LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

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Design Equations for Initially Flat Blade Springs at Arbitrary Mounting Angles

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1 Introduction

The purpose of this document is to describe the design equations for a cantilevered flat blade spring, mounted at an arbitrary tilt angle, which takes a precise circular curvature under load with the tip of the spring having a horizontal slope. The test results of two specimen blade spring designs are shown.

2 Design Equations

The bending and deflection of a flat blade cantilever spring are calculated from fundamental principles.[1]

The geometry of the cantilever blade spring is shown in Figure 1.

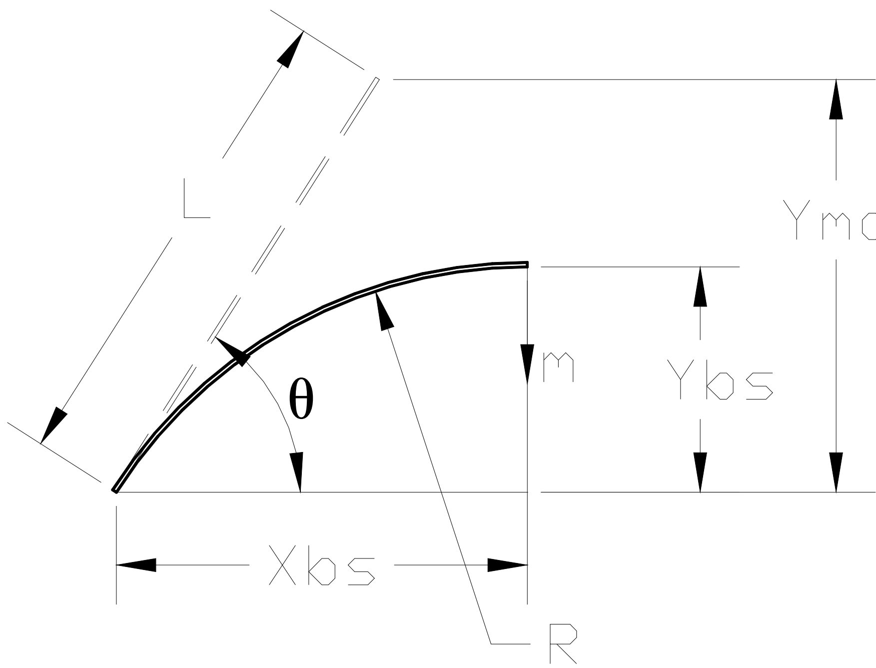

Figure 1: Geometry of the Cantilever Blade Spring

[1] Design of Machine Elements, M.R. Spotts, 2nd ed. 1953, Prentice-Hall, N.J

2.1 Theory

2.1.1 Beam Bending

The maximum normal stress at any cross section of the blade occurs at the outer surface of the blade and is proportional to applied moment, to the distance from the neutral axis of the cross section to the outer surface, and inversely proportional to the area moment of inertia of the cross section.

Bending moment	M
Thickness of blade	t
Width of blade	b
Moment of inertia of cross section	I

For a rectangular cross section

$$I := \frac{b \cdot t^3}{12}$$

Maximium normal stress

$$S := \frac{M \cdot \frac{t}{2}}{I}$$

The maximum strain at the outer surface of the blade is inversely proportional to the bending radius of the blade at the particular cross section.

maximum strain

$$\varepsilon := \frac{\frac{t}{2}}{R}$$

Introducing the modulus of elasticity through the stress/strain relationship for a material that obeys Hook's law,

$$S := E \cdot \varepsilon$$

$$S := E \cdot \frac{t}{2 \cdot R}$$

In general, the width of the blade is >> than the thickness, and the effective modulus of elasticity must be increased to account for the additional stiffness caused by the Poisson's ratio, μ.

$$E_{eff} := \frac{E}{\left(1 - \mu^2\right)}$$

The deflected shape of the blade spring will be circular with a constant radius, R, if the stress is equal at every cross section. The design will use a safe working stress for the material, which does not exceed the elastic limit.

$$S := S_w$$

2.1.2 Vertical Bounce Frequency

The vertical bounce frequency of a mass on a spring with a spring constant k is

$$f_0 := \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{k}{m}}$$

The spring constant is defined as the ratio of vertical force to vertical displacement.

$$k := \frac{m \cdot g}{y}$$

Substituting the spring constant into the frequency equation, we obtain

$$f_0 := \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{g}{y}}$$

which is the equation for the frequency of a pendulum of length y. This result relies on the spring being linear with a single spring const.

Strangely enough, even though the blade spring is non-linear and the spring constant varies with deflection, the measured vertical bounce frequency of the blade spring can be predicted precisely by using the pendulum formula and by choosing y = ymax, the total deflection of the blade spring under load.

2.2 Design Equations

Blade spring Test 45 deg
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acceleration of gravity, m/s^2 $g := 9.8$

BLADE SPRING TEST @ 45 deg

Faraday upper blade spring design

yield strength of 1095 spring steel, psi $S_{yieldpsi} := 84000$

Poisson's ratio $\mu := 0.30$

modulus of elasticity, psi $E_{psi} := 30 \cdot 10^{6}$

modulus of elasticity, Pa $E := E_{psi} \cdot 6895$ $E = 2.0685 \times 10^{11}$

working stress of 1095 spring steel, psi $S_{wpsi} := S_{yieldpsi} \cdot 0.28$ $S_{wpsi} = 2.352 \times 10^{4}$

working stress of 1095 spring steel, Pa $S_w := \dfrac{S_{wpsi}}{1.45 \cdot 10^{-4}}$ $S_w = 1.62207 \times 10^{8}$

radius of blade spring, m $R_{bs} := 14 \cdot .0254$ $R_{bs} = 0.3556$

radius of blade spring, in $R_{bsin} := \dfrac{R_{bs}}{.0254}$ $R_{bsin} = 14$

thickness of blade spring, m $t := \dfrac{2 \cdot R_{bs} \cdot S_w \cdot \left(1 - \mu^2\right)}{E}$ $t = 5.07513 \times 10^{-4}$

thickness of blade spring, in $t_{in} := \dfrac{t}{.0254}$ $t_{in} = 0.01998$

arc of blade spring, rad $\theta_m := \dfrac{\pi}{4}$

blade arc angle, deg $\theta_{mdeg}(\theta_m) := \theta_m \cdot \dfrac{180}{\pi}$ $\theta_{mdeg}(\theta_m) = 45$

length of blade spring, in	$l_{bsin} := R_{bsin} \cdot \theta_m$	$l_{bsin} = 10.99557$
length of blade spring, m	$l_{bs} := l_{bsin} \cdot .0254$	$l_{bs} = 0.27929$
horizontal distance of suspension point from blade spring mount, in	$x_{bsin} := R_{bsin} \cdot \sin(\theta_m)$	$x_{bsin} = 9.89949$
vertical height of suspension from blade spring mount, m	$y_{bsin} := R_{bsin} \cdot (1 - \cos(\theta_m))$	$y_{bsin} = 4.10051$
unloaded height of blade spring, m	$y_{max} := l_{bs} \cdot \sin(\theta_m)$	
vertical distance blade moves, m	$y_r := y_{max} - y_{bsin} \cdot 0.0254$ $y_r = 0.09333$	
approximate vertical resonant frequency based on blade depression, Hz	$f_0 := \frac{\sqrt{\frac{g}{y_r}}}{2 \cdot \pi}$	$f_0 = 1.63085$
distance along spring, m	$l := 0$	
distance along spring, in	$l_{in} := \frac{l}{.0254}$	
mass supported by each blade spring, kg	$m_{bs} := 0.2$	
load on blade spring, N	$P := m_{bs} \cdot 9.8$	$P = 1.96$

Blade Shape

blade width at l from end, m

$$b(l_{in}) := \frac{6\cdot P\cdot R_{bs}\cdot \sin\left(\frac{l_{in}}{R_{bsin}}\right)}{S_w\cdot t^2} \qquad b(l) = 0$$

$$C := \frac{6\cdot P\cdot R_{bs}}{.0254\, S_w\cdot t^2} \qquad C = 3.94069$$

blade width at l from end, in

$$b_{in}(l_{in}) := \frac{b(l_{in})}{.0254} \qquad b_{in}(l) = 0$$

$$b_{in}\left(\frac{l_{bsin}}{4}\right) = 0.76879 \qquad \frac{l_{bsin}}{4} = 2.74889$$

$$b_{in}\left(\frac{l_{bsin}}{2}\right) = 1.50804 \qquad \frac{l_{bsin}}{2} = 5.49779$$

$$b_{in}(l_{bsin}\cdot 0.75) = 2.18933 \qquad l_{bsin}\cdot 0.75 = 8.24668$$

$$b_{in}(l_{bsin}) = 2.78649 \qquad l_{bsin} = 10.99557$$

alternative formula

$$b_{inmax} := \frac{3\cdot P\cdot E^2\cdot \sin\left(\frac{l_{bs}}{R_{bs}}\right)}{2\cdot R_{bs}\cdot S_w{}^3\cdot\left(1-\mu^2\right)^2\cdot 0.0254} \qquad b_{inmax} = 2.78649$$

$$b_{max} := b_{inmax}\cdot 0.0254 \qquad b_{max} = 0.07078$$

$$\frac{C}{2} = 1.97034 \qquad R_{bsin} = 14 \qquad t_{in} = 0.01998$$

$$x := 0$$

Solid Works equation

$$y_{down} := -1.97034\sin\left(\frac{x}{14}\right)$$

$$y_{up} := 1.97034 \sin\left(\frac{x}{14}\right)$$

stress at l_{in} Pa

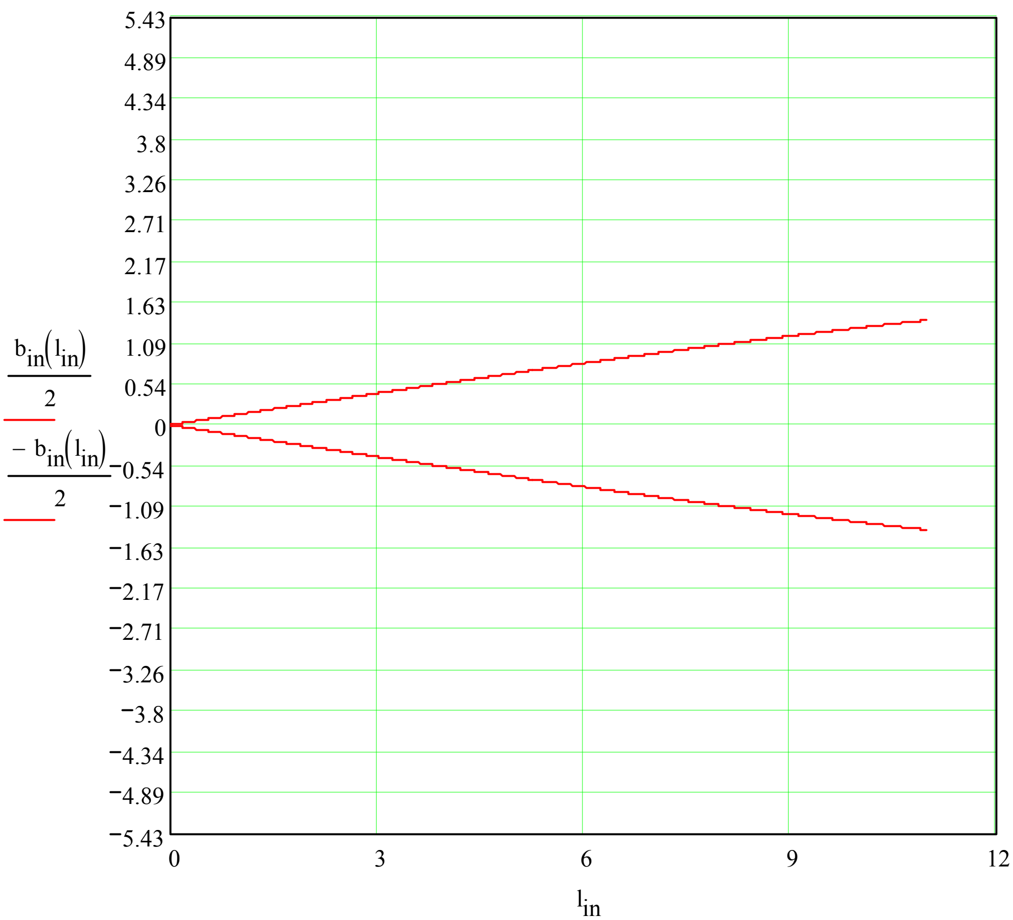
$$S(l_{in}) := \frac{6\cdot P\cdot R_{bs}\cdot \sin\left(\frac{l_{in}}{R_{bsin}}\right)}{b(l_{in})\cdot t^2}$$

$S_w = 1.62207\times 10^8$

$$S\left(\frac{l_{bsin}}{1}\right) = 1.62207\times 10^8$$

$$S_{psi}(l_{bsin}) := S(l_{bsin})\cdot\left(1.45\cdot 10^{-4}\right) \quad S_{psi}(l_{bsin}) = 2.352\times 10^4$$

$l_{in} := 0, 0.01 .. l_{bsin}$



3 Test Results

A drawing of the loaded blade spring was placed behind and adjacent to the spring under test to measure the deviation from the designed radius of curvature. A vertical scale was read to measure the height of the tip of the blade spring above the mounting point. The spring was loaded with mass until the height of the loaded spring matched the design height. With this amount of load, the spring was given a vertical impulse, and the oscillation period was measured with a stop watch.

3.1 45 Degree, 11 inch Blade Spring

3.1.1 Design Values

Material	1095 steel
Modulus of elasticity, psi	30 E6
Length of flat blade, in	11.00
Width at base, in	2.786
Thickness, in	0.020
Vertical height of loaded spring, in	4.10
Mass load, kg	0.200
Radius of curvature, in	14.00
Vertical bounce frequency, Hz	1.63

3.1.2 Experimental Results

The spring loaded with mass at the design height is shown in Figure 2. With this amount of load, the spring was given a vertical impulse, and the oscillation frequency was determined.

The following experimental results were obtained.

Mass load for design height	0.186 kg	error = 0.07
Radius of curvature	14.00 +/- 0.02	
Frequency	1.65	error = 0.01

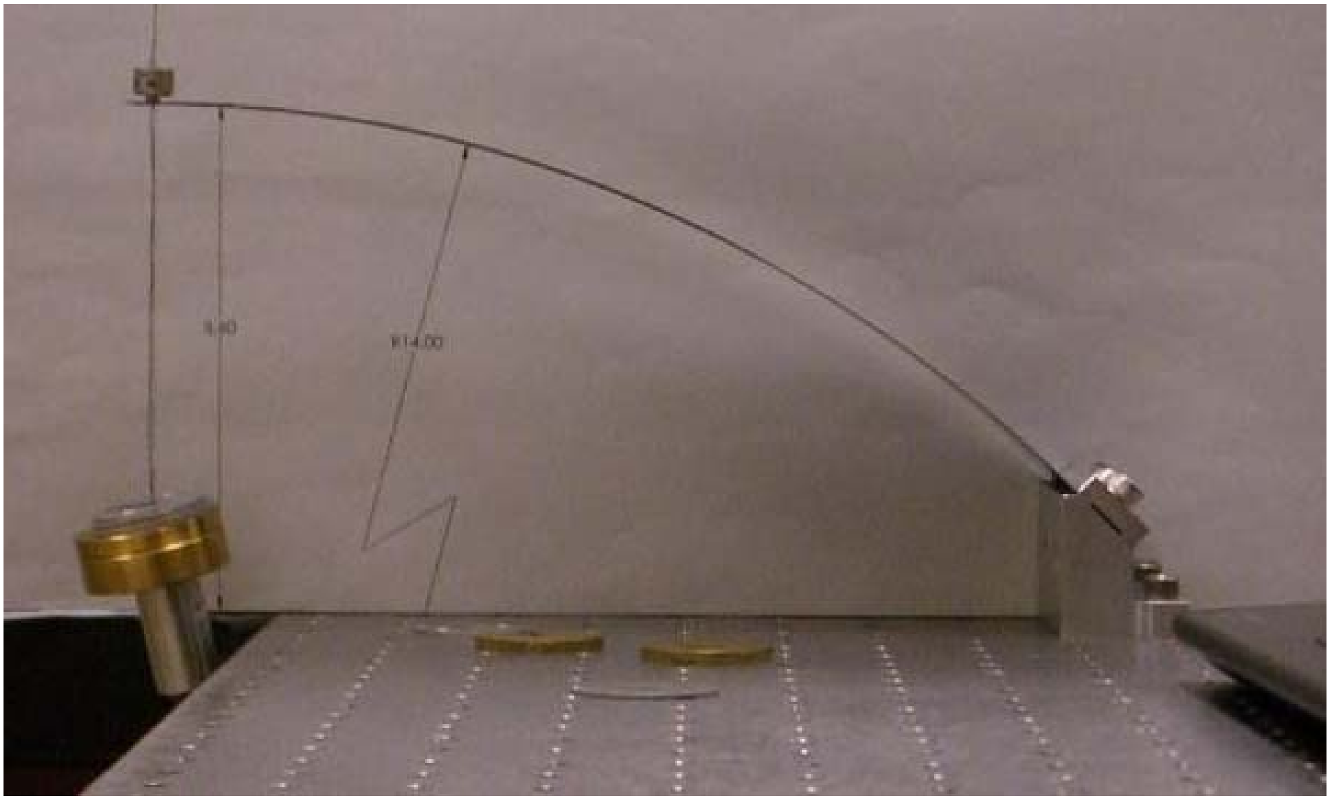


Figure 2: Measured Curvature of 11 in Blade Spring under Balanced Load

3.2 45 Degree, 6.7 inch Blade Spring

3.2.1 Design Values

Material	1095 steel
Modulus of elasticity, psi	30 E6
Length of flat blade, in	6.68
Width at base, in	2.350
Thickness, in	0.015
Vertical height of loaded spring, in	2.49
Mass load, kg	0.200
Radius of curvature, in	8.50
Vertical bounce frequency, Hz	2.09

3.2.2 Experimental Results

The spring loaded with mass at the design height is shown in Figure 3. With this amount of load, the spring was given a vertical impulse, and the oscillation frequency was determined.

The following experimental results were obtained.

Mass load for design height	0.184 kg	error = 0.08
Frequency	2.09	error = 0.00
Radius of curvature	6.68 +/- 0.04	

The radius of curvature matched the printed design profile within twice the thickness of the blade, as shown in the photograph in Figure 3.

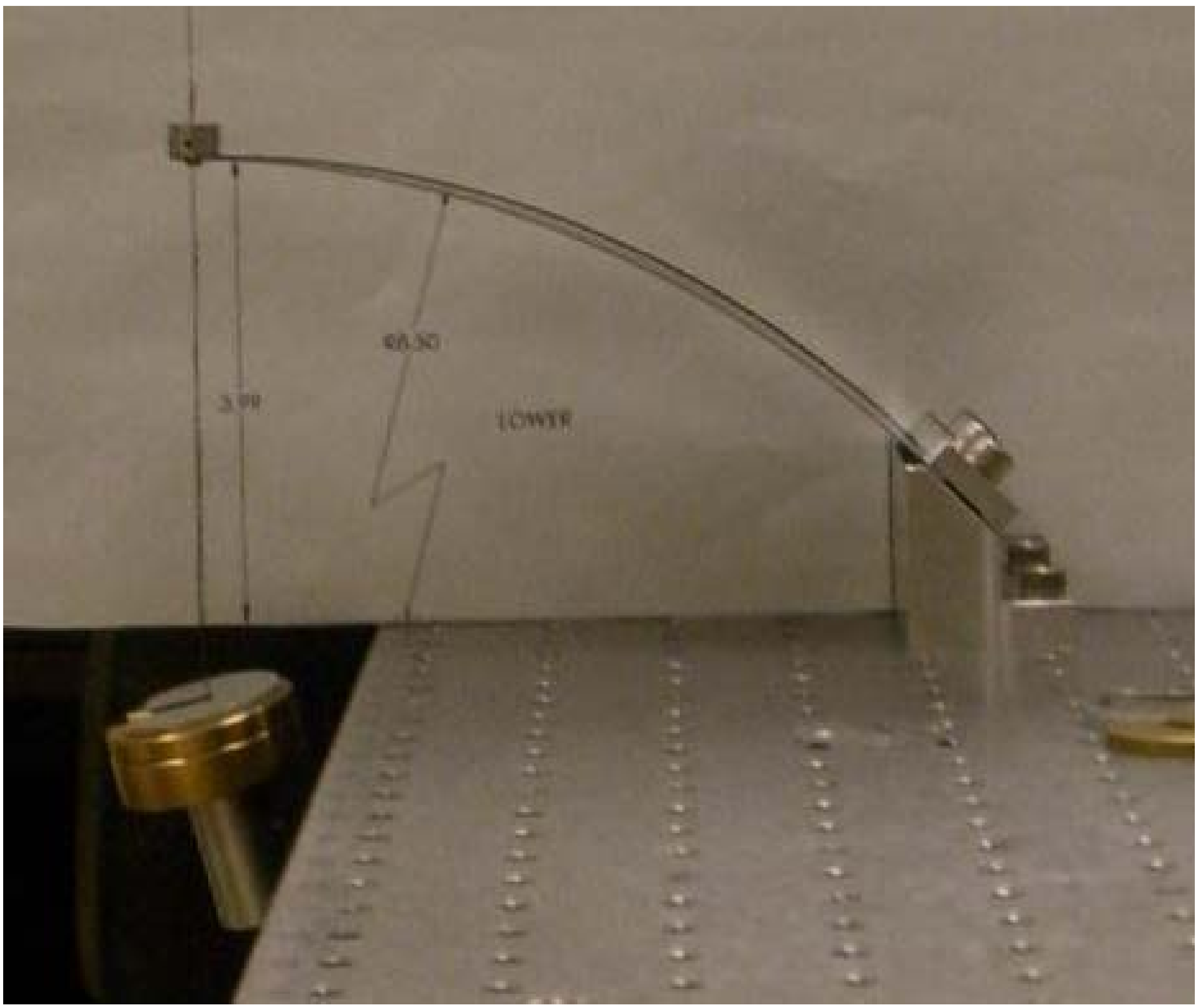


Figure 3: Measured Curvature of 6.7 in Blade Spring under Balanced Load

3.3 22.5 Degree, 28 inch Blade Spring

3.3.1 Design Values

Design Values:

Length of flat blade	11.00 in
Vertical height of loaded spring	4.10 in
Mass load	0.200 kg
Radius of curvature	14.00 in

3.3.2 Experimental Results

TBD