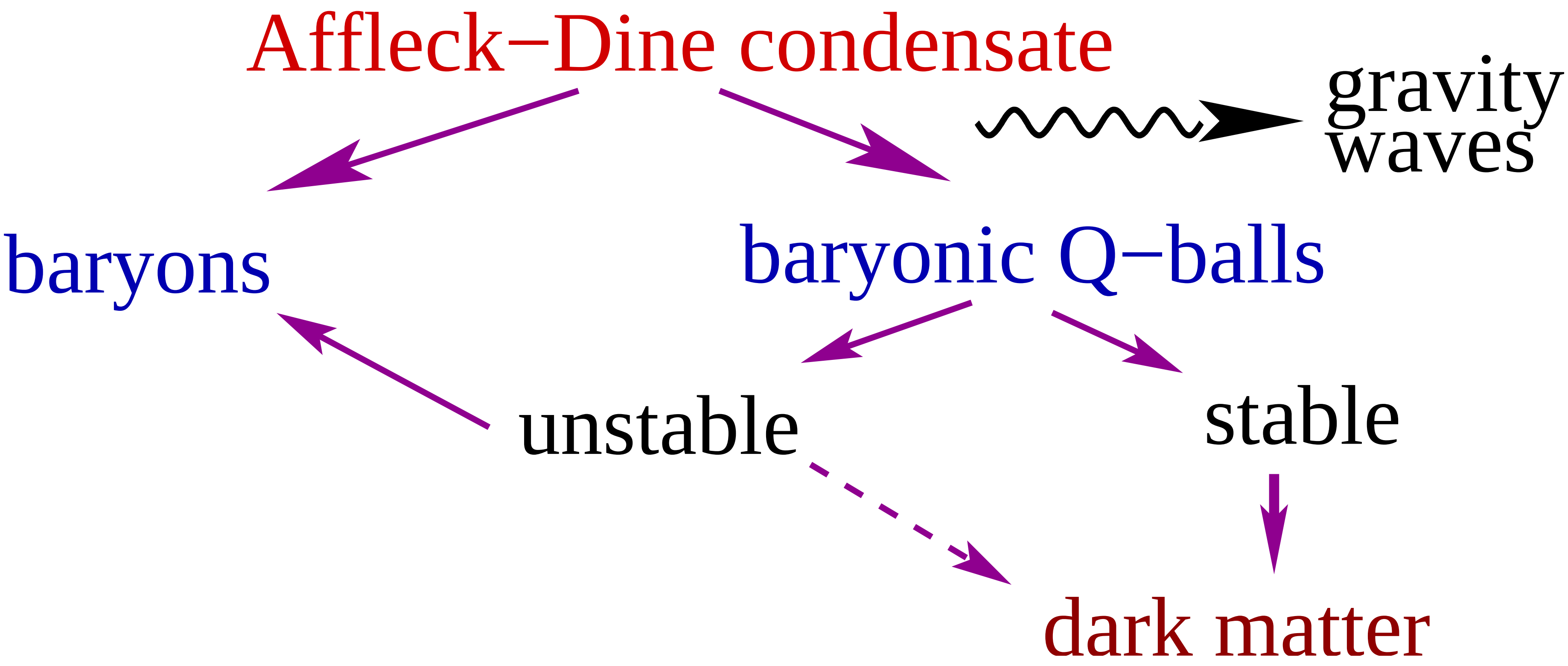
Echoes of supersymmetry: BAU, relic Q-balls, and gravity waves

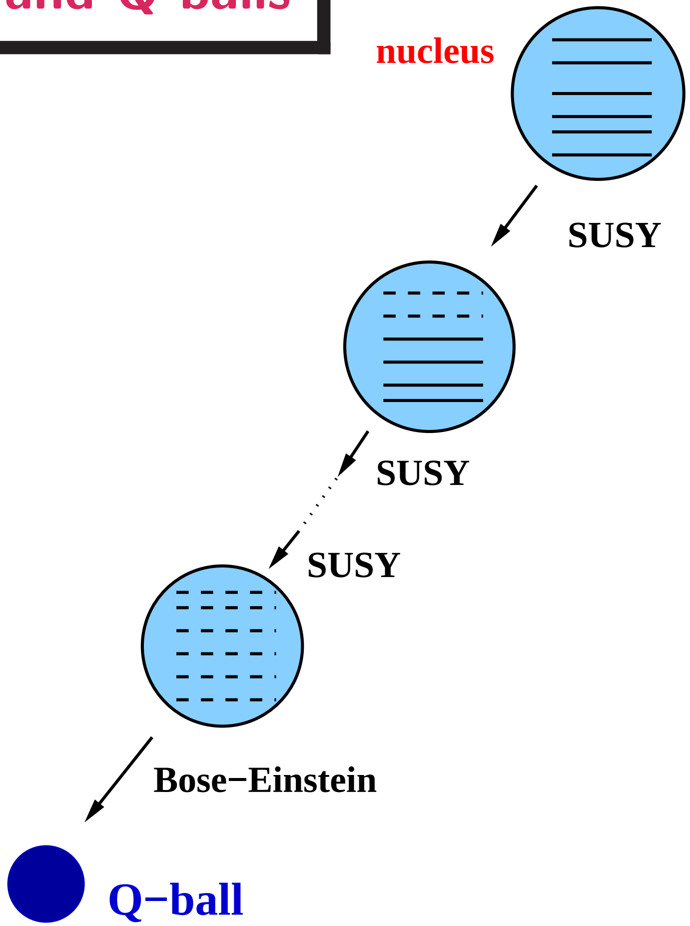
- Introduction: SUSY Q-balls
- Inflation+SUSY$\Rightarrow$ Q-balls
- stable Q-balls as dark matter
- constrains
- gravitational waves

Echoes of supersymmetry: BAU, relic Q-balls, and gravity waves



SUSY and Q-balls

Why would one suspect that
SUSY $\Rightarrow$ Q-balls?



Q-balls

Let us consider a complex scalar field $\phi(x,t)$ in a potential that respects a U(1) symmetry: $\phi \to e^{i\theta}\phi$.

vacuum: $\phi = 0$

conserved charge: $Q = \frac{1}{2i}\int \left(\phi^\dagger \overleftrightarrow{\partial}_0 \phi\right) d^3x$

$Q \neq 0 \Rightarrow \phi \neq 0$ in some finite domain

$\Rightarrow$ Q-ball [Rosen; Friedberg, Lee, Sirlin; Coleman]

Q-balls exist if

$$U(\phi)\,/\phi^2 = \min, \quad \text{for } \phi = \phi_0 > 0$$

[Coleman]

Finite ϕ_0: $M(Q) \propto Q$

Flat potential ($U(\phi) \sim \phi^p, p < 2$); $\phi_0 = \infty$:

$$M(Q) \propto Q^\alpha, \alpha < 1$$

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Q-balls exist in (softly broken) SUSY because

- the theory has scalar fields
- the scalar fields carry conserved global charge (baryon and lepton numbers)
- attractive scalar interactions (tri-linear terms, flat directions) force $(U(\phi)/\phi^2) = \min$ for non-vacuum values.

MSSM, gauge mediated SUSY breaking

Baryonic Q-balls (B-balls) are entirely stable if their mass per unit baryon charge is less than the proton mass.

$M(Q) = M_S Q^{3/4} \Rightarrow$

$\frac{M(Q_B)}{Q_B} \sim M_S Q^{-1/4} < 1\mathrm{GeV}$

$$\text{for } Q_B \gg \left(\frac{M_S}{1\ \mathrm{TeV}}\right)^4 \gtrsim 10^{12}$$

Such B-balls are entirely stable.

Baryon asymmetry

$$\eta \equiv \frac{n_B}{n_\gamma} = \left(6.1\,^{+0.3}_{-0.2}\right) \times 10^{-10} (\mathrm{WMAP})$$

What happened right after the Big Bang?

- Inflation probably took place
- Baryogenesis – definitely *after* inflation

Standard Model is not consistent
with the observed baryon asymmetry (assuming inflation)

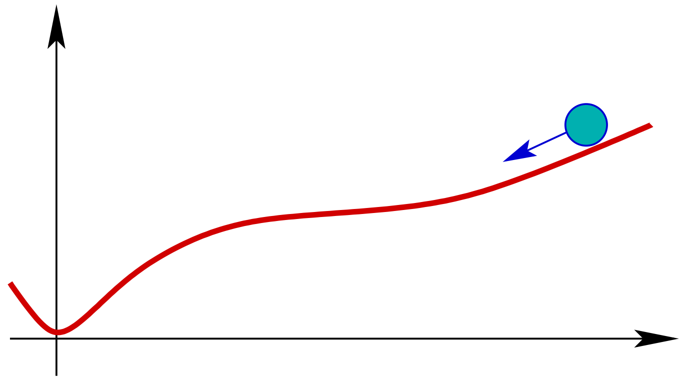
Affleck–Dine baryogenesis

- Natural if SUSY+Inflation
- Can explain matter
- Can explain **dark** matter
- Predictions can be tested soon

Inflation

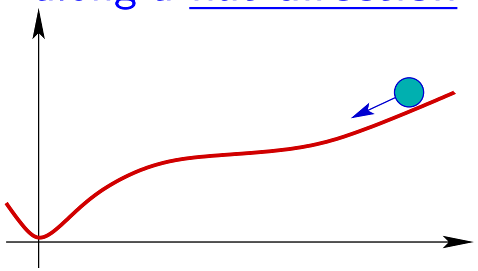
All matter is produced during reheating after inflation.

SUSY $\Rightarrow$ flat directions.
During inflation, scalar fields
are displaced from their minima.



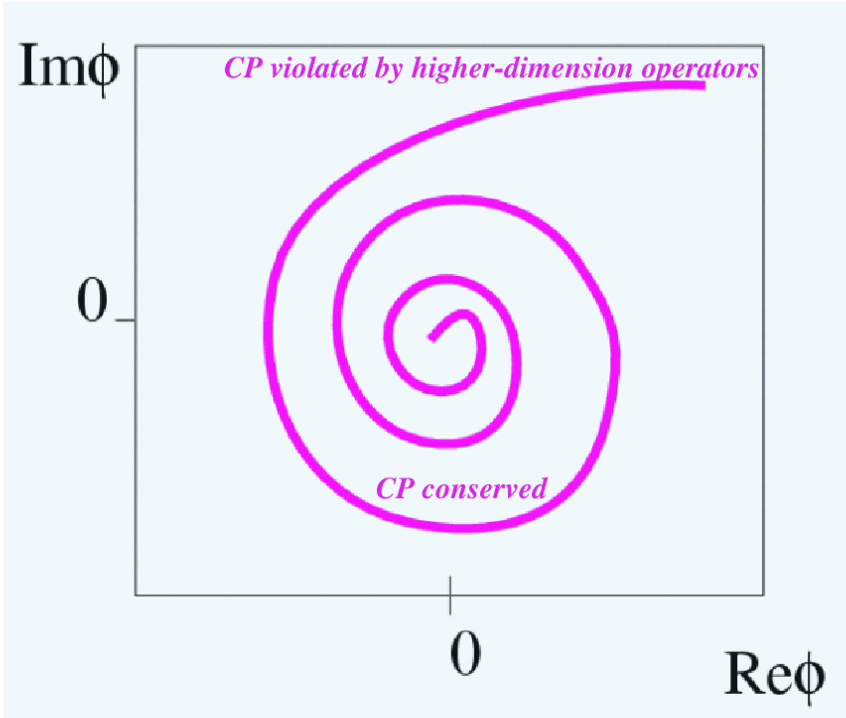
Affleck – Dine baryogenesis

at the end of inflation
a scalar condensate
develops a large VEV
along a **flat direction**

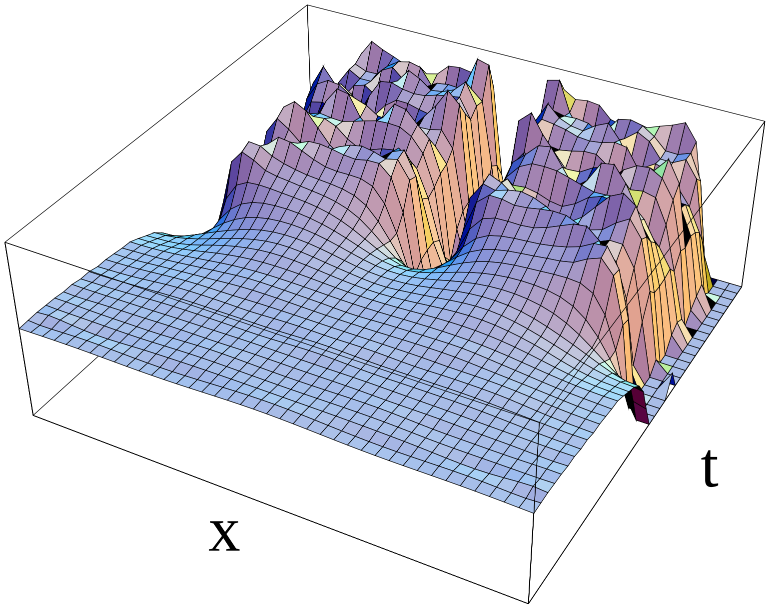


CP violation is due to time-dependent background.

Baryon asymmetry: $\phi = |\phi| e^{i\omega t}$



Fragmentation of the Affleck-Dine condensate



[AK, Shaposhnikov]

small inhomogeneities can grow

unstable modes:

$$0 < k < k_{\max} = \sqrt{\omega^2 - U''(\phi)}$$

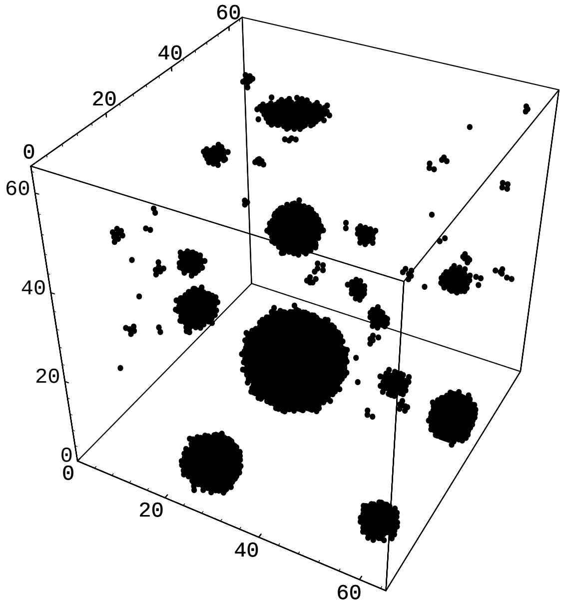
$\Rightarrow$ **Lumps of baryon condensate**

$\Rightarrow$ **Q-balls**

Fragmentation ≈ pattern formation

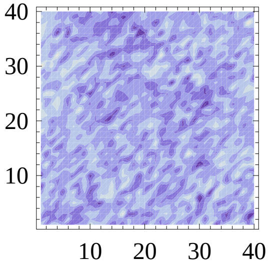
Familiar example:

Numerical simulations of the fragmentation

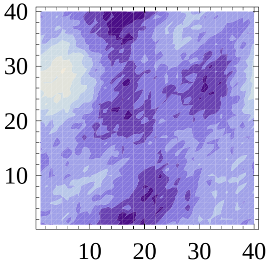


[Kasuya, Kawasaki]

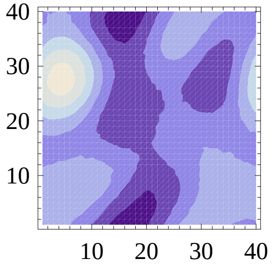
Two-dimensional charge density plots [Multamaki].



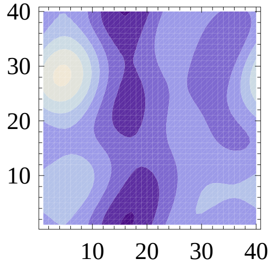
(a) $mt = 0$



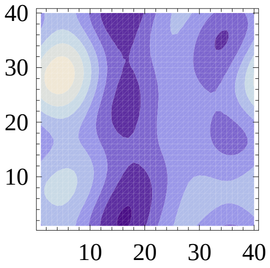
(b) $mt = 75$



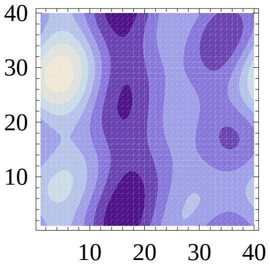
(c) $mt = 150$



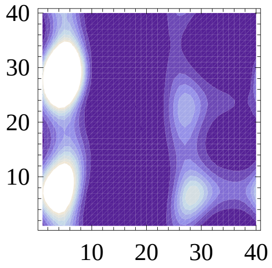
(d) $mt = 375$



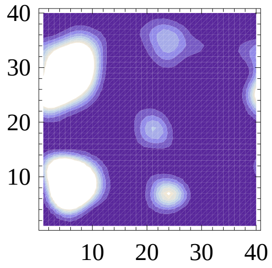
(e) $mt = 525$



(f) $mt = 675$

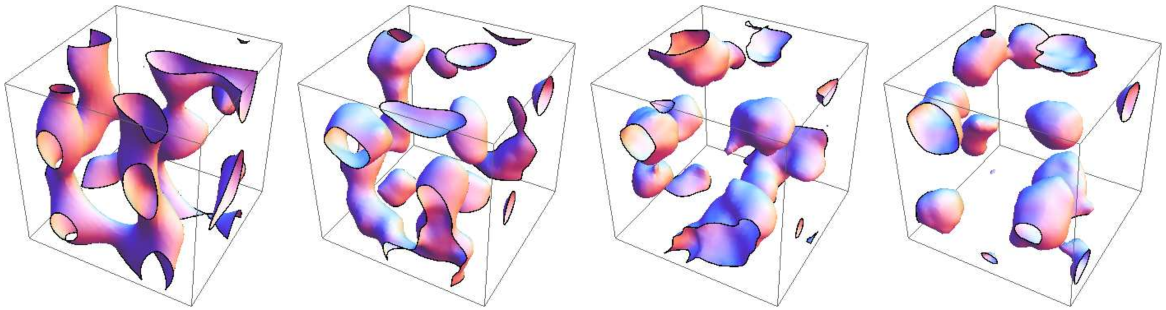


(g) $mt = 825$



(h) $mt = 900$

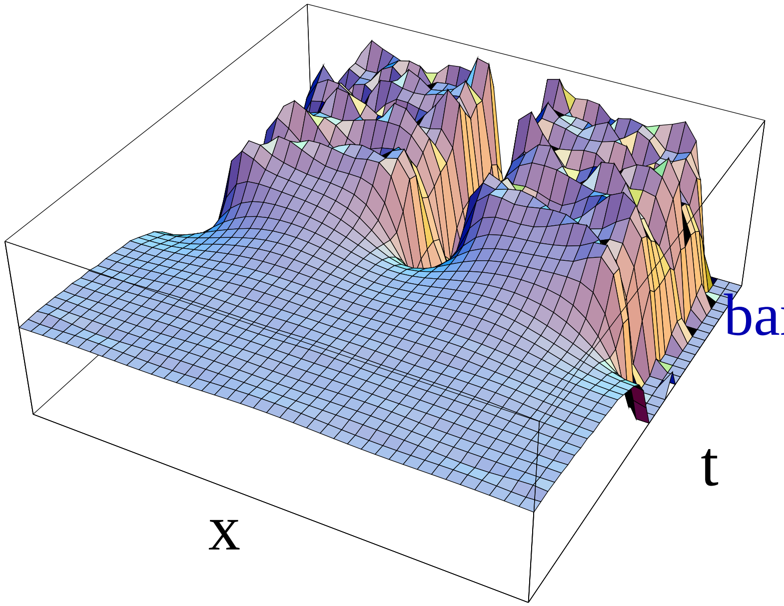
Three-dimensional charge density plots [Multamaki].

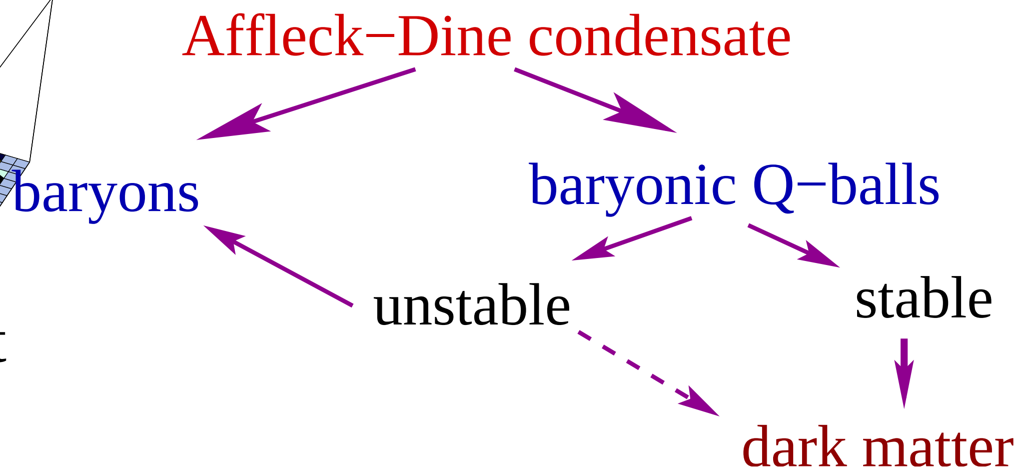


(i) $mt = 900$ (j) $mt = 1050$ (k) $mt = 1200$ (l) $mt = 1350$

Fragmentation of AD condensate can produce Q-balls

SUSY Q-balls may be stable or unstable
if stable ⇒ **dark matter**





[AK, Shaposhnikov; Enqvist, McDonald]

Stable Q-balls as dark matter

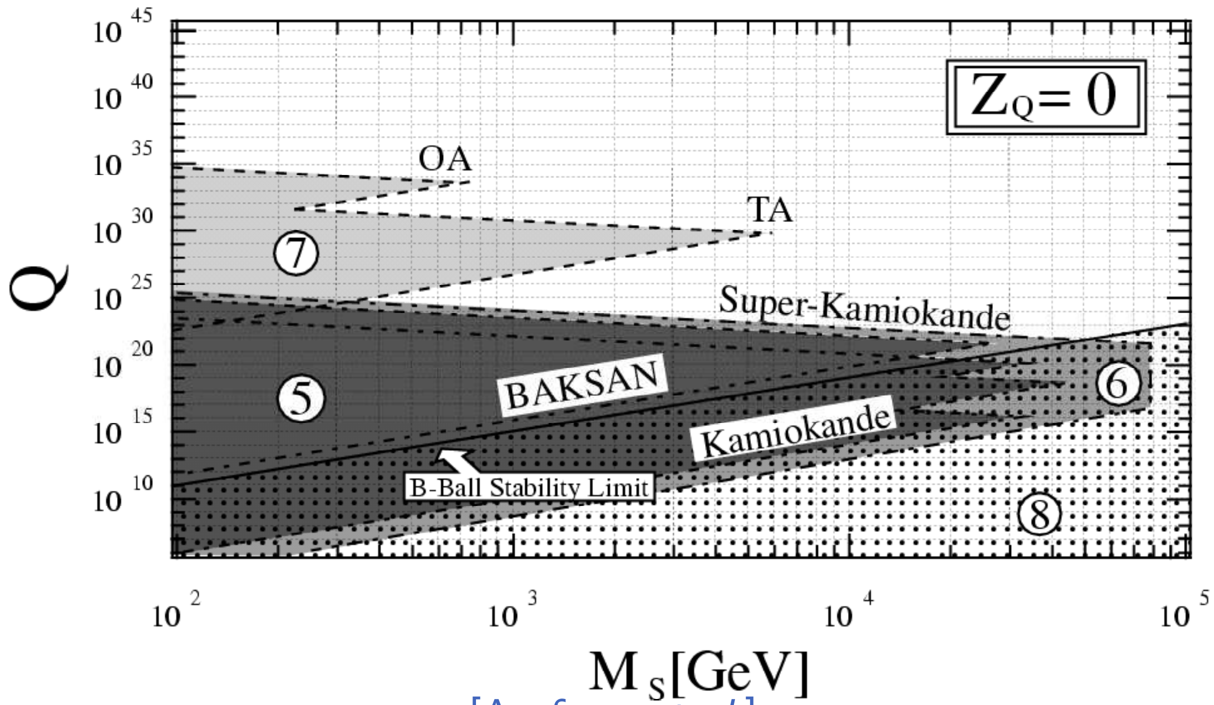
Q-balls can accommodate baryon number at lower energy than a nucleon $\Rightarrow$ **B-Balls catalyze proton decay** Signal:

$$\frac{dE}{dl} \sim 100 \left(\frac{\rho}{1\,\mathrm{g/cm^3}}\right) \frac{\mathrm{GeV}}{\mathrm{cm}}$$

Heavy $\Rightarrow$ low flux

$\Rightarrow$ **experimental limits from Super-Kamiokande and other *large* detectors**

Present experimental limits



[Arafune *et al.*];

A "candidate event"

C.M.G. Lattes et al., Hadronic interactions of high energy cosmic-ray observed by emulsion chambers

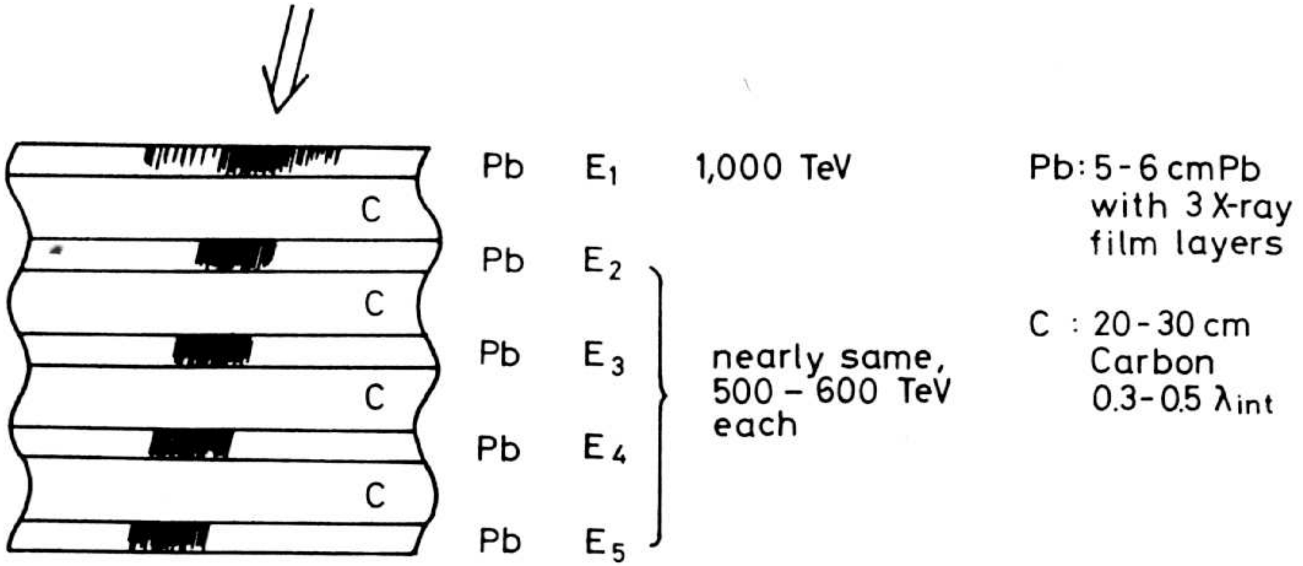


Fig. 47. Illustration of penetrating cores of Pamir experiment.

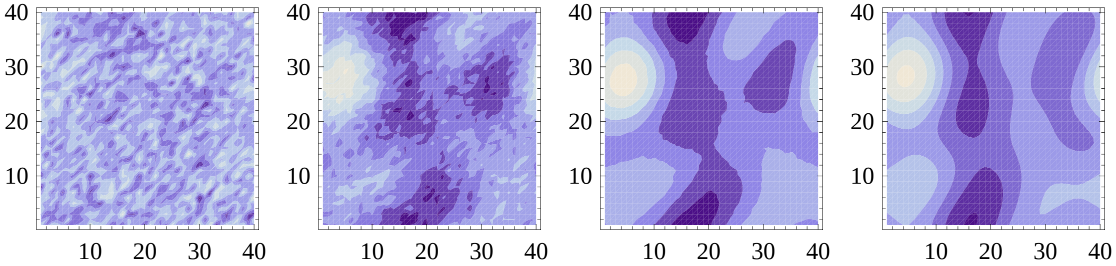
[Lattes, Fujimoto and Hasegawa, Phys.Rept. **65**, 151 (1980)]

Gravitational radiation from the fragmentation process

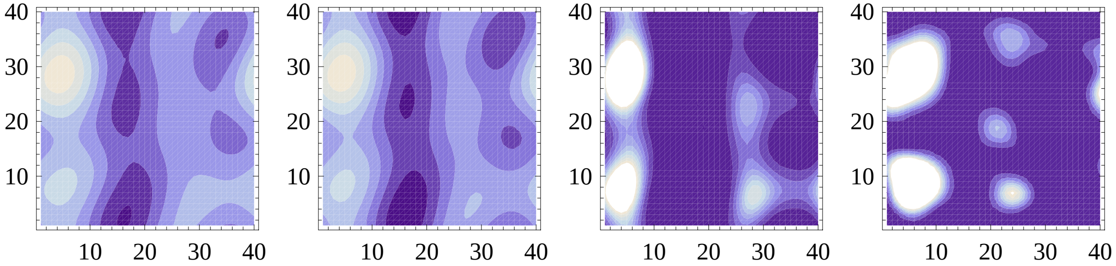
One can expect gravitational waves if

- large masses move around
- relativistic velocities
- no spherical symmetry

All of these conditions can be satisfied for *some* flat directions.

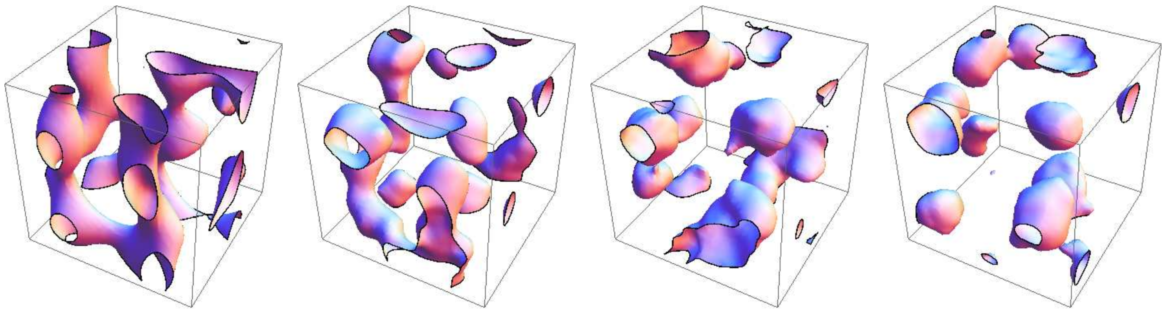


(m) $mt = 0$ (n) $mt = 75$ (o) $mt = 150$ (p) $mt = 375$



(q) $mt = 525$ (r) $mt = 675$ (s) $mt = 825$ (t) $mt = 900$

Two-dimensional charge density plots.



(u) $mt = 900$ (v) $mt = 1050$ (w) $mt = 1200$ (x) $mt = 1350$

Three-dimensional charge density plots.

The lack of spherical symmetry in the early steps of fragmentation means gravity waves can be produced.

Analytical estimates

The mass density of the condensate undergoing fragmentation can be written as $\rho(x,t) = \rho_0 + \rho_1(x,t)$, where

$$\rho_1(x,t) = \epsilon\rho_0 \int d^3k\, e^{\alpha_k t} \cos(\omega t - \vec{k}\cdot\vec{x}) \,.$$

The quadrupole moment that generates gravity waves:

$$D_{ij} = \int d^3x\; x_i x_j\, T^{00}(x,t) \,,$$

where the energy-momentum tensor $T^{00}(x,t) \approx \rho(x,t)$.

Based on the analytical and numerical calculations of the condensate fragmentation

[Kawasaki et al.],

$$k \sim \xi_k \times 10^2 H_*, \; \omega_k \sim vk \sim \xi_k \times 10^2 \, vH_*,$$

where H_* is the Hubble constant at the time of the condensate.

For $\omega \sim 10^2 v H_*$, the power in gravitational waves in a Hubble volume:

$$P \sim 10^4 \xi_k^{-2} \, G \frac{\rho_0^2 v^6}{H_*^4} \, .$$

For mode $\phi(x,t) \approx R(t) \exp\{\alpha_k t\} \cos(\omega_k t - kx)$, where $R(t)$ is a slowly changing function of time, At the time of production [AK, Mazumdar],

$$\Omega_{GW*} \sim 10^{-3} \xi_k^{-3} \xi_v^6 \frac{\rho_0^2}{(H_* M_{\rm Pl})^4}$$

The energy density depends on the type of SUSY breaking and the type of flat direction.

Strong gravitational waves:

- gravity mediated SUSY breaking (more mass per scalar)
- not the flat direction of AD baryogenesis: $\eta_B = n_B/n_\gamma \sim 10^{-10}$ too small
- $(B+L)$ flat directions OK: sphalerons destroy $(B+L)$, so there is no constraint on the initial density carried by the $(B+L)$ flat directions.

Predictions:

Peak frequency of the gravitational radiation observed today, $f_* = \omega_k/2\pi$:

$$f = f_* \frac{a_*}{a_0} = f_* \left(\frac{a_*}{a_{\rm rh}}\right) \left(\frac{g_{s,0}}{g_{s,{\rm rh}}}\right)^{1/3} \left(\frac{T_0}{T_{\rm rh}}\right)$$

$$\approx 0.6 \,{\rm Hz}\, \xi_k \xi_v \left(\frac{g_{s,{\rm rh}}}{100}\right)^{1/6} \left(\frac{T_{\rm rh}}{10^3 \,{\rm TeV}}\right) \left(\frac{f_*}{10 H_*}\right) ,$$

$T_{\rm rh} \sim 1 \,{\rm TeV} \Rightarrow$ mHz frequency, accessible to LISA $T_{\rm rh} \sim 10^3 - 10^5 \,{\rm TeV} \Rightarrow$ 10–100 Hz frequency, accessible to LIGO and BBO

Spectral signature: signal is peaked at the longest wavelength, determined by the size of the Q-balls, and it falls off as $1/f^3$ for larger frequencies.

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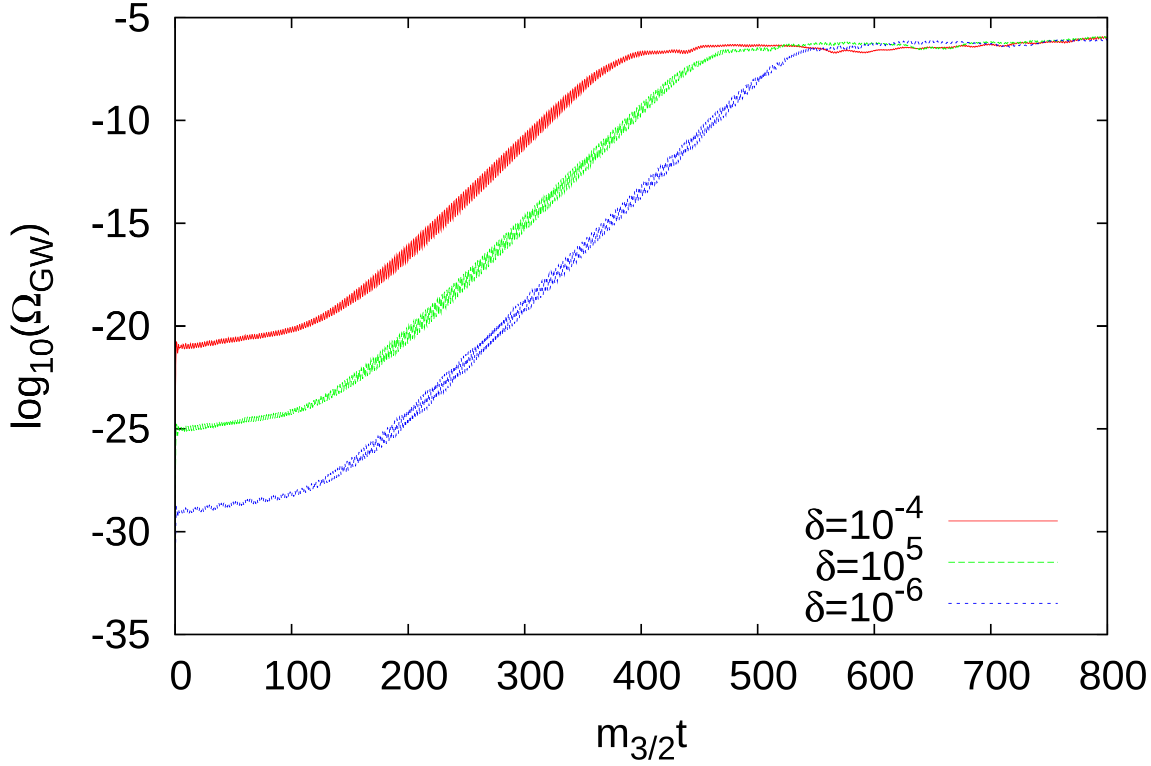
The fraction of the critical energy density ρ_c stored in the gravity waves today is

$$\begin{aligned} \boldsymbol{\Omega}_{\mathbf{GW}} &= \Omega_{\mathrm{GW}*} \left(\frac{a_*}{a_0}\right)^4 \left(\frac{H_*}{H_0}\right)^2 \\ &\approx \frac{1.67 \times 10^{-5}}{h^2} \left(\frac{100}{g_{s,*}}\right)^{1/3} \Omega_{\mathrm{GW}*} \approx \mathbf{10^{-8}}\, \boldsymbol{\xi_k^{-3} \xi_v^6 \, h^{-2}} \end{aligned}$$

LISA band: $\Omega_{\mathrm{GW}} h^2 \sim 10^{-11}$ at mHz frequencies

LIGO band: $\Omega_{\mathrm{GW}} h^2 \sim (10^{-5} - 10^{-11})$ in the $(5 - 10^3)$ Hz frequency band.

Numerical simulations [AK, Mazumdar, Multamäki]



Conclusion

- SUSY + Inflation $\Rightarrow$ Q-balls, some may be stable, may be dark matter
- Typical size large $\Rightarrow$ typical density small $\Rightarrow$ need large detectors to search for relic Q-balls
- Gravitational waves from the fragmentation of $(B+L)$ flat directions may be observed by LIGO and LISA.
- **Gravitational waves detectors can detect echoes of primordial supersymmetry**