

Progress on FE modelling for Newtonian noise estimates

David Rabeling

Mark Beker, Eric Hennes, and
Jo van den Brand

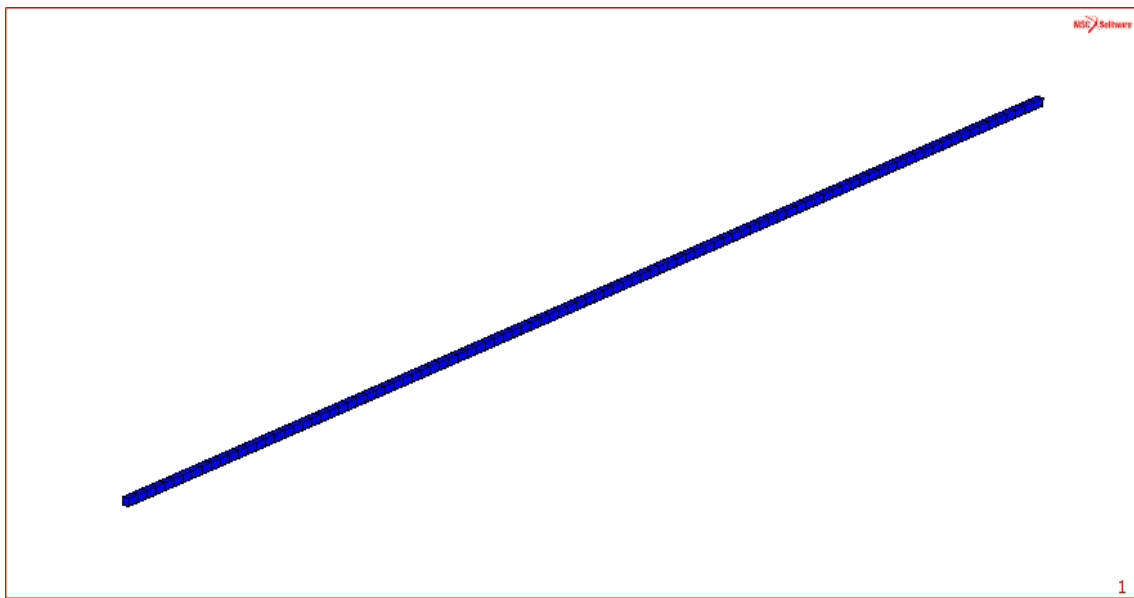
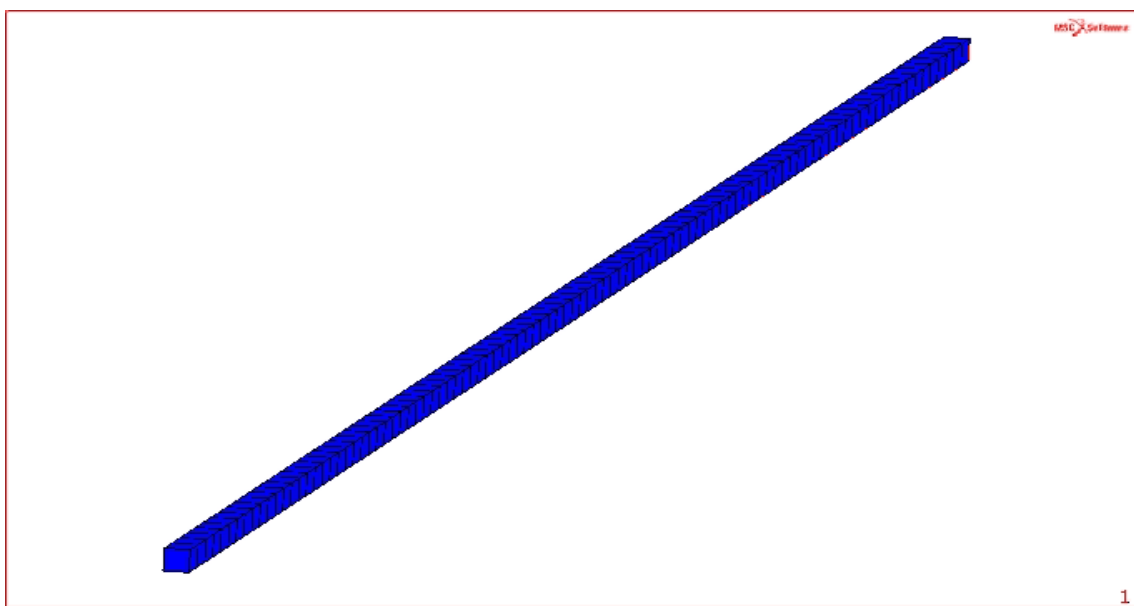
First and second generation detectors:

- Saulson made first predictions and set upper limits to the expected GG noise levels in first generation detectors.
- Beccaria *et. al.* created a more accurate estimate of GG noise for VIRGO.
- Thorne and Hughes published a full analytic analysis of GG noise and human interaction with the detector.

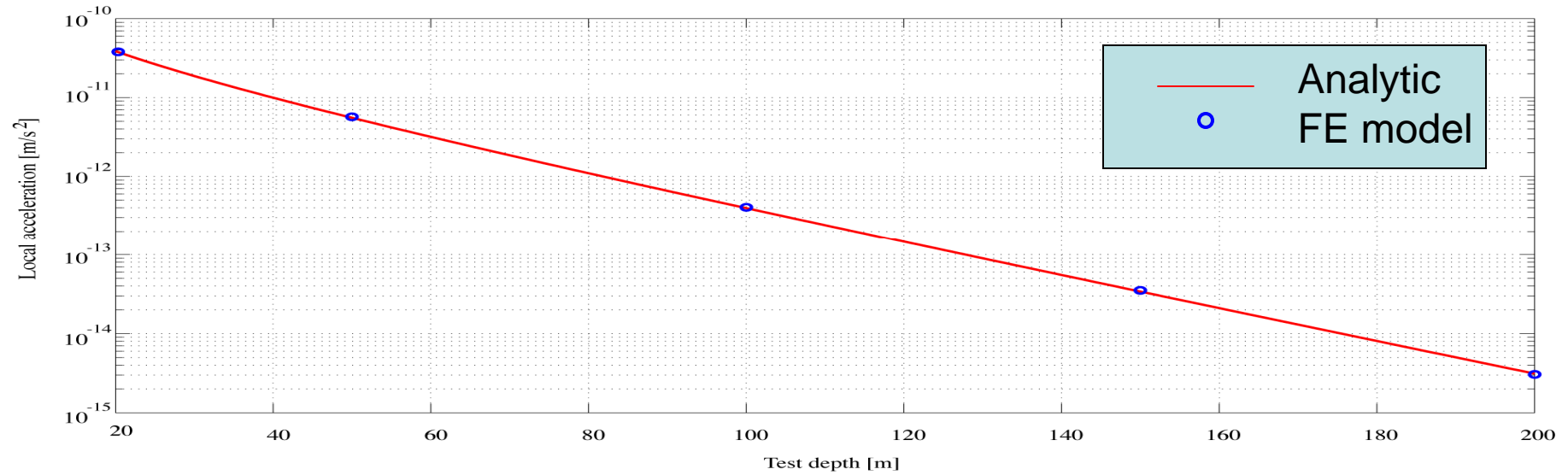
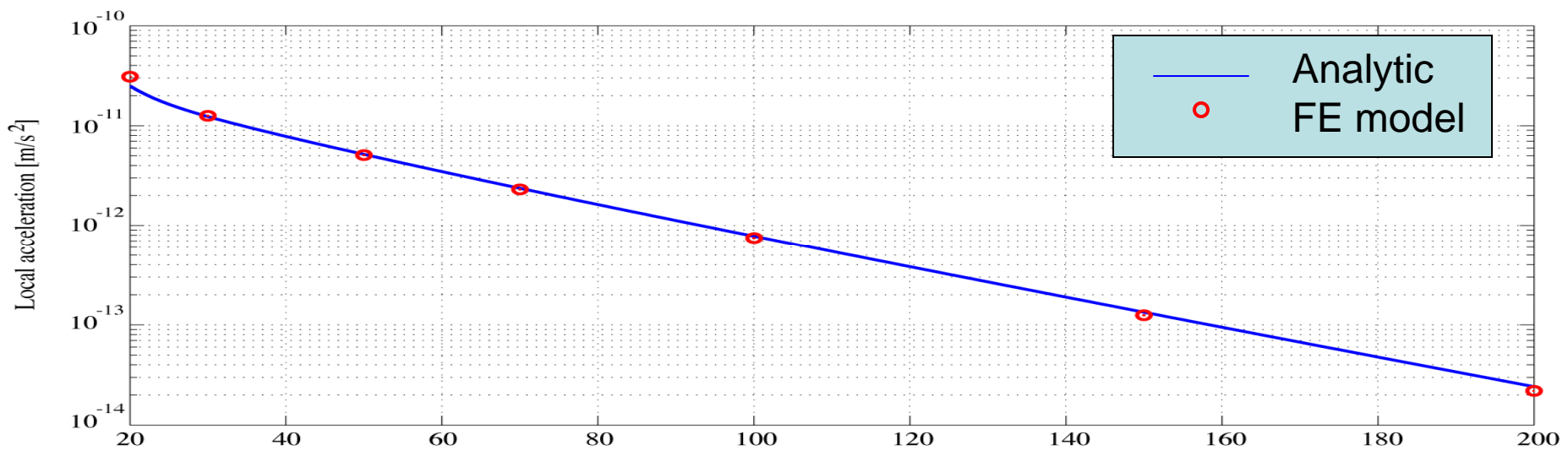
Third generation detectors:

- Cella presented various studies on subterranean gravitational wave detectors and accompanying noise reduction due to placing the detector test masses at various depths and in different types of cavities.

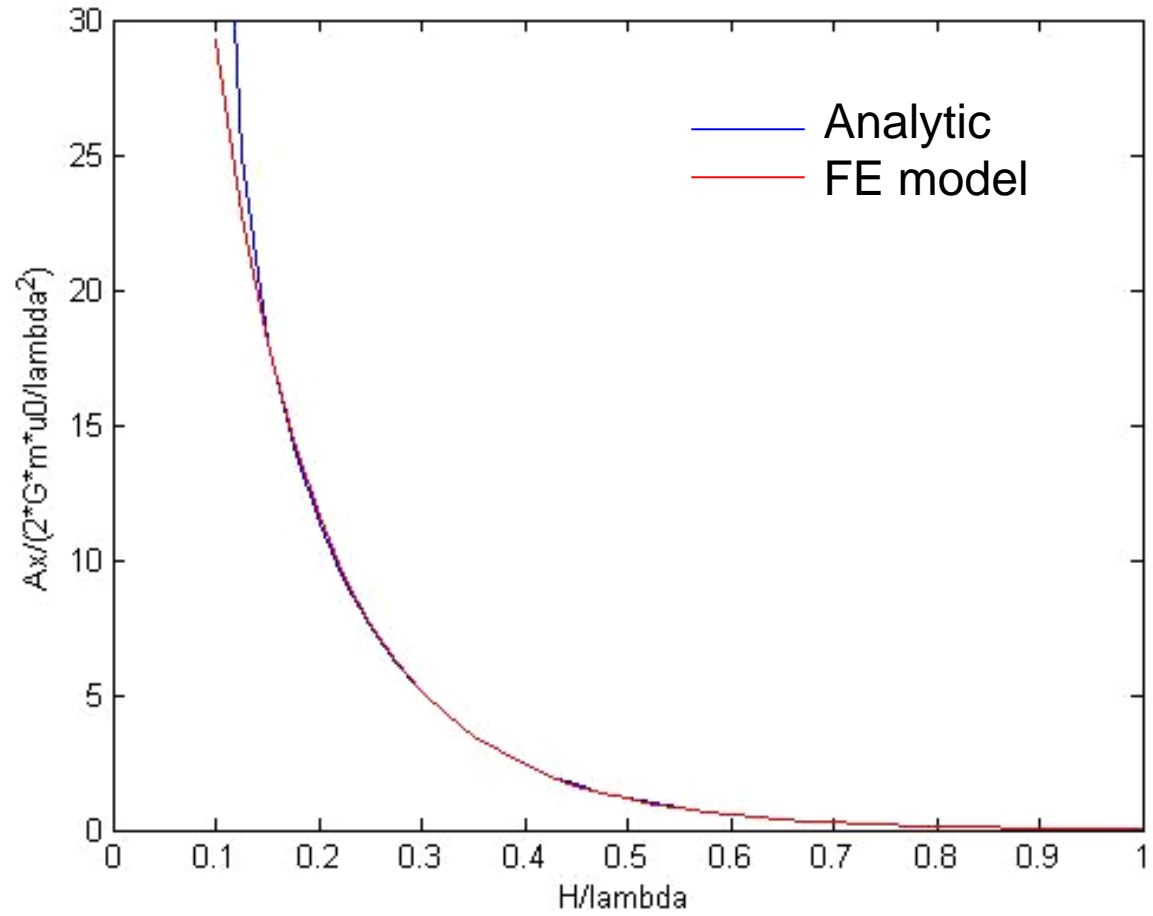
We are now working on a FE model to verify more complex cavity models and soil compositions.



- We want to probe the Newtonian noise at a height H from a source of pressure or shear waves and compare that to our analytic model



Converting this to dimensionless parameters



Harmonic Rayleigh wave: $\lambda=123\text{m}$,

$$c = 0.61 \sqrt{E / \rho}$$

MSC Software

Model parameters:

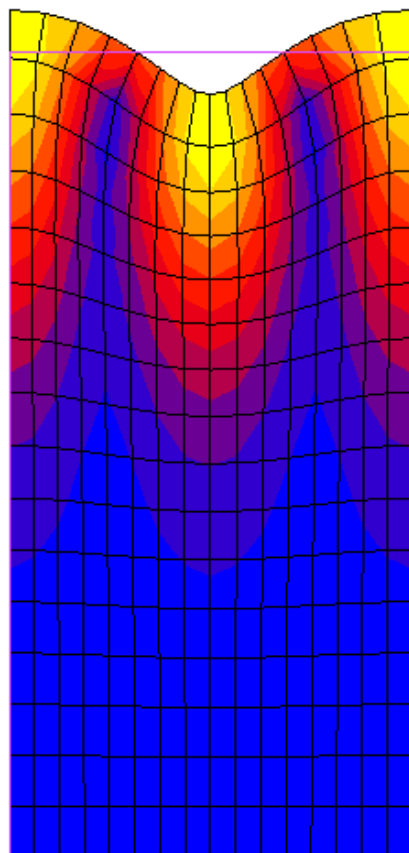
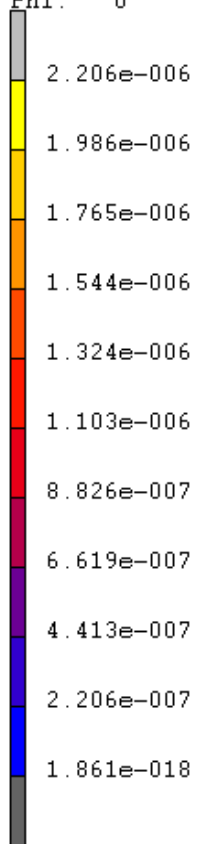
$$\rho = 2000\text{kg/m}^3$$

$$E = 80\text{MPa}$$

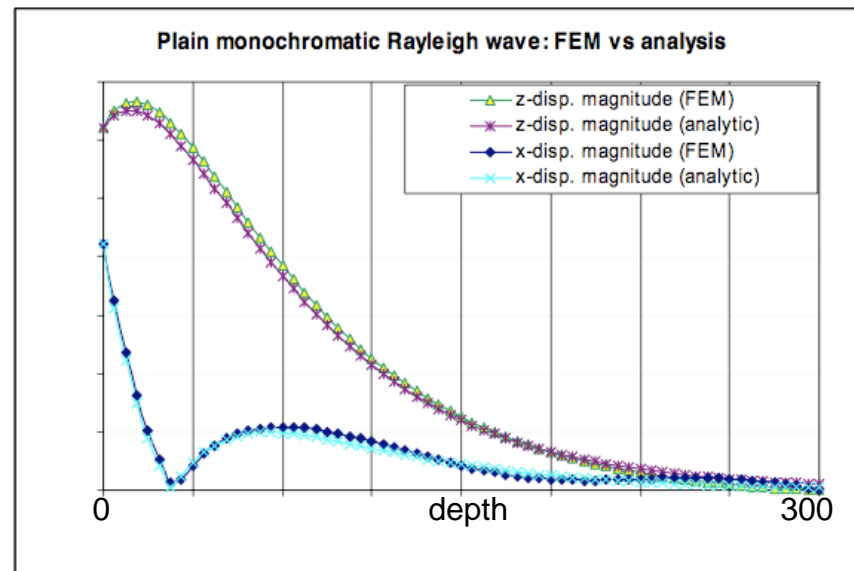
$$\nu = 0$$

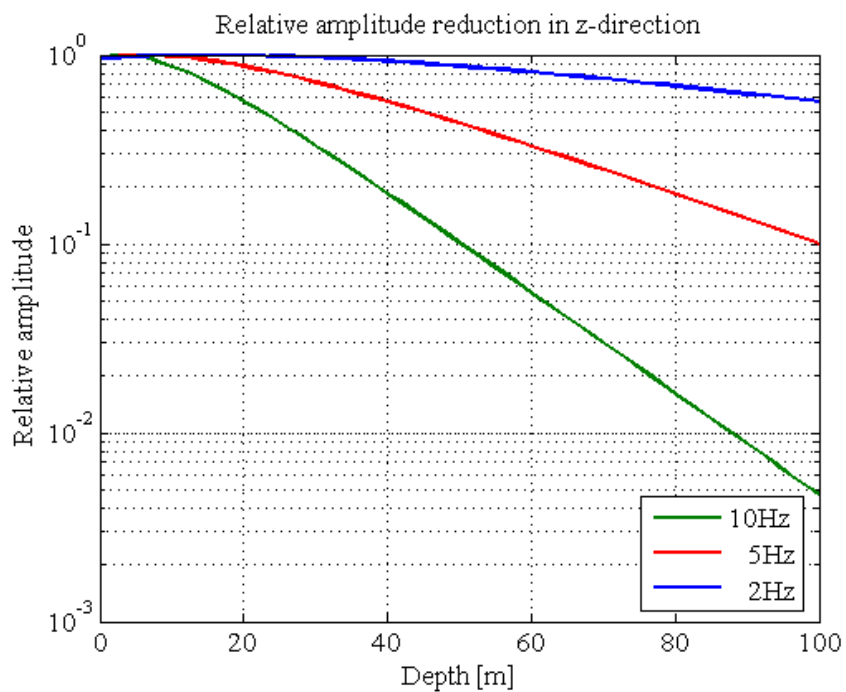
$$f = 1\text{Hz}$$

Inc: 0:16
Time: 0.000e+000
Freq: 9.350e-001
Phi: 0



lcase1
Displacement

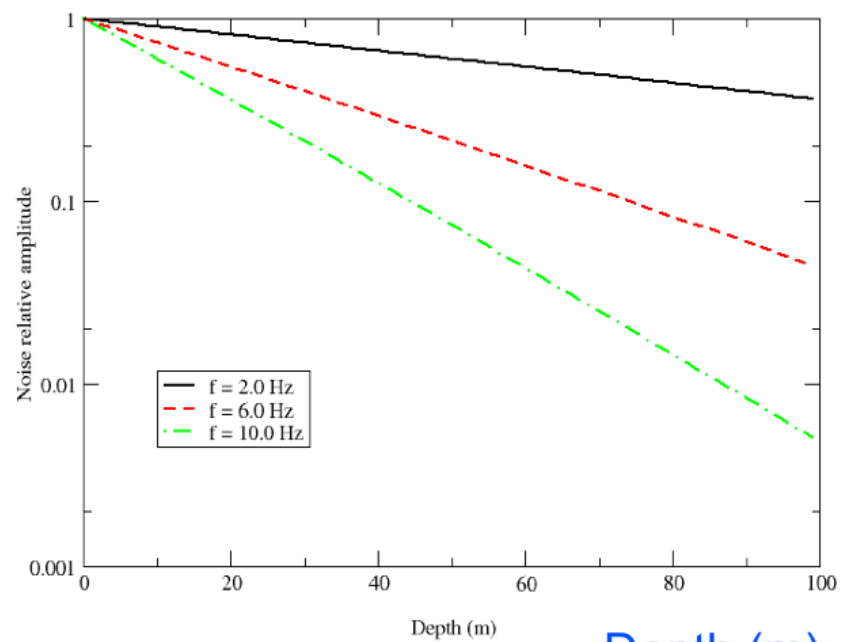




← Rayleigh wave: $\lambda=400\text{m}$,

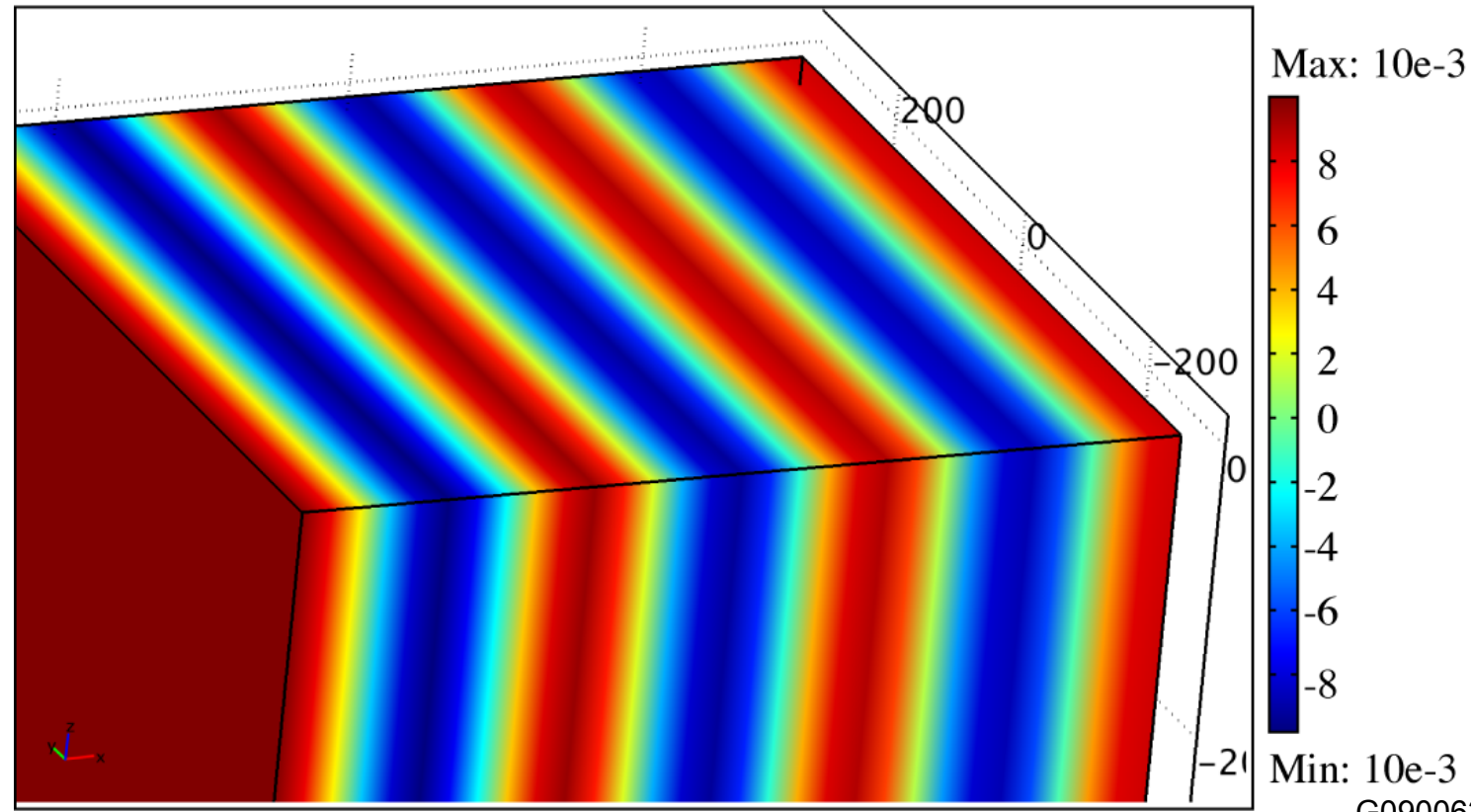
Seismic suppression due to depth (Cella)

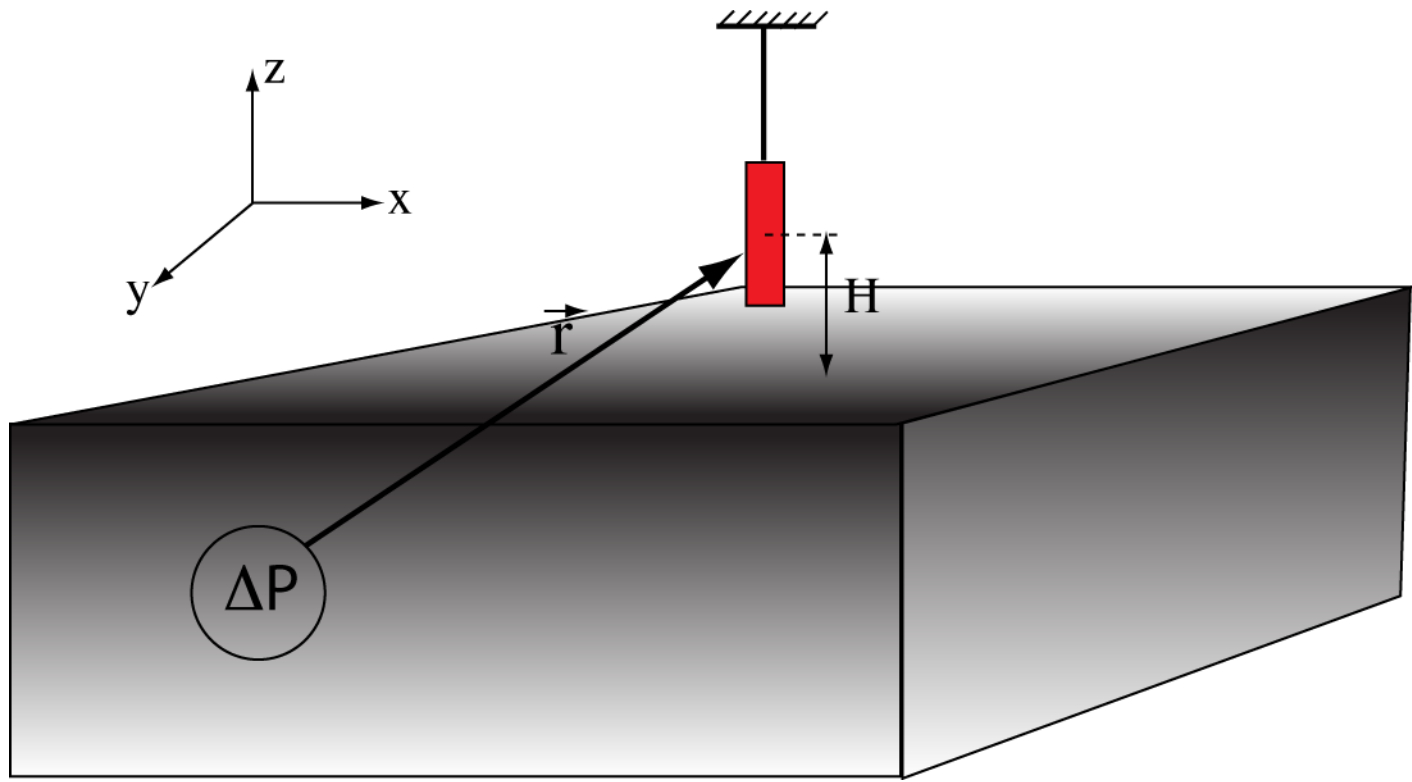
Rayleigh wave: λ approx 400 →



Verify analytic calculation of the NN due to a continuous pressure wave through a homogeneous half space.

Time = 5.25 Boundary X-displacement [m]

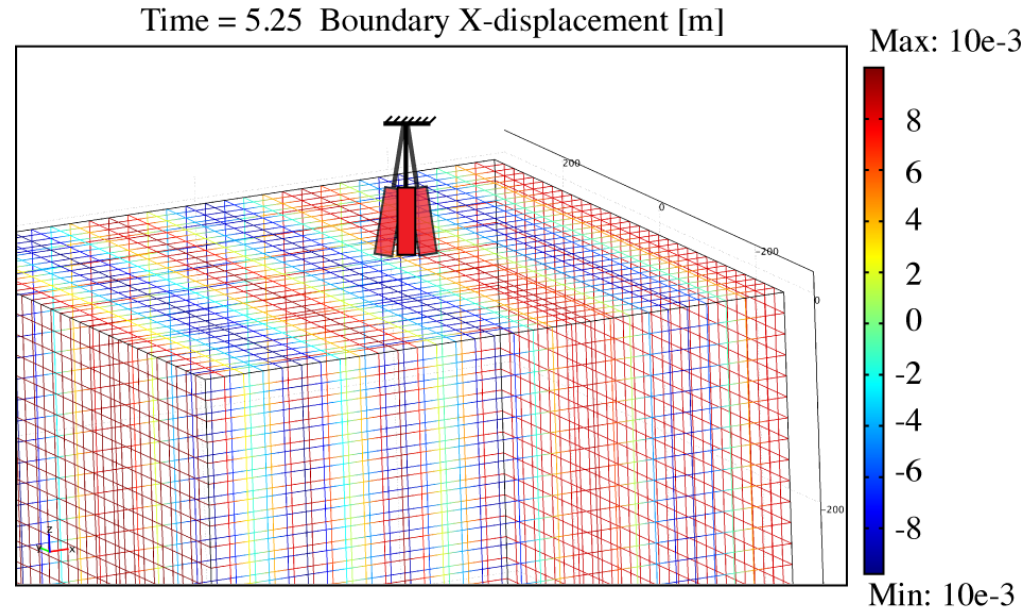




Pressure wave through a homogeneous half space in the positive x-direction:

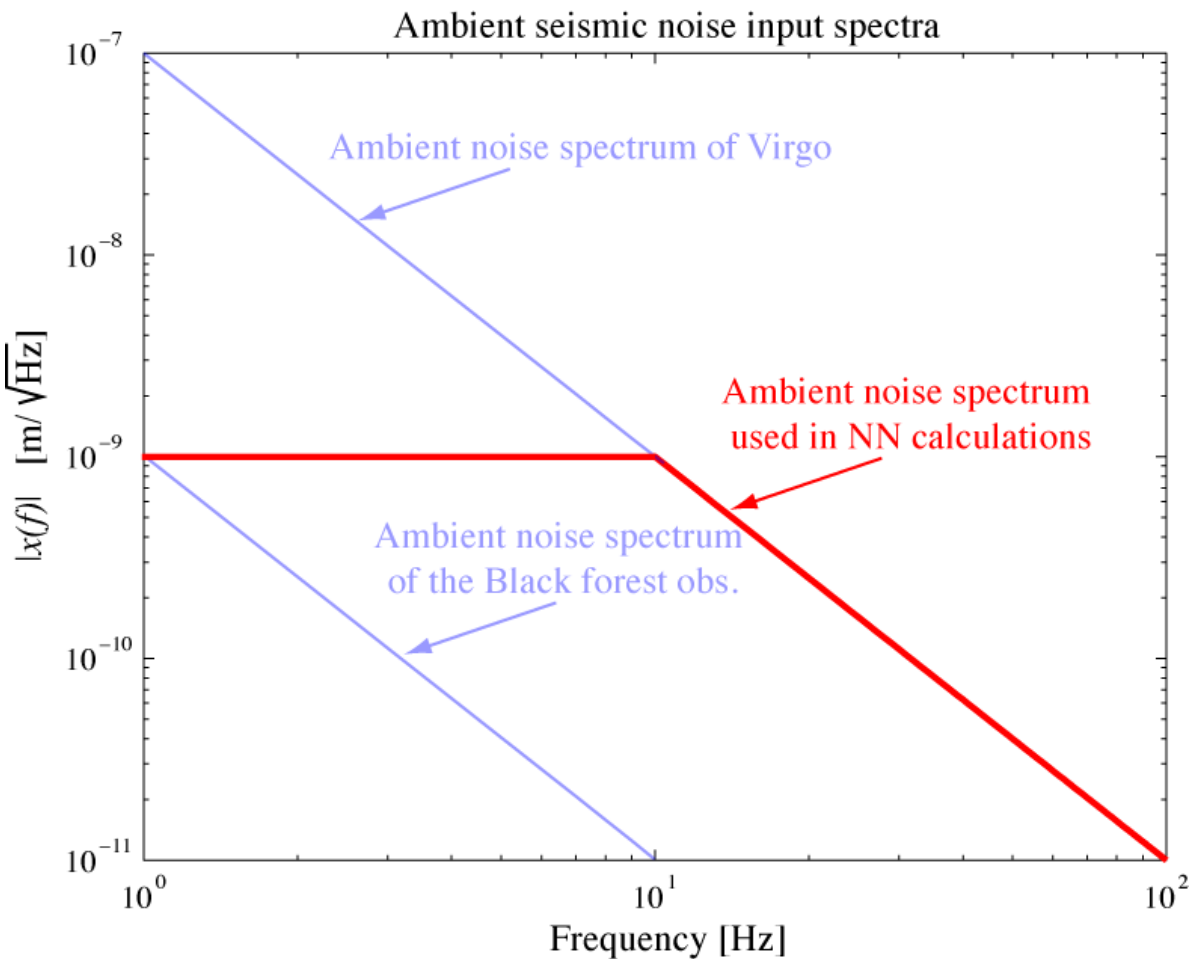
$$a_x = 2\pi G \rho u_0 e^{-2\pi(\mathcal{H}/\lambda)}$$

- For the FE model we can calculate the acceleration at our test point as:

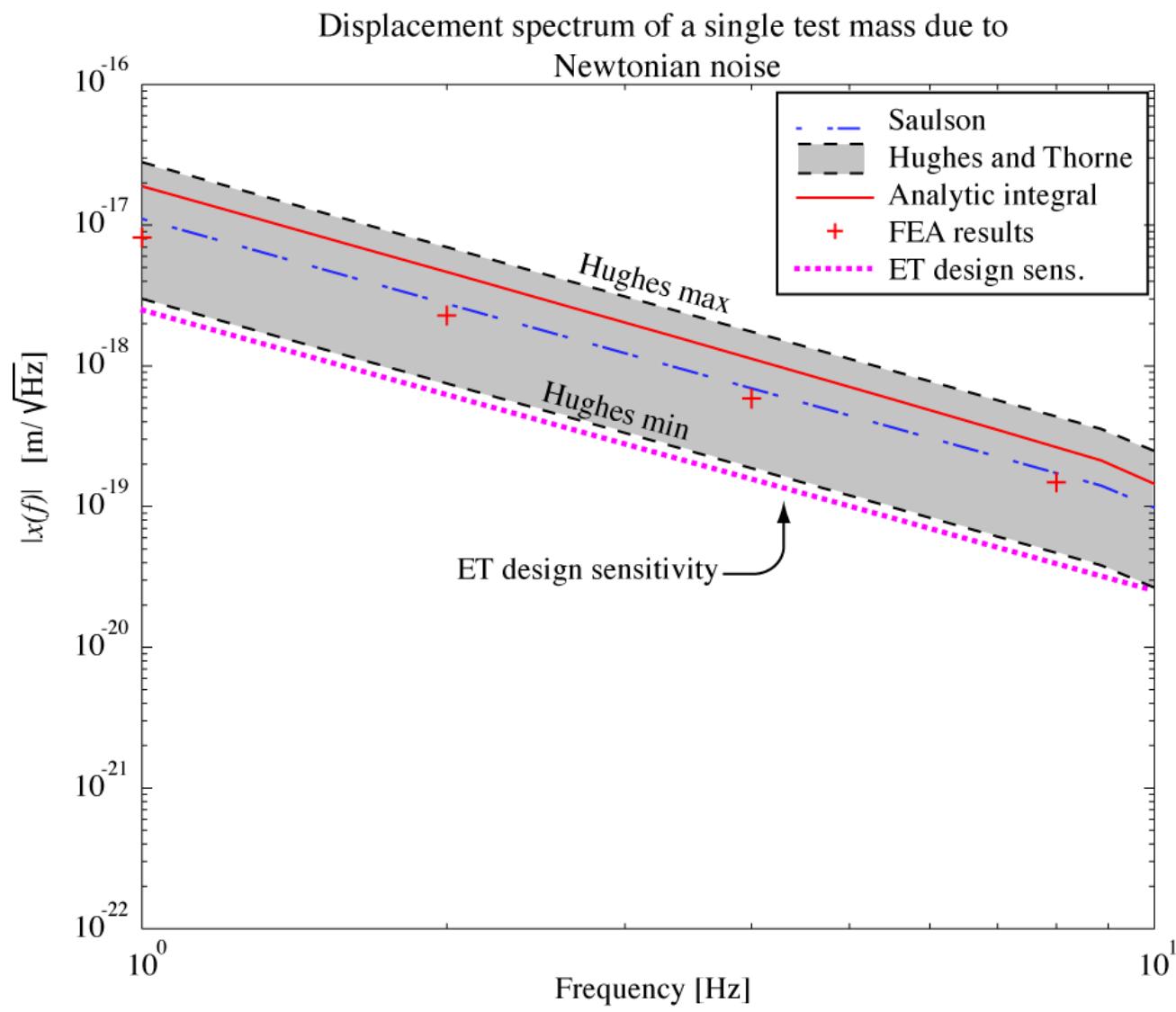


$$\vec{a}(t) = \sum_{x,y,z} \frac{-G\rho \Delta V}{(x^2 + y^2 + z^2)^{(5/2)}} \times \begin{bmatrix} 2x^2 - y^2 - z^2 & 3xy & 3xz \\ 3yx & 2y^2 - x^2 - z^2 & 3yz \\ 3zx & 3zy & 2z^2 - x^2 - y^2 \end{bmatrix} \begin{bmatrix} u(t) \\ v(t) \\ w(t) \end{bmatrix}$$

Surface detector input spectra



- $1e-9$ m/sqrt(Hz) between 1 and 10Hz
- $1e-9$ m/sqrt(Hz) ($1/f^2$) from 10Hz
- We know that for Virgo this does not represent the ambient seismic spectrum below 10Hz.

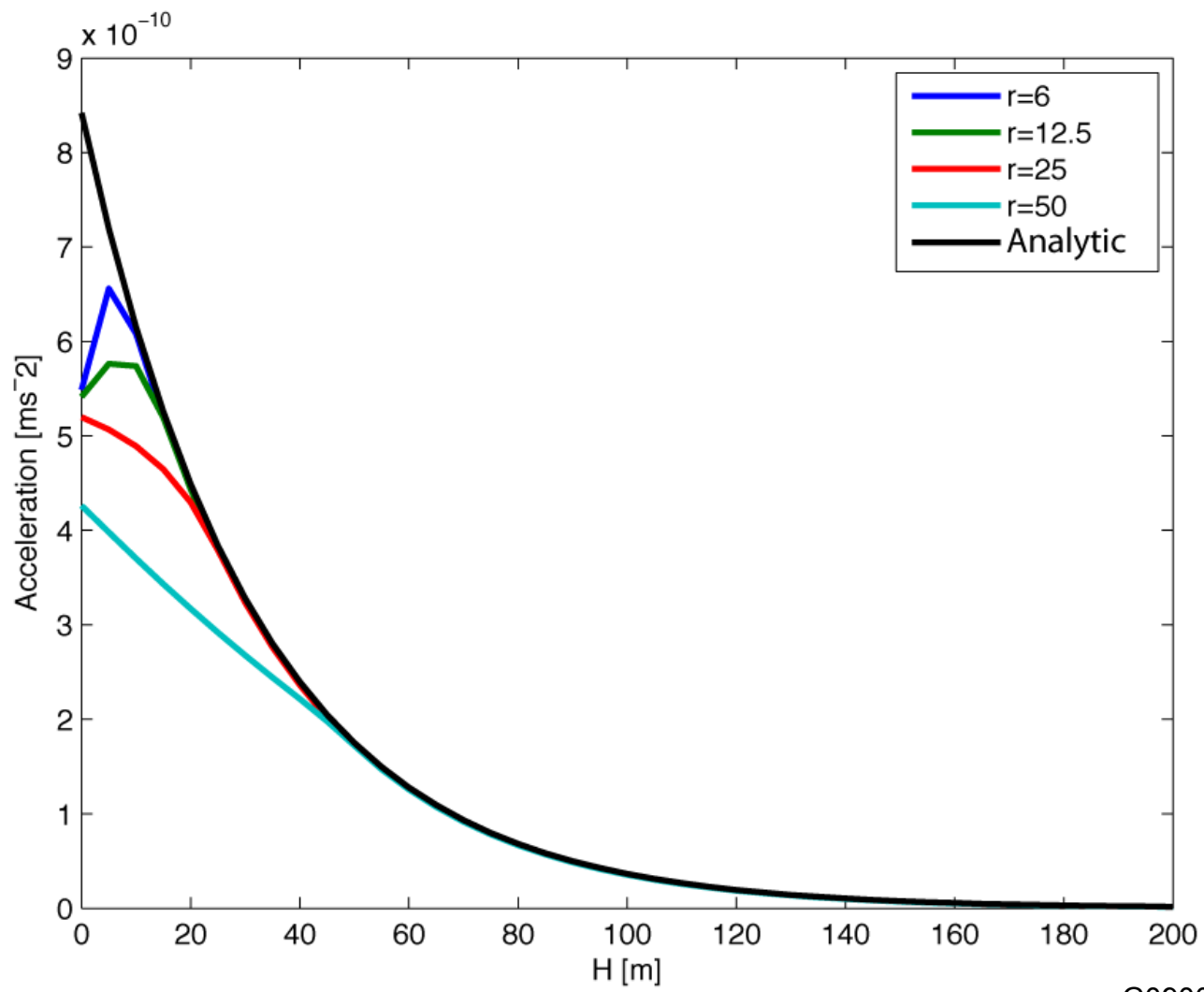


FE model input:

- Model dimensions: 1000 x 1000 x 1000m
- $E = 80\text{MPa}$
- $\rho = 2000 \text{ kg/m}^3$
- Integration cut-off 50m

FE mode Output:

- Elements volumes
- Node coordinates: x_i, y_i, z_i
- Node displacement: $u_i(t), v_i(t), w_i(t)$



G0900633-v1



Geometric suppression

- Why underground?

- Limit: contributions consistent with LNM

- 0.1 nm/rtHz at 1 Hz

- Cultural noise: dominated by surface waves

- Geometric suppression through integration

- Geometric suppression

- Surface layer: 3 km x 3 km x 50 m

- Grid of 20 m x 20 m x 20 m

- Cut-off 50 m

- 2D compression waves

- Wavelength 200 m

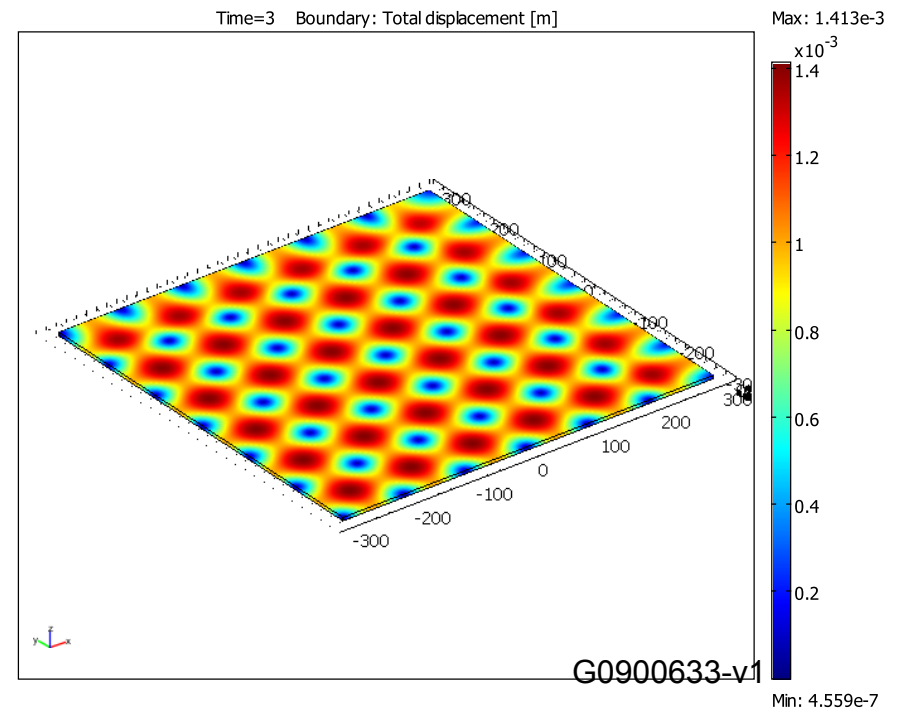
- Integrate NN versus distance

- More realistic model

- Surface wave amplitudes decay exponentially with depth

- Include compression waves

- Include incoherent sources



Geometric suppression

- Why underground?

Limit: contributions consistent with LNM

0.1 nm/rtHz at 1 Hz

Cultural noise: dominated by surface waves

Geometric suppression through integration

- Geometric suppression

Surface layer: 3 km x 3 km x 50 m

Grid of 20 m x 20 m x 20 m

Cut-off 50 m

2D compression waves

Wavelength 200 m

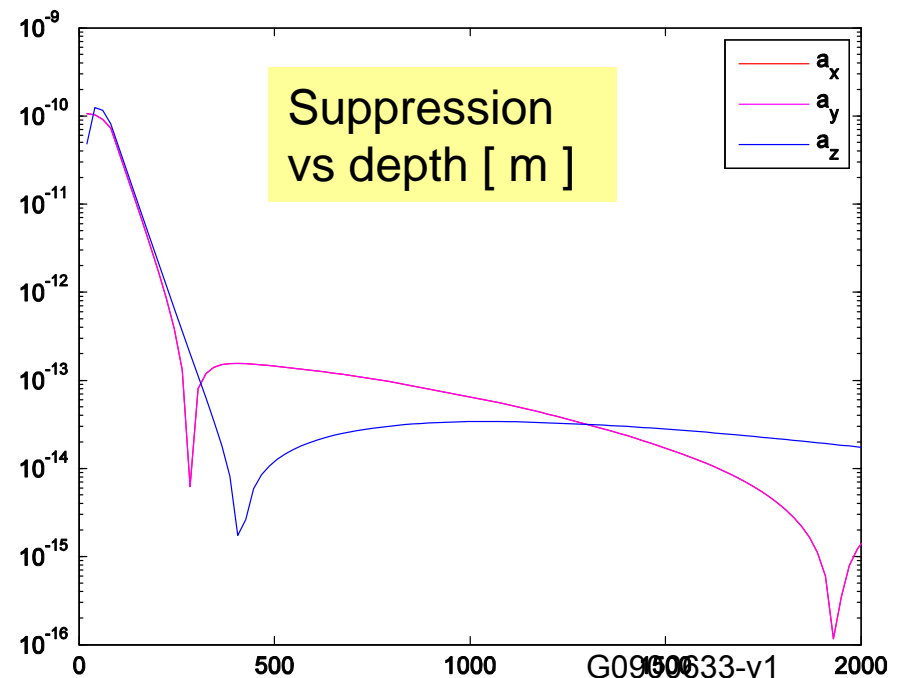
Integrate NN versus distance

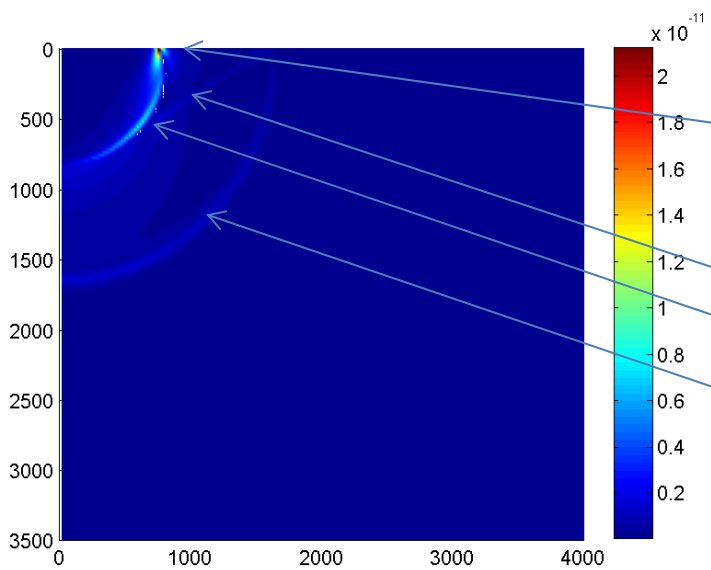
- More realistic model

Surface wave amplitudes decay exponentially with depth

Include compression waves

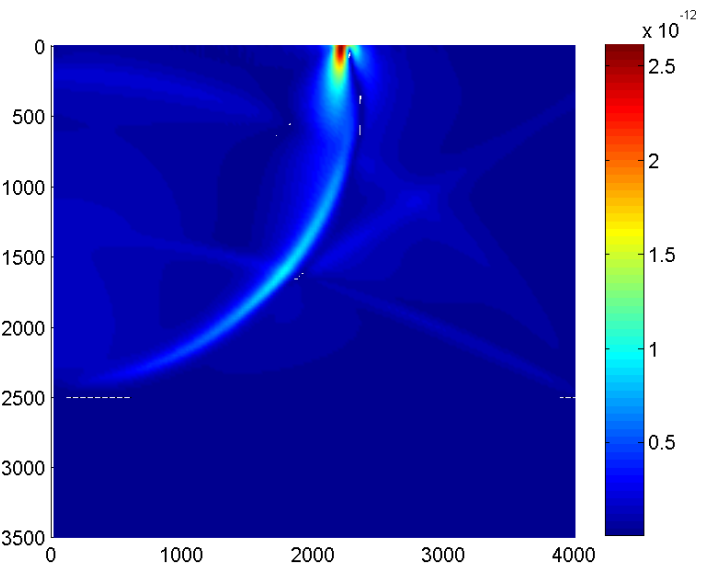
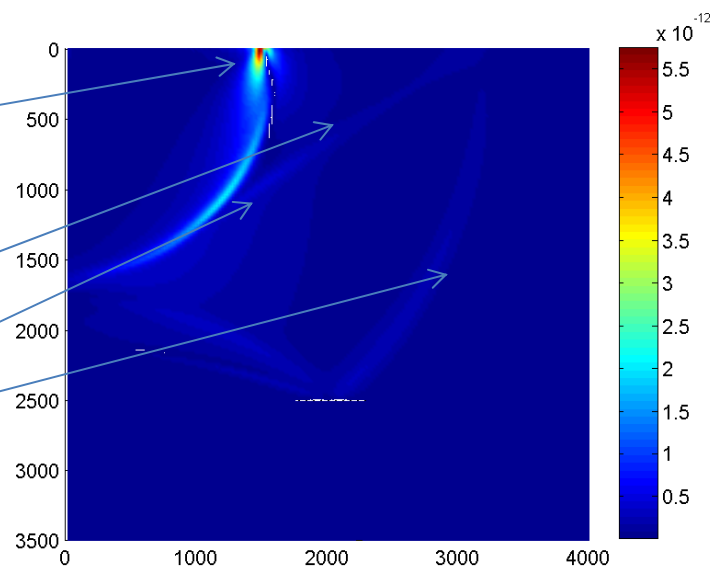
Include incoherent sources





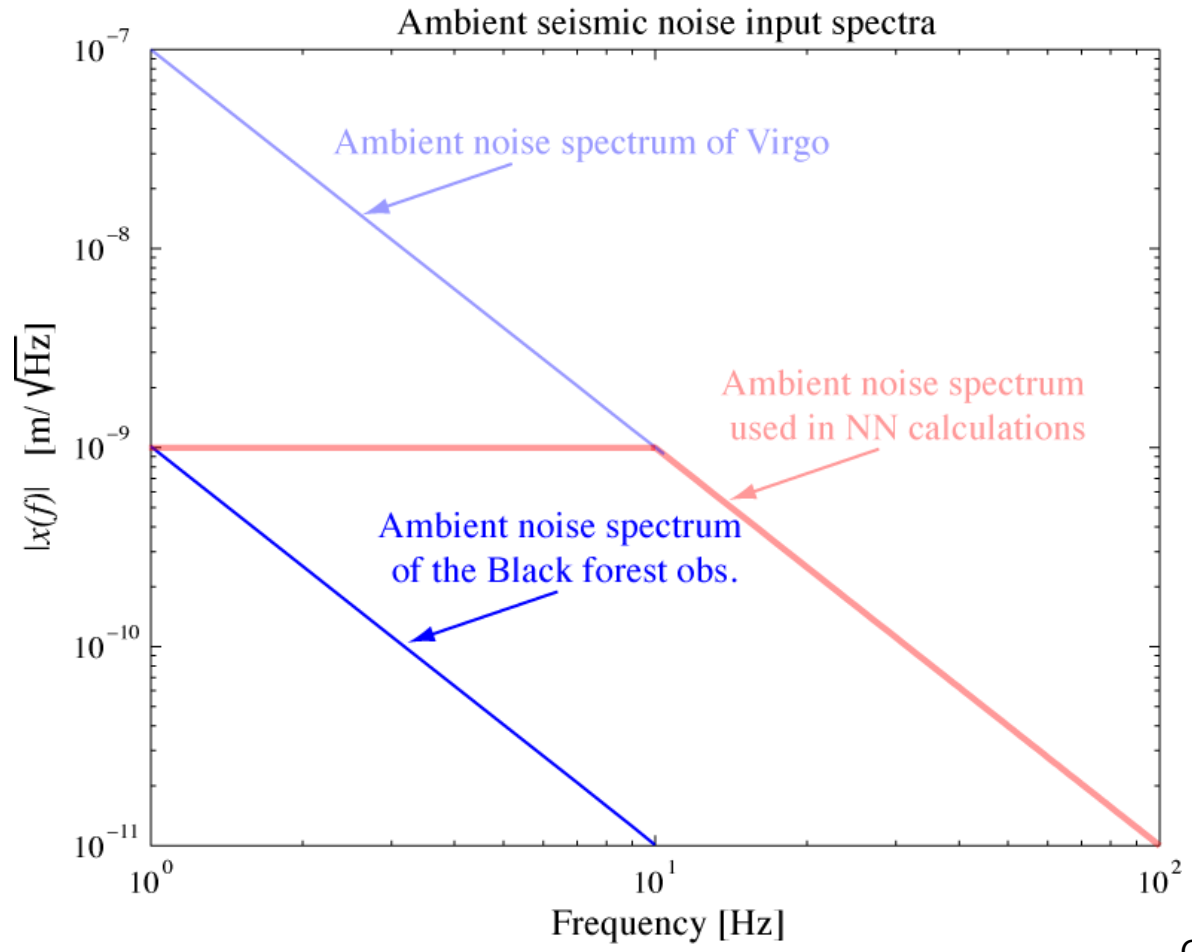
Surface waves
Rayleigh

Body waves
Head,
Shear, &
Pressure



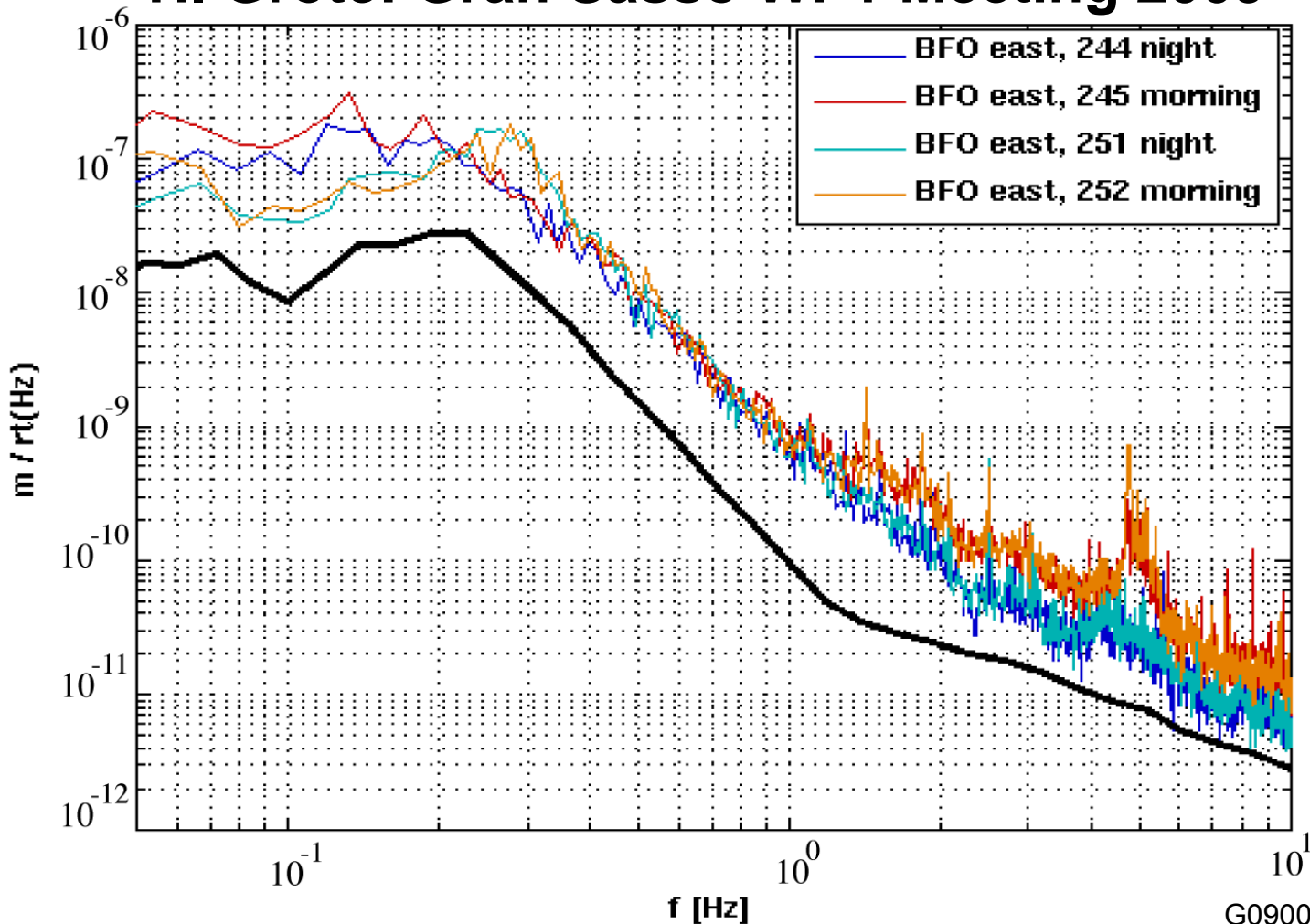
- Wave attenuation has two components
 - Geometrical (expansion of wave fronts) $\sim r^n$
 - Raleigh, $n=-1/2$
 - Body waves on surface, $n=-2$
 - Body waves at depth, $n=-1$
 - Material (damping).

Dummy calculation: Subterranean detector input spectra



BLACK FOREST OBERVATORY

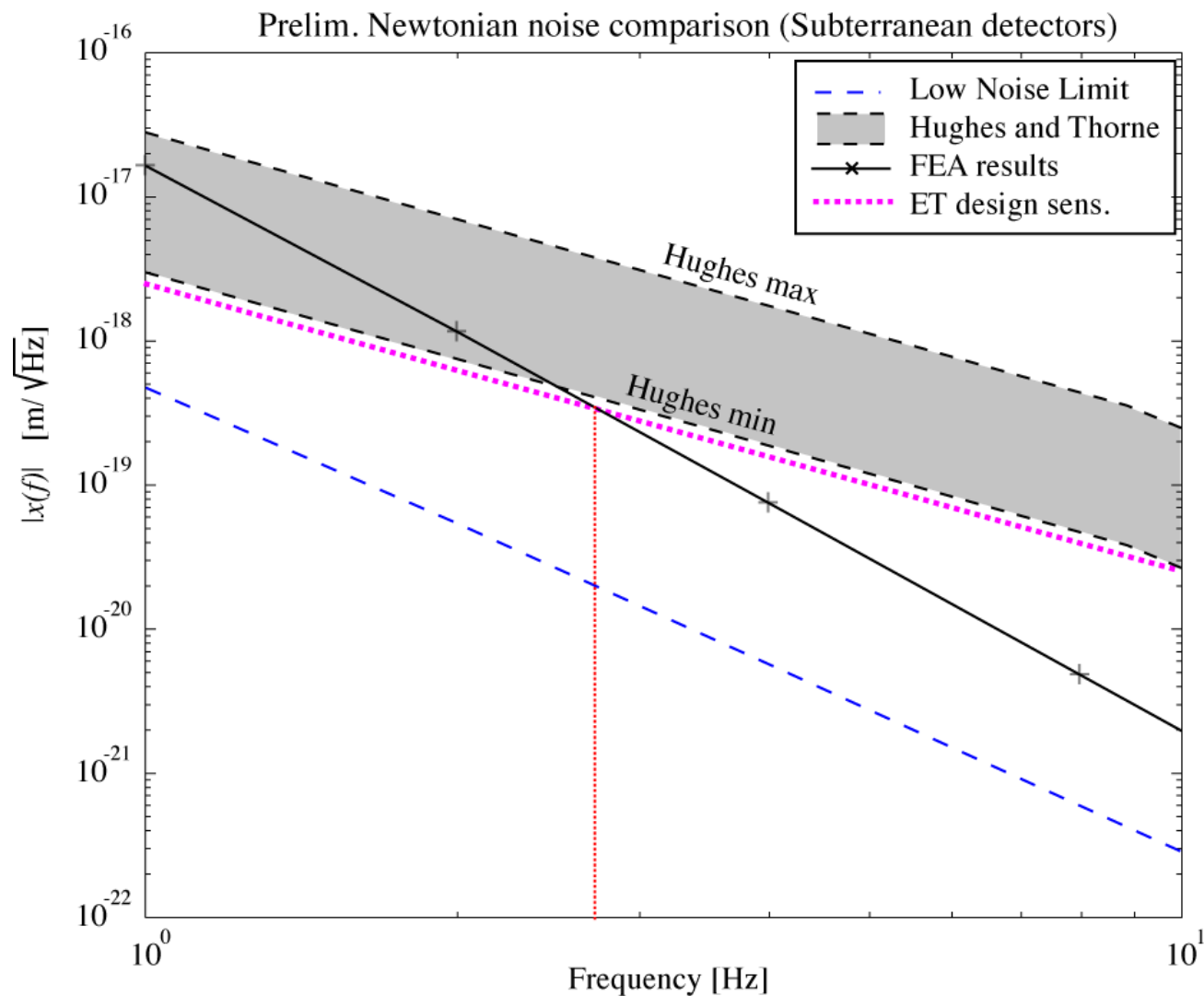
H. Grote: Gran Sasso WP1 Meeting 2009



G0900633-v1



Dummy calc: Displacement sensitivity for a single test mass



FE model input:

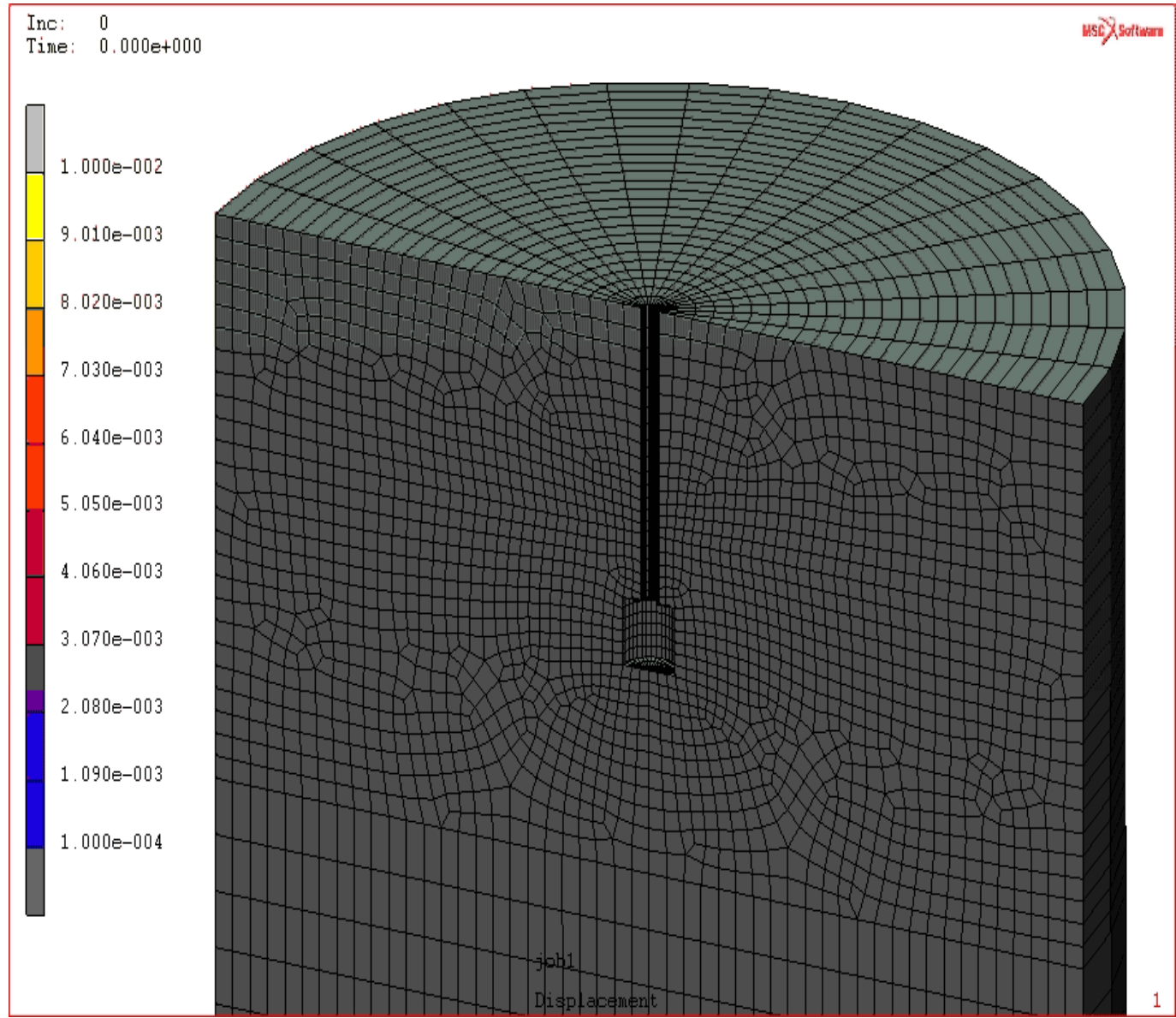
- Model dimensions: 1000 x 1000 x 1000m
- $E = 80\text{MPa}$
- $\rho = 2000 \text{ kg/m}^3$
- Integration cut-off 50m

FE mode Output:

- Elements volumes
- Node coordinates: x_i, y_i, z_i
- Node displacement: $u_i(t), v_i(t), w_i(t)$

- Future models should represent realistic geology (apply a Young's mod variation in soil layers).
- Test cavity geometries in homogeneous and inhomogeneous half spaces.
- Using complex a geology, test “seismometer” arrays to look at active subtraction schemes.

- Future models should represent realistic geology (apply a Young's mod variation in soil layers).
- Test cavity geometries in homogeneous and inhomogeneous half spaces.
- Using complex a geology, test “seismometer” arrays to look at active subtraction schemes.



FEM input:

- 200m clay:
 $E = 80\text{MPa}$
 $\rho = 3000\text{ kg/m}^3$
- 1300m granite:
 $E = 20\text{GPa}$
 $\rho = 3000\text{ kg/m}^3$
- cavity:
depth 260m
 $\text{Ø}50\text{m}$

FEM Output:

- Elements volumes
- Node coordinates:
 x_i, y_i, z_i
- Node displacement:
 $u_i(t), v_i(t), w_i(t)$

- Future models should represent realistic geology (apply a Young's mod variation in soil layers).
- Test cavity geometries in homogeneous and inhomogeneous half spaces.
- Using complex a geology, test “seismometer” arrays to look at active subtraction schemes.

Checking the acceleration at various test depths

