the angular dependence of the total response function of the LIGO interferometer to the magnetic component of the $\times$ polarization of GWs for $f = 100Hz$

can be identificated without ambiguity in the longh-wavelenght regime (see also [13]), is sufficient only for ground based interferometers, for which the condition $f \ll 1/L$ is in general satisfed. For space-based interferometers for which the above condition is not satisfed in the high-frequency portion of the sensivity band [13, 14, 22, 23] the full theory of gravitational waves has to be used.

If one removes the low-frequencies approximation, to compute the total response functions of interferometers generalized in their full frequency dependence, an analysis parallel to the one used for the first time in [16] can be used: the so called "bounching photon metod". We emphasize that this metod has been generalized to scalar waves, angular dependence and massive modes of GWs in [12]. This is also a part of a more general problem of finding the null geodesic of light in the presence of a weak gravitational wave [13, 15, 20, 21, 22, 23].

In this section we compute the variaton of the proper distance that a photon covers to make a round-trip from the beam-splitter to the mirror of an interferometer [12, 16] with the gauge choice (4). In this case one does not have the necessity of introducting the frame of the local observer (see also Section 5 of [13]). In this way, with a treatment parallel to the one of [12, 16], the analysis is translated in the frequency domain and the general response functions are obtained.

A special property of the TT gauge is that an inertial test mass initially at rest in these coordinates, remains at rest throughout the entire passage of the GW [15, 16, 18]. Here we have to clarify the use of words " at rest": we want to mean that the coordinates of the test mass do not change in the presence of the GW. The proper distance between the beam-splitter and the mirror of our interferometer changes even though their coordinates remain the same [15, 16].

We start from the + polarization. In this case the interval (4) takes the form (i.e. in this Section the coordinates of the TT gauge are called t, x, y, z):

$$ds^2 = -dt^2 + dz^2 + [1 + h^+(t + z)]dx^2 + [1 + h^+(t + z)]dy^2. \tag{26}$$

But we recall that the arms of our interferometer are in the $\overrightarrow{u}$ and $\overrightarrow{v}$ directions, while the x, y, z frame is the proper frame of the incoming GW.

We can write for the metric tensor (see Chap. (10) of [17]):

$$g^{ik} = \frac{\partial x^i}{\partial x'^l}\frac{\partial x^k}{\partial x'^m}g'^{lm}. \tag{27}$$

By using eq. (11), eq. (12) and eq. (27), in the new rotated frame, the line element (26) in the $\overrightarrow{u}$ direction becomes:

$$ds^2 = -dt^2 + [1 + (\cos^2\theta\cos^2\phi - \sin^2\phi)h^+(t + u\sin\theta\cos\phi)]du^2. \tag{28}$$

It has to be emphasized that in the line element (28), differently from that in eq. 2 of ref. [16], where, because of the simplest geometry, there is a purely time dependence, there are a spatial dependence in the u direction and an angular dependence too. Thus our analysis is more general than the analysis of [16], and parallel to the one of Section 7 of [12] for the angular response function of the scalar component.

A good way to analyze variations in the proper distance (time) is by means of "bouncing photons" (see [12, 13, 16, 20, 21, 22] and figure 1). A photon can be launched from the beam-splitter to be bounced back by the mirror.

The condition for null geodesics ($ds^2 = 0$) in eq. (28) gives the coordinate velocity of the photon:

$$v^2 \equiv (\frac{du}{dt})^2 = \frac{1}{[1 + (\cos^2\theta\cos^2\phi - \sin^2\phi)h^+(t + u\sin\theta\cos\phi)]}, \qquad (29)$$

which is a convenient quantity for calculations of the photon propagation time between the the beam-splitter and the mirror [12, 16]. We remember that the beam splitter is located in the origin of the new coordinate system (i.e. $u_b = 0$, $v_b = 0$, $w_b = 0$). We know that the coordinates of the beam-splitter $u_b = 0$ and of the mirror $u_m = L$ do not changes under the influence of the GW, thus one can find the duration of the forward trip as

$$T_1(t) = \int_0^L \frac{du}{v(t' + u\sin\theta\cos\phi)}, \qquad (30)$$

with

$$t' = t - (L - u).$$

In the last equation t' is the retardation time (i.e. t is the time at which the photon arrives in the position L, so $L - u = t - t'$).

To first order in h^+ this integral can be approximated with

$$T_1(t) = T + \frac{\cos^2\theta\cos^2\phi - \sin^2\phi}{2}\int_0^L h^+(t' + u\sin\theta\cos\phi)du, \qquad (31)$$

where

$$T = L$$

is the transit time of the photon in the absence of the GW. Similiary, the duration of the return trip will be

$$T_2(t) = T + \frac{\cos^2\theta\cos^2\phi - \sin^2\phi}{2}\int_L^0 h^+(t' + u\sin\theta\cos\phi)(-du), \qquad (32)$$

though now the retardation time is

$$t' = t - (u - l).$$

The round-trip time will be the sum of $T_2(t)$ and $T_1[t - T_2(t)]$. The latter can be approximated by $T_1(t - T)$ because the difference between the exact and the approximate values is second order in h^+. Then, to first order in h^+, the duration of the round-trip will be

$$T_{r.t.}(t) = T_1(t-T) + T_2(t). \tag{33}$$

By using eqs. (31) and (32) one sees immediatly that deviations of this round-trip time (i.e. proper distance) from its imperurbated value are given by

$$\begin{array}{c} \delta T(t) = \frac{\cos^2\theta\cos^2\phi - \sin^2\phi}{2}\int_0^L [h^+(t-2T-u(1-\sin\theta\cos\phi))+ \\ +h^+(t+u(1+\sin\theta\cos\phi))]du. \end{array} \tag{34}$$

Now, using the Fourier transform of the + polarization of the field, defined by eq. (18) one obtains , in the frequency domain:

$$\delta\tilde{T}(\omega) = (\cos^2\theta\cos^2\phi - \sin^2\phi)\tilde{H}_u(\omega,\theta,\phi)\tilde{h}^+(\omega) \tag{35}$$

where

$$\begin{array}{c} \tilde{H}_u(\omega,\theta,\phi) = \frac{-1+\exp(2i\omega L)}{2i\omega(1+\sin^2\theta\cos^2\phi)}+ \\ +\frac{-\sin\theta\cos\phi((1+\exp(2i\omega L)-2\exp i\omega L(1-\sin\theta\cos\phi)))}{2i\omega(1+\sin\theta\cos^2\phi)}, \end{array} \tag{36}$$

and we immediately see that $\tilde{H}_u(\omega,\theta,\phi) \to L$ when $\omega \to 0$.

Thus, the total response function of the arm of the interferometer in the $\overrightarrow{u}$ direction to the + component of the GW is:

$$\Upsilon_u^+(\omega) = \frac{(\cos^2\theta\cos^2\phi - \sin^2\phi)}{2L}\tilde{H}_u(\omega,\theta,\phi), \tag{37}$$

where $2L = 2T$ is the round trip time in absence of gravitational waves (note that in [16] the Laplace transform is used. Here the Fourier one is used because we are going to grafic the frequency response functions of the Virgo and LIGO interferometer for the two polarizations of the GW, see also [12]).

In the same way the line element (26) in the $\overrightarrow{v}$ direction becomes:

$$ds^2 = -dt^2 + [1 + (\cos^2\theta\sin^2\phi - \cos^2\phi)h^+(t + v\sin\theta\sin\phi)]dv^2, \tag{38}$$

and the response function of the v arm of the interferometer to the + polarization of the GW is:

$$\Upsilon_v^+(\omega) = \frac{(\cos^2\theta\sin^2\phi - \cos^2\phi)}{2L}\tilde{H}_v(\omega,\theta,\phi) \tag{39}$$

where now it is

$$\begin{array}{c} \tilde{H}_v(\omega,\theta,\phi) = \frac{-1+\exp(2i\omega L)}{2i\omega(1+\sin^2\theta\sin^2\phi)}+ \\ +\frac{-\sin\theta\sin\phi((1+\exp(2i\omega L)-2\exp i\omega L(1-\sin\theta\sin\phi)))}{2i\omega(1+\sin^2\theta\sin^2\phi)}, \end{array} \tag{40}$$

with $\tilde{H}_v(\omega,\theta,\phi) \rightarrow L$ when $\omega \rightarrow 0$. In this case the variation of the distance (time) is

$$\delta\tilde{T}(\omega) = (\cos^2\theta\cos^2\phi - \cos^2\phi)\tilde{H}_v(\omega,\theta,\phi)\tilde{h}^+(\omega). \tag{41}$$

From equations (35) and (41), the total distances of the two arms in presence of the + polarization of the GW and in the frequency domain are:

$$\tilde{T}_u(\omega) = (\cos^2\theta\cos^2\phi - \sin^2\phi)\tilde{H}_u(\omega,\theta,\phi)\tilde{h}^+(\omega) + T. \tag{42}$$

$$\tilde{T}_v(\omega) = (\cos^2\theta\cos^2\phi - \cos^2\phi)\tilde{H}_v(\omega,\theta,\phi)\tilde{h}^+(\omega) + T, \tag{43}$$

that are particular cases of the more general equation (39) of [13].

Thus the total frequency-dependent response function (i.e. the detector pattern) of an interferometer to the + polarization of the GW is:

$$\begin{array}{c} \tilde{H}^+(\omega) = \Upsilon_u^+(\omega) - \Upsilon_v^+(\omega) = \\ \\ = \frac{(\cos^2\theta\cos^2\phi - \sin^2\phi)}{2L}\tilde{H}_u(\omega,\theta,\phi) + \\ \\ -\frac{(\cos^2\theta\sin^2\phi - \cos^2\phi)}{2L}\tilde{H}_v(\omega,\theta,\phi) \end{array} \tag{44}$$

that in the low frequencies limit ($\omega \rightarrow 0$), if one retains the first two terms of the expansion, is in perfect agreement with the detector pattern of eq. (46) of [13] for the + polarization:

$$\begin{array}{c} \tilde{H}^+(\omega \rightarrow 0) = \frac{1}{2}(1+\cos^2\theta)\cos 2\phi + \\ \\ -\frac{1}{4}i\omega L\sin\theta[(\cos^2\theta + \sin 2\phi\frac{1+\cos^2\theta}{2})](\cos\phi - \sin\phi). \end{array} \tag{45}$$

This result also confirms that the magnetic contribution to the distance is an universal phenomenon because it has been obtained starting by the full theory of gravitational waves in the TT gauge (see also [13]).

Now the same analysis can be made for the $\times$ polarization. In this case, from eq. (4) it is:

$$ds^2 = -dt^2 + dz^2 + dx^2 + dy^2 + 2h^\times(t+z)dxdy \tag{46}$$

for the line element, and, by using eq. (11), eq. (12) and eq. (27), in the new rotated frame, the line element (46) in the $\overrightarrow{u}$ direction becomes:

$$ds^2 = -dt^2 + [1 - 2\cos\theta\cos\phi\sin\phi h^\times(t + u\sin\theta\cos\phi)]du^2. \tag{47}$$

In this way the response function of the u arm of the interferometer to the $\times$ polarization of the GW is:

$$\Upsilon_u^\times(\omega) = \frac{-\cos\theta\cos\phi\sin\phi}{L}\tilde{H}_u(\omega,\theta,\phi). \tag{48}$$

In the same way the line element (46) in the $\overrightarrow{v}$ direction becomes:

$$ds^2 = -dt^2 + [1 + 2\cos\theta\cos\phi\sin\phi h^{\times}(t + u\sin\theta\sin\phi)]dv^2 \qquad (49)$$

and the response function of the v arm of the interferometer to the $\times$ polarization of the GW is:

$$\Upsilon_v^{\times}(\omega) = \frac{\cos\theta\cos\phi\sin\phi}{L}\tilde{H}_v(\omega,\theta,\phi) \qquad (50)$$

Thus the total frequency-dependent response function of an interferometer to the $\times$ polarization of the GW is:

$$\tilde{H}^{\times}(\omega) = \frac{-\cos\theta\cos\phi\sin\phi}{L}[\tilde{H}_u(\omega,\theta,\phi) + \tilde{H}_v(\omega,\theta,\phi)] \qquad (51)$$

that in the low frequencies limit ($\omega \to 0$) is in perfect agreement with the detector pattern of eq. (46) of [13] for the $\times$ polarization::

$$\tilde{H}^{\times}(\omega \to 0) = -\cos\theta\sin 2\phi - i\omega L\sin 2\phi(\cos\phi + \sin\phi)\cos\theta, \qquad (52)$$

while the total distances of the two arms in presence of the $\times$polarization of the GW and in the frequency domain are:

$$\tilde{T}_u(\omega) = (\cos\theta\cos\phi\sin\phi)\tilde{H}_u(\omega,\theta,\phi)\tilde{h}^{\times}(\omega) + T. \qquad (53)$$

$$\tilde{T}_v(\omega) = (-\cos\theta\cos\phi\sin\phi)\tilde{H}_v(\omega,\theta,\phi)\tilde{h}^{\times}(\omega) + T, \qquad (54)$$

that also are particular cases of the more general equation (39) of [13]. We also emphasize that the total low frequencies response functions of eqs. (45) and (45) are more accurate than the ones of [24, 25] because our equations include the "magnetic" contribution (see also [13]).

Then, we have shown that a generalization of the analysis of [12, 16] works in the computation of the response functions of interferometers and that our results in the frequency domain are totally consistent with the results of [13]. Thus our results confirm the presence and importance of the so-called "magnetic" components of GWs and the fact that they have to be taken into account in the context of the total response functions of interferometers for GWs propagating from arbitrary directions.

In figs. 11 and 12 the absolute values of the total response functions of the Virgo interferometer for the $+$ and $\times$ polarizations of gravitational waves propagating from the direction $\theta = \frac{\pi}{4}$ and $\phi = \frac{\pi}{3}$ are shown respectively. The same for the LIGO interferometer is shown in figs. 13 and 14. We can see from the figures that at high frequencies the absolute values of the response functions decreases respect to the constant value of the low frequencies approximation. Finally, in figs. 15 and 16 the angular dependence of the total response functions of the Virgo interferometer to the $+$ and $\times$ polarizations of GWs for $f = 100Hz$ are shown, while in figs. 17 and 18 the same angular dependences are shown for the LIGO interferometer.

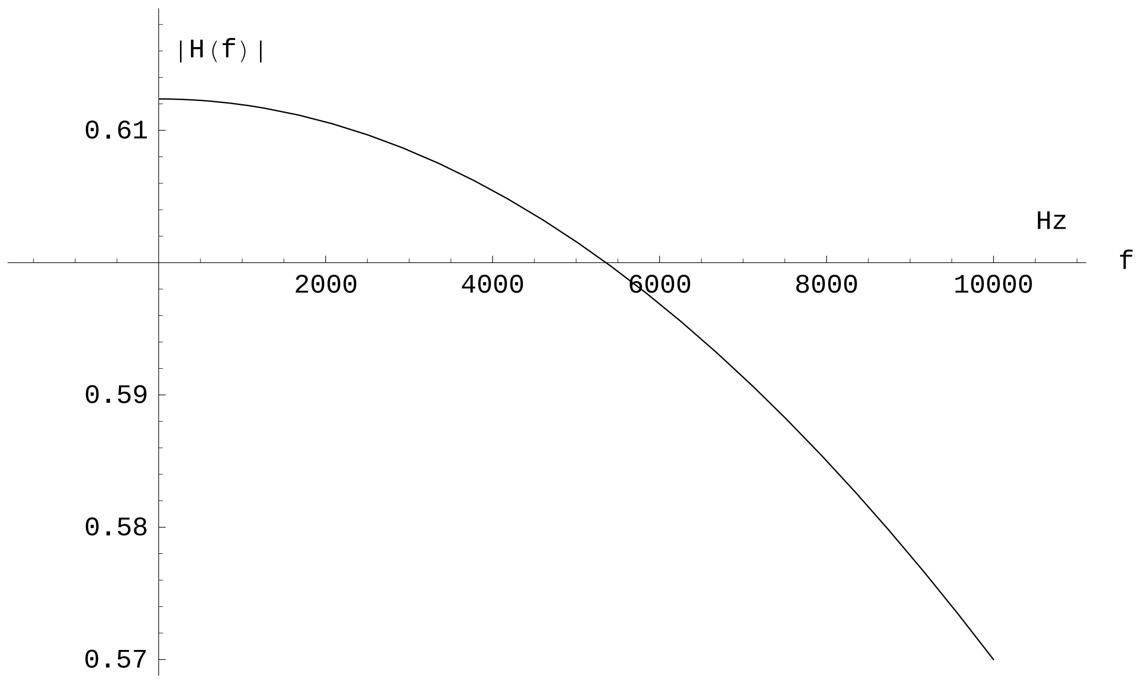

Figure 11: the absolute value of the total response function of the Virgo interferometer to the + polarization of the gravitational waves for $\theta = \frac{\pi}{4}$ and $\phi = \frac{\pi}{3}$.

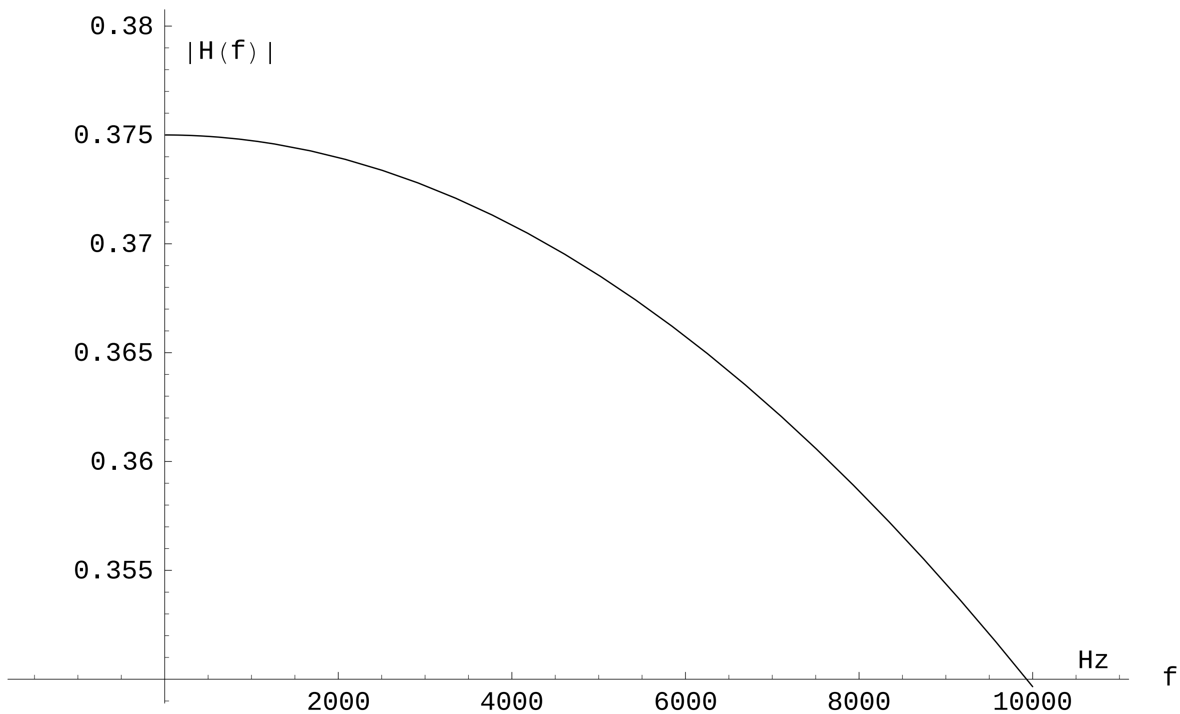

Figure 12: the absolute value of the total response function of the Virgo interferometer to the $\times$ polarization of the gravitational waves for $\theta = \frac{\pi}{4}$ and $\phi = \frac{\pi}{3}$.

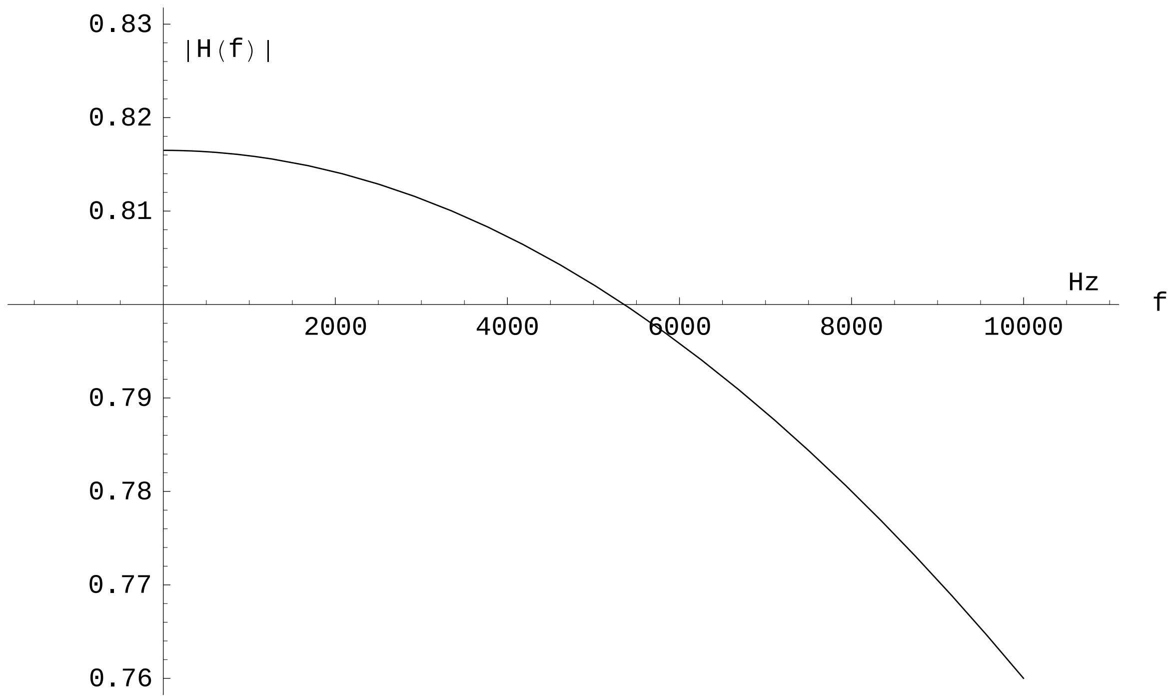

Figure 13: the absolute value of the total response function of the LIGO interferometer to the + polarization of the gravitational waves for $\theta = \frac{\pi}{4}$ and $\phi = \frac{\pi}{3}$.

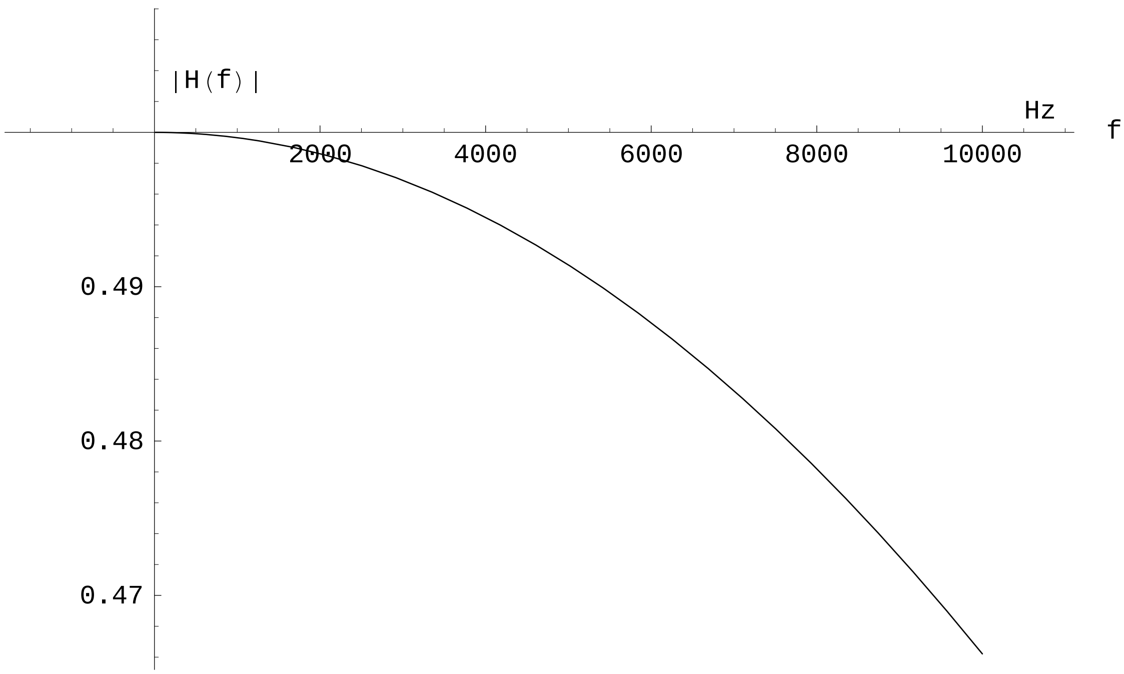

Figure 14: the absolute value of the total response function of the LIGO interferometer to the $\times$ polarization of the gravitational waves for $\theta = \frac{\pi}{4}$ and $\phi = \frac{\pi}{3}$.

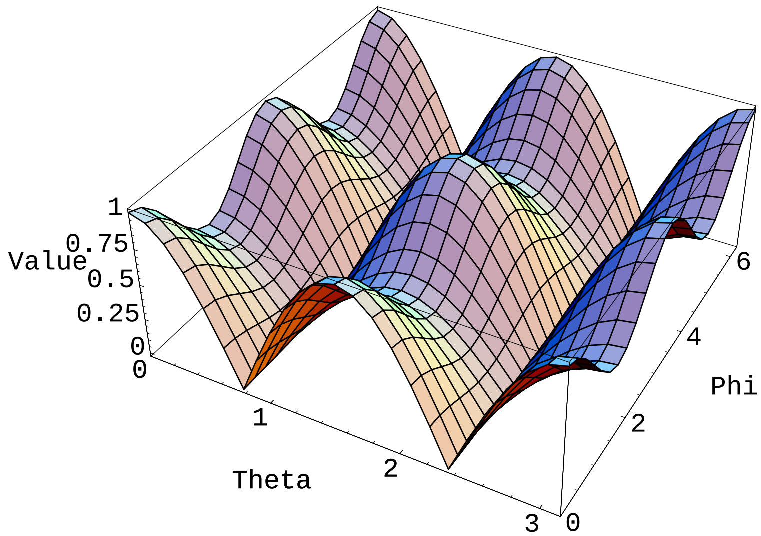

Figure 15: the angular dependence of the total response function of the Virgo interferometer to the + polarization of GWs for $f = 100Hz$

6 Conclusion remarks

In this paper more detailed angular and frequency dependences of the response functions for the magnetic components of GWs have been given in the approximation of wavelength much larger than the linear dimensions of the interferometer, with a specific application to the parameters of the LIGO and Virgo interferometers. Our results agree with the work of [13] in which it has been shown that the identification of "electric" and "magnetic" contributions is unambiguous in the longh-wavelenght approximation. At the end of this paper the angular and frequency dependences of the total response functions of the LIGO and Virgo interferometers have been given. In the high-frequency regime the division on "electric" and "magnetic" components become ambiguous, thus the full theory of gravitational waves has been used. Our results are consistent with the ones of [13] in this case too.

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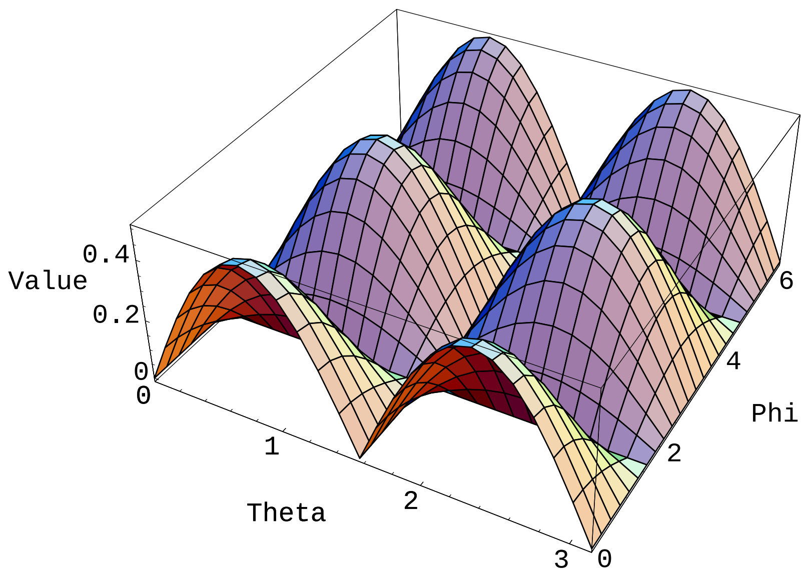

Figure 16: the angular dependence of the total response function of the Virgo interferometer to the $\times$ polarization of GWs for $f = 100Hz$

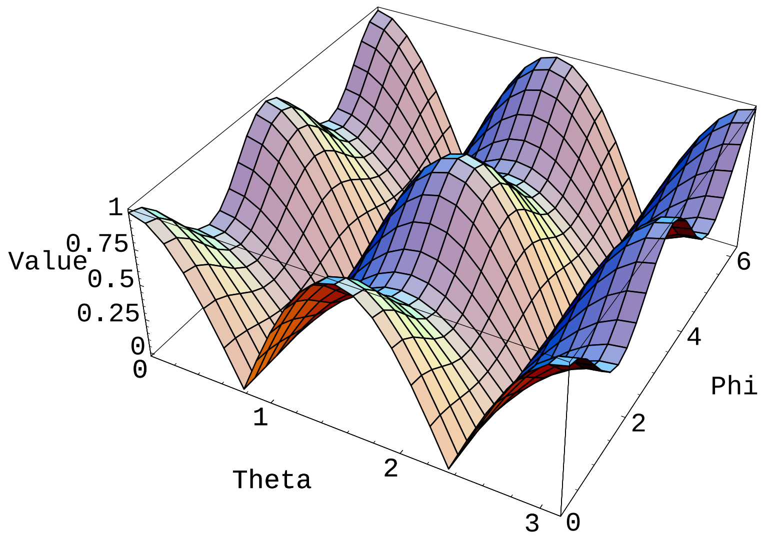

Figure 17: the angular dependence of the total response function of the LIGO interferometer to the + polarization of GWs for $f = 100Hz$

GWs. The European Gravitational Observatory (EGO) consortium has also to be thanked for the using of computing facilities.

References

[1] Acernese F et al. (the Virgo Collaboration) - Class. Quant. Grav. **23** 19 S635-S642 (2006)

[2] Acernese F et al. (the Virgo Collaboration) - Class. Quant. Grav. **23** 8 S63-S69 (2006)

[3] Hild S (for the LIGO Scientific Collaboration) - Class. Quant. Grav. **23** 19 S643-S651 (2006)

[4] Willke B et al. - Class. Quant. Grav. **23** 8S207-S214 (2006)

[5] Sigg D (for the LIGO Scientific Collaboration) - www.ligo.org/pdf_public/P050036.pdf

[6] Abbott B et al. (the LIGO Scientific Collaboration) - Phys. Rev. D 72, 042002 (2005)

[7] Ando M and the TAMA Collaboration - Class. Quant. Grav. **19** 7 1615-1621 (2002)

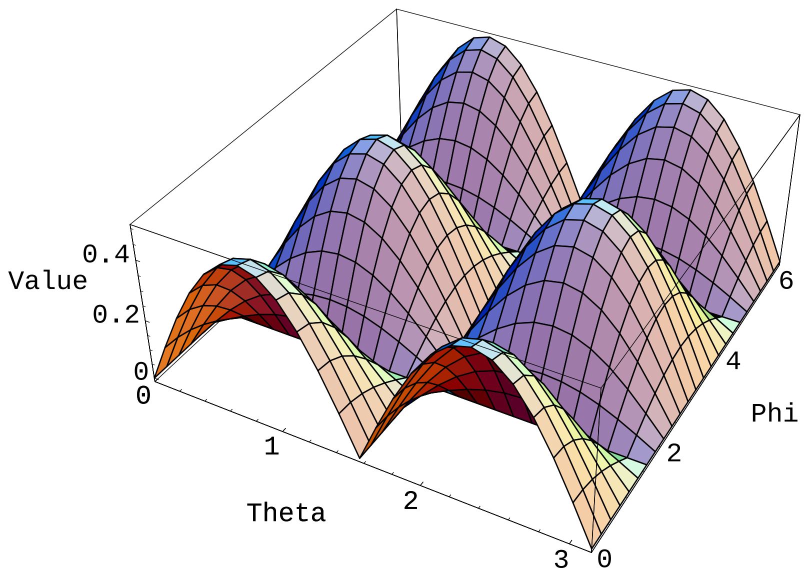

Figure 18: the angular dependence of the total response function of the LIGO interferometer to the $\times$ polarization of GWs for $f = 100Hz$

[8] Tatsumi D, Tsunesada Y and the TAMA Collaboration - Class. Quant. Grav. **21** 5 S451-S456 (2004)

[9] Capozziello S - *Newtonian Limit of Extended Theories of Gravity* in *Quantum Gravity Research Trends* Ed. A. Reimer, pp. 227-276 Nova Science Publishers Inc., NY (2005) - also in arXiv:gr-qc/0412088 (2004)

[10] Capozziello S and Troisi A - Phys. Rev. D **72** 044022 (2005)

[11] Will C M *Theory and Experiments in Gravitational Physics*, Cambridge Univ. Press Cambridge (1993)

[12] Capozziello S and Corda C - Int. J. Mod. Phys. D **15** 1119 - 1150 (2006); Corda C - *Response of laser interferometers to scalar gravitational waves*- talk in the *Gravitational Waves Data Analysis Workshop in the General Relativity Trimester of the Institut Henri Poincare* - Paris 13-17 November 2006, on the web in www.luth2.obspm.fr/IHP06/workshops/gwdata/corda.pdf

[13] Baskaran D and Grishchuk LP - Class. Quant. Grav. **21** 4041-4061 (2004)

[14] Private Communication with the referee

[15] Misner CW, Thorne KS and Wheeler JA - "Gravitation" - W.H.Feeman and Company - 1973

[16] Rakhmanov M - Phys. Rev. D **71** 084003 (2005)

[17] Landau L and Lifsits E - "Teoria dei campi" - Editori riuniti edition III (1999)

[18] Maggiore M - Physics Reports **331**, 283-367 (2000)

[19] Grishchuk LP - Sov. Phys. Usp. **20** 319 (1977)

[20] Grishchuk LP - Sov. Phys. JETP **39** 402 (1974)

[21] Estabrook FB and Wahlquist HD - Gen. Relativ. Gravit. **6** 439 (1975)

[22] Thorne KS - *Proc. Snowmass'94 Summer Study On Particle and Nuclear Astrophysics and Cosmology* - Ed. Kolb EW and Peccei R - World Scientific, Singapore, p.398 (1995)

[23] Tinto M, Estabrook FB and Armstrong JW - Phys. Rev. D **65** 084003 (2002)

[24] Thorne KS - *300 Years of Gravitation* - Ed. Hawking SW and Israel W Cambridge University Press p. 330 (1987)

[25] Saulson P - *Fundamental of Interferometric Gravitational Waves Detectors* - World Scientific, Singapore (1994)