Estimating the structure of a Stochastic Gravitational Wave background

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Amaldi 8

New York

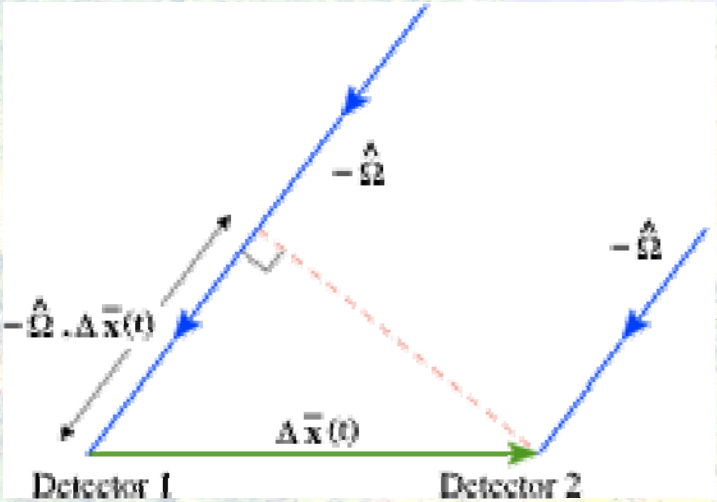
Stefan Ballmer, Caltech
For the LIGO Scientific Collaboration

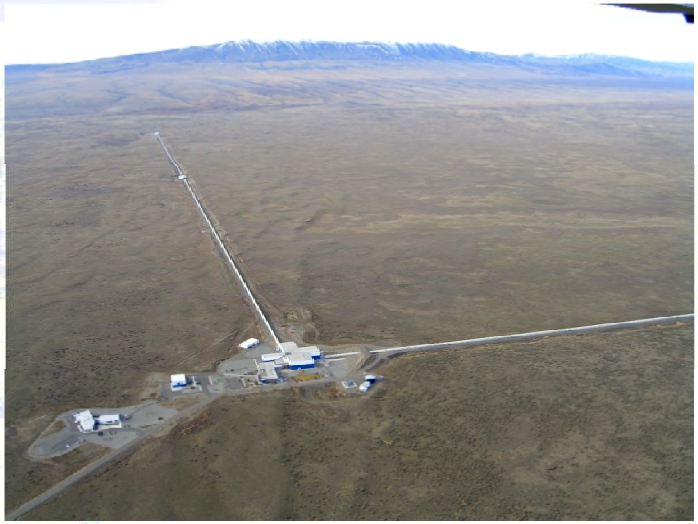
A stochastic GW background

- Correlations between separated detectors

➔Time shift and X-correlate!

$$\int \mathrm{d}t\, s_1(t)\, s_2(t - \hat{\mathbf{q}} \cdot \boldsymbol{\Delta}\mathbf{x}(t)/c) \ldots$$





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LIGO

Why a new method?

- Up to now:
  - Isotropic search (Vuk Mandic's talk yesterday)
    - One number as result
    - But isotropy an assumption
  - Point source search (radiometer)
    - Optimal SNR for point sources
    - But maps are rather blurred ("dirty")
    - "Un-blurring" required…
- Can we do all of the above at once AND get a good estimate of a diffuse background?

Maximum likelihood formalism

- The Measurements:
  - Detector X-Power as function of time and frequency (SFT products).

$$C_{ft} = s_1^*(f,t)s_2(f,t)$$

  - They are all independent, i.e.

$$N_{ft,f't'} := < (C_{ft} - < C_{ft} >)^* (C_{ft} - < C_{ft} >) > \approx \delta_{tt'}\delta_{ff'}P_1(f,t)P_2(f,t)$$

Maximum likelihood formalism

- Model:
  - Distribution:

$$P(\Omega) = \sum_{lm} P_{lm} Y_{lm}$$

Expectation value:

$$< C_{ft} >= H(f) \gamma_{ft,lm} P_{lm}$$

- Probability distribution: …just Gaussian…

$$p(C_{ft}|P_{lm}) \propto \exp\left[-\frac{1}{2}\left(C^*_{ft} - H(f)\gamma^*_{ft,lm}P^*_{lm}\right)(N^{-1})_{ft,f't'}\left(C_{f't'} - H(f')\gamma_{f't',l'm'}P_{l'm'}\right)\right]$$

Maximum likelihood formalism

- Probability distribution:

$$p(C_{ft}|P_{lm}) \propto \exp\left[-\frac{1}{2}\left(C^*_{ft} - H(f)\gamma^*_{ft,lm}P^*_{lm}\right)(N^{-1})_{ft,f't'}\left(C_{f't'} - H(f')\gamma_{f't',l'm'}P_{l'm'}\right)\right]$$

- Is maximal at

$$P_{lm} = (\Gamma^{-1})_{ml,l'm'}X_{l'm'}$$

$$\Gamma_{lm,l'm'} = \gamma^*_{ft,lm}\frac{H^2(f)}{P_1(f,t)P_2(f,t)}\gamma_{ft,l'm'}$$

$$X_{lm} = \gamma^*_{ft,lm}\frac{H(f)}{P_1(f,t)P_2(f,t)}C_{ft}$$

- H(f): Spectral shape
- $\gamma_{ft,lm}$: Geometry factor

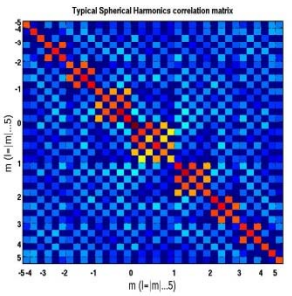
Extension to multiple baseline

- If we have more than two detectors:
  - Repeat analysis for every pair (baseline)
  - Quantities X and Γ are additive!

$$X_\alpha^{\mathcal{N}} = \sum_I \sum_{J>I} X_\alpha^{IJ}, \quad \Gamma_{\alpha\beta}^{\mathcal{N}} = \sum_I \sum_{J>I} \Gamma_{\alpha\beta}^{IJ}$$

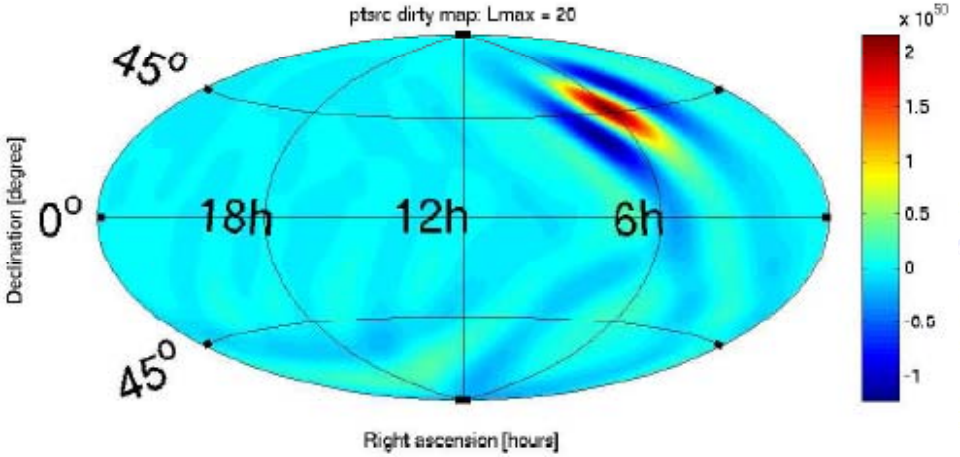
Choice of basis

- Any basis of S_2 works in principle
  - Pixel, Spherical Harmonics, …
- We chose:
  Spherical Harmonics, z parallel earth axis
  - Respects 2 symmetries of the problem:
    - Parity, exact
      ➔ covariance $\Gamma_{lm,l'm'} = 0$ for all odd (l-l')
    - Z-rotation, broken by daily sensitivity variations
      ➔ Block diagonally dominant covariance matrix $\Gamma_{lm,l'm'}$
  - Has natural resolution cut-off l_{max}

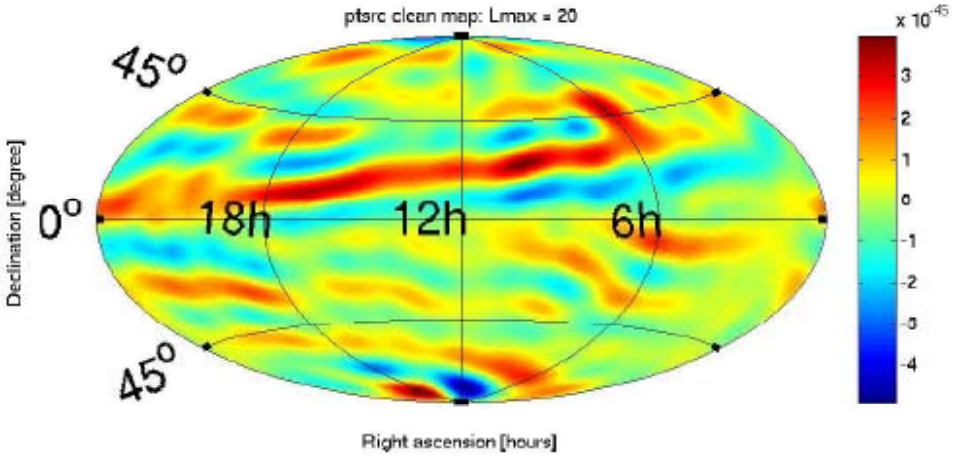


Naïve application point source

Dirty map X



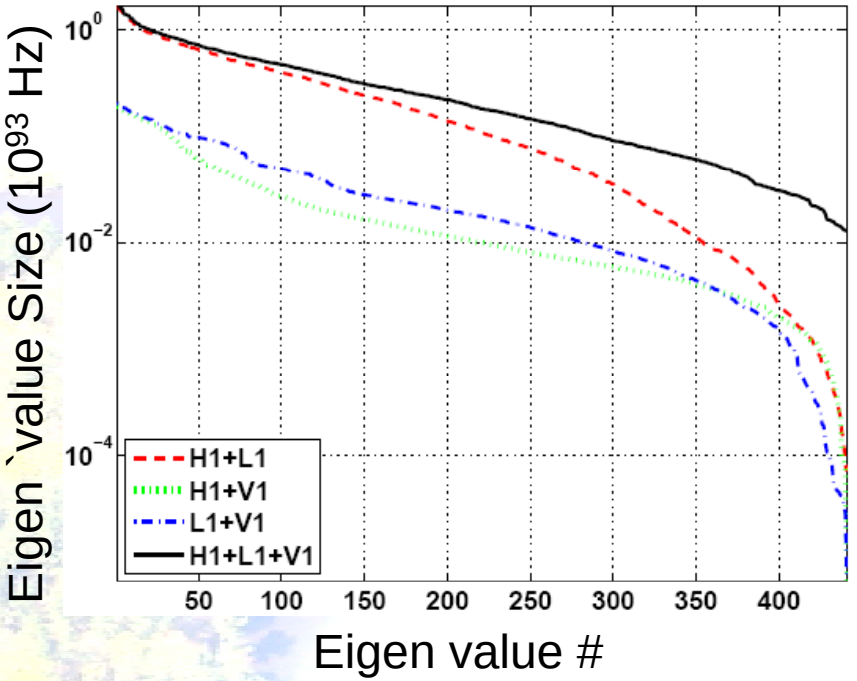
"Clean" map $\hat{\mathcal{P}}$



What's wrong?

Regularization

- Is Γ invertible?
- Small Eigen-values problematic: $X_\alpha = \Gamma_{\alpha\beta}\hat{\mathcal{P}}_\beta \approx 0$
- Singular Value Dec…
- Ignore the small EVs (e.g. 1/3 of all EVs)
  - Effect. blind to corresp. Eigen-vectors $\hat{\mathcal{P}}_\beta$

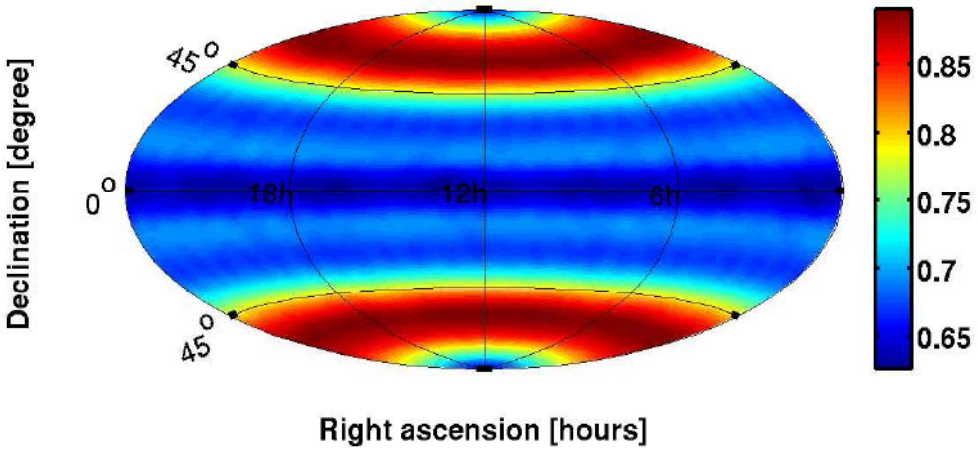


Implication of Regularization

- Blind to some distributions $\hat{\mathcal{P}}_\beta$

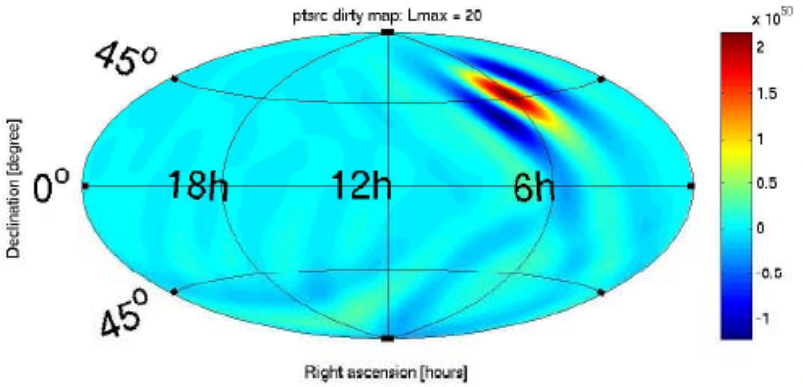
  ➔Introduces a Bias (some power missed)

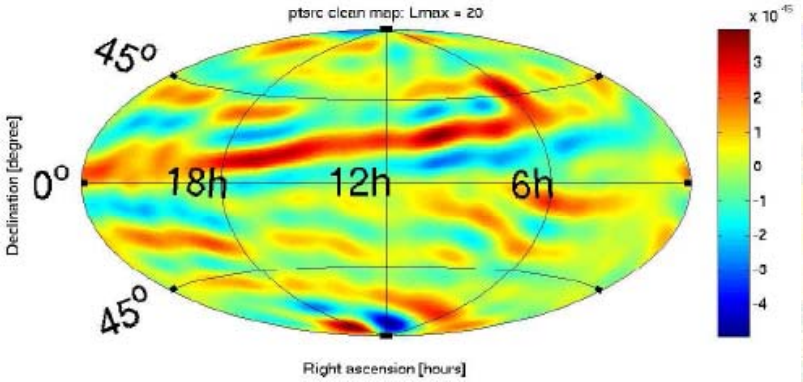
  – Bias can be calculated IF we know the actual distribution:

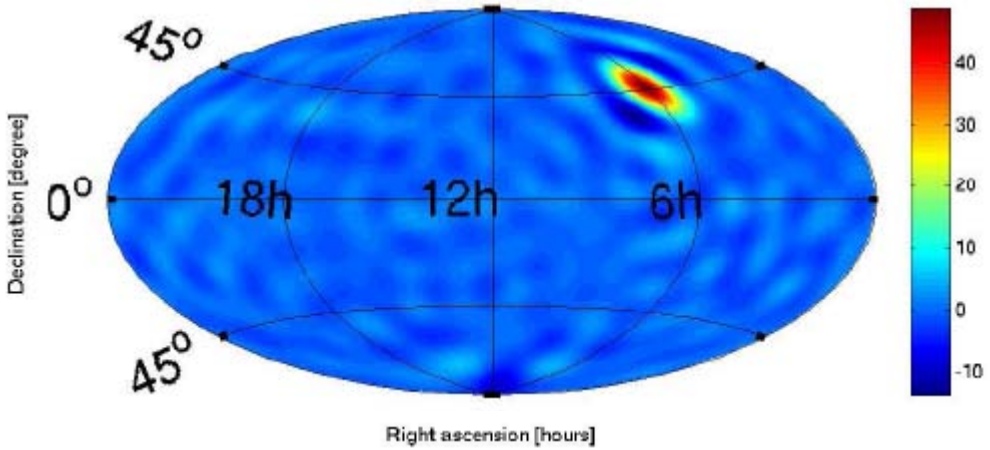


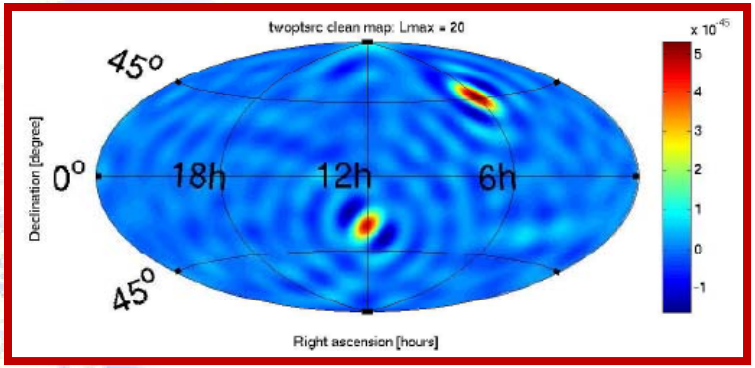
Simulation: Point source

SNR



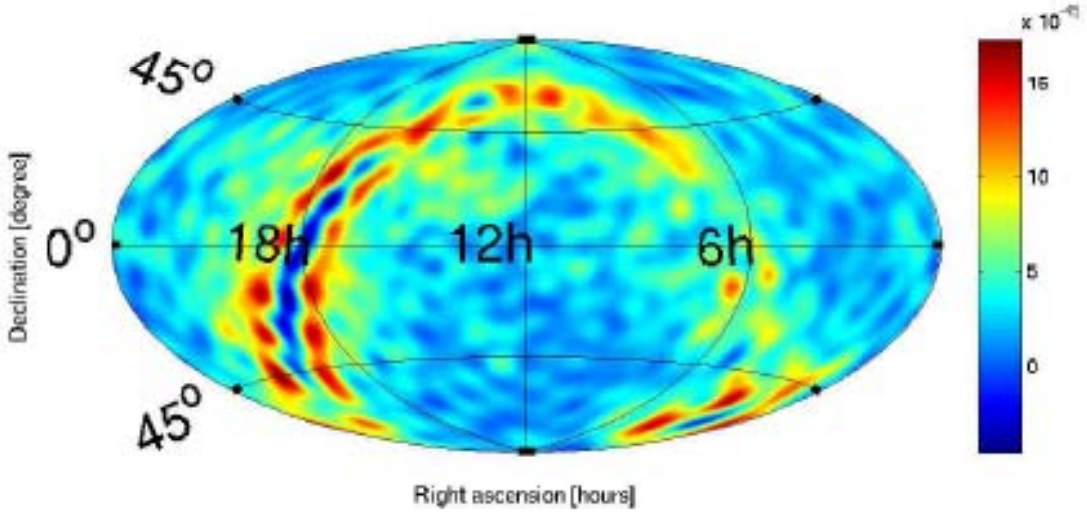




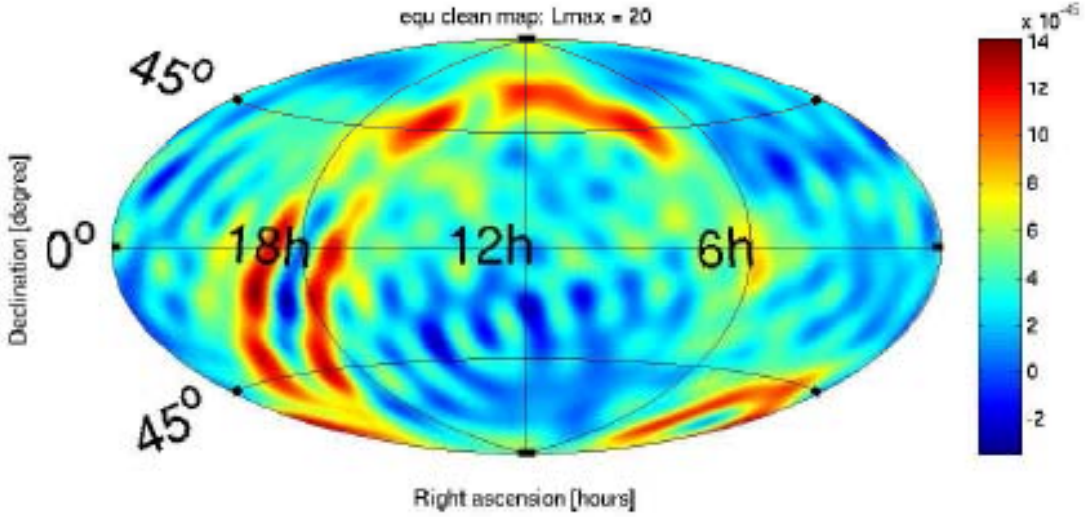


Simulation: "Diffuse Background"

Injected



Recovered



Conclusion

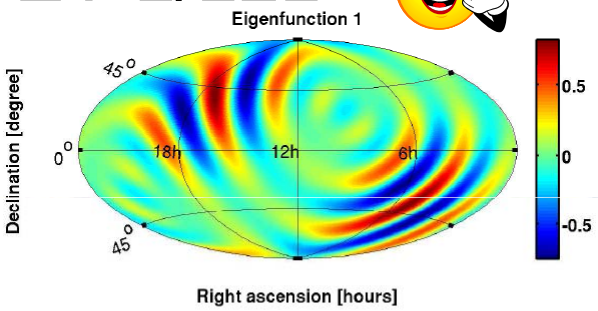
- The ML method
  - Includes existing analysis as special cases
  - Can recover spatial structure well
- Method paper in preparation
- S5 analysis under way

The End

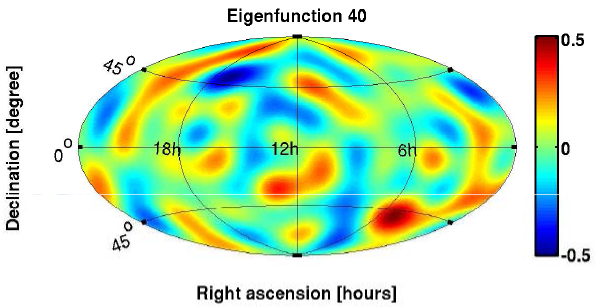
Extra Slides

We are partly blind…

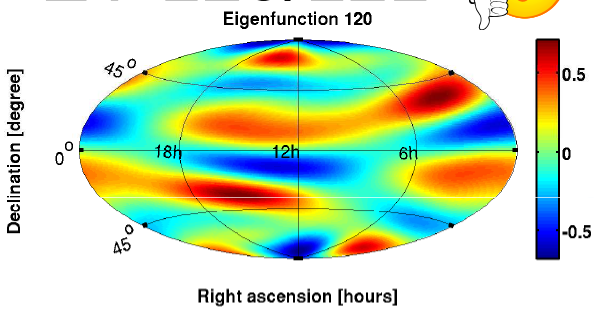
EV 1/121



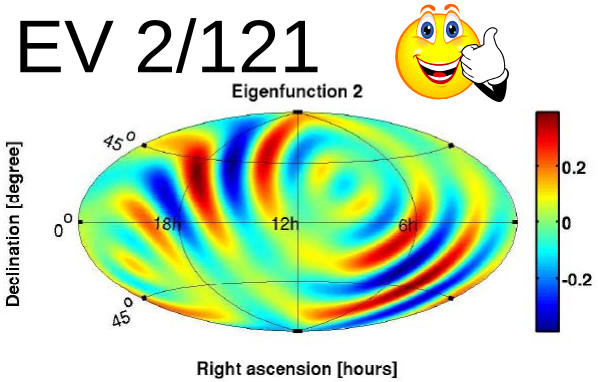
EV 40/121



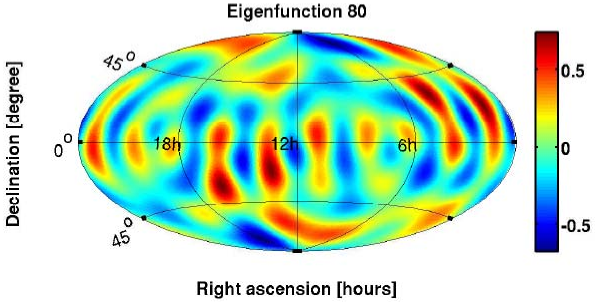
EV 120/121



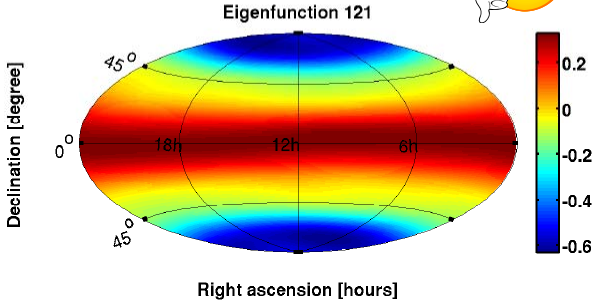
EV 2/121



EV 80/121



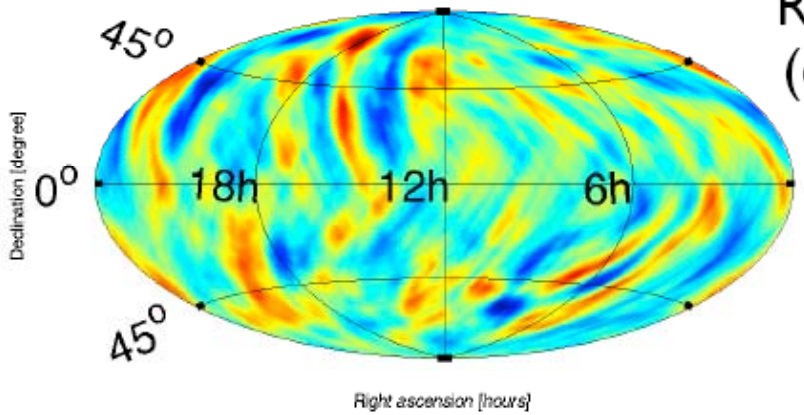
EV 121/121



Special case

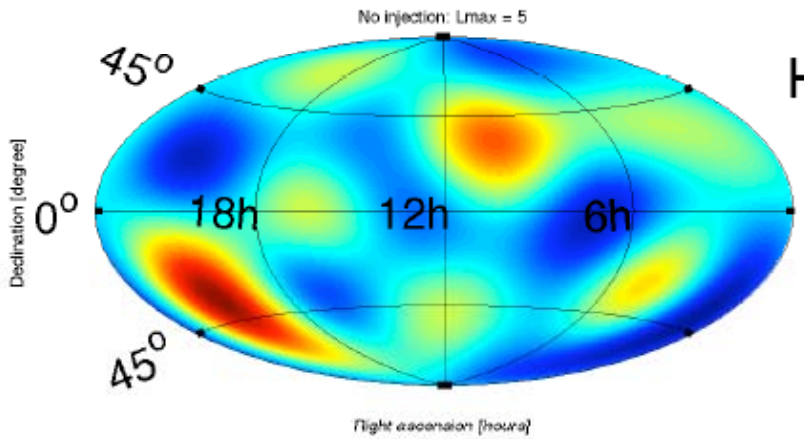
Spherical Harmonics
Lmax = 0

Isotropic result:
One number

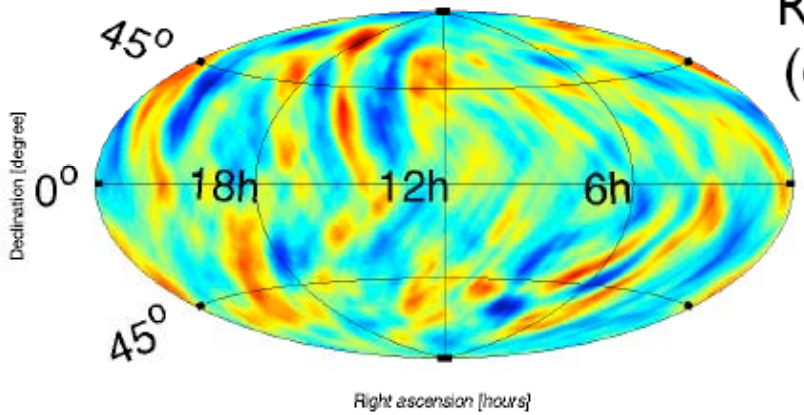


Radiometer (dirty map)

Special case

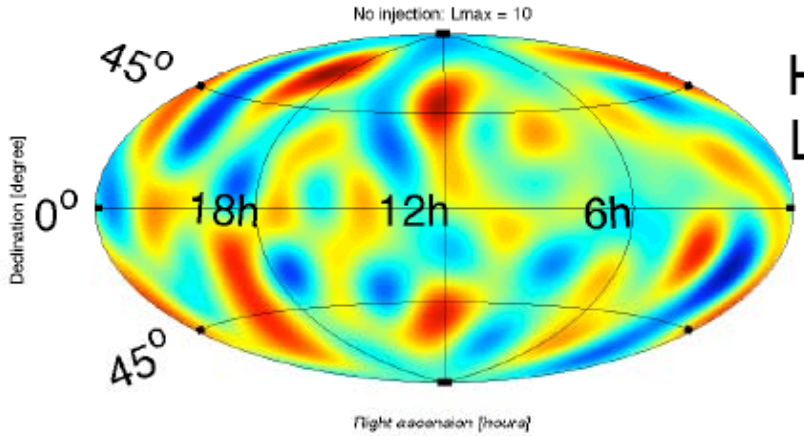


Spherical Harmonics Lmax = 5

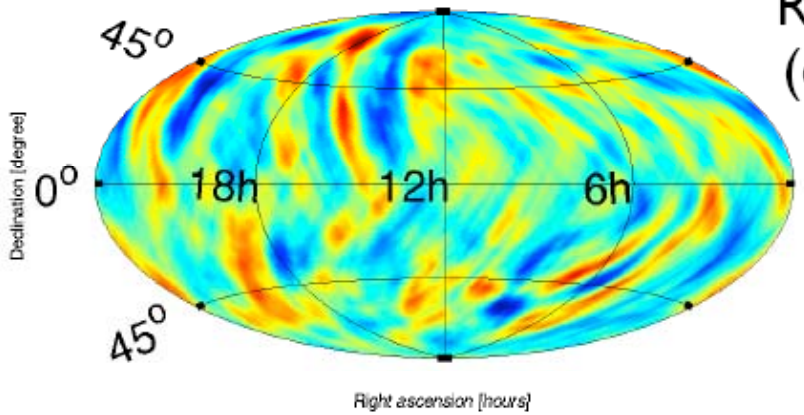


Radiometer (dirty map)

Special case

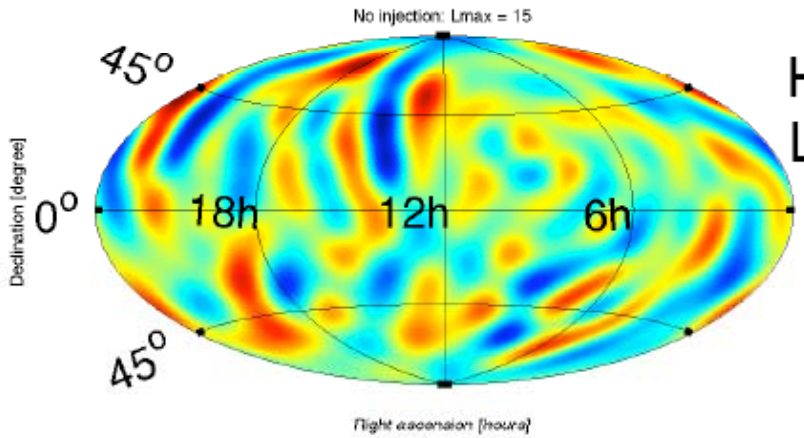


Spherical Harmonics Lmax = 10

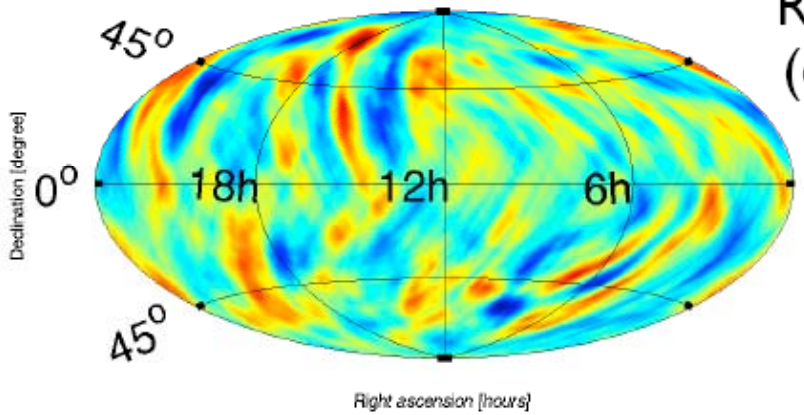


Radiometer (dirty map)

Special case

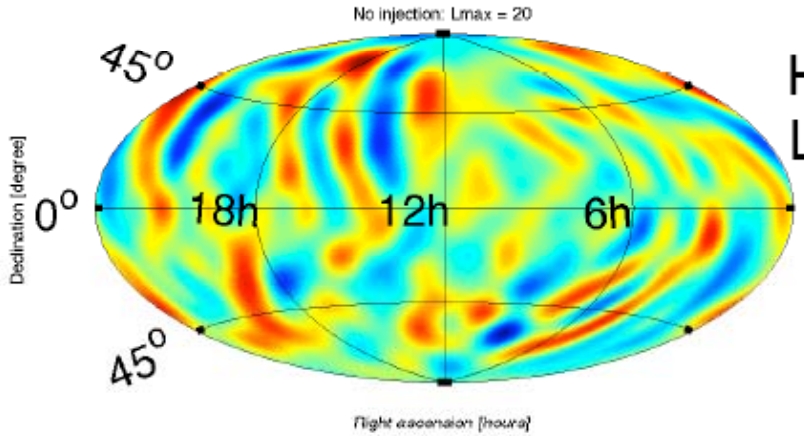


Spherical Harmonics Lmax = 15

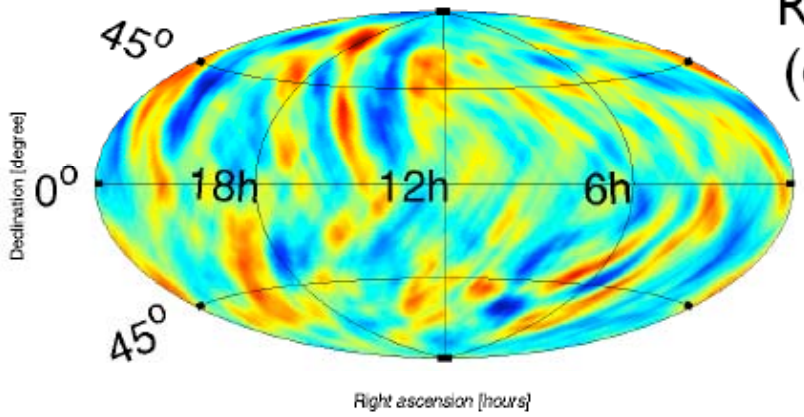


Radiometer (dirty map)

Special case

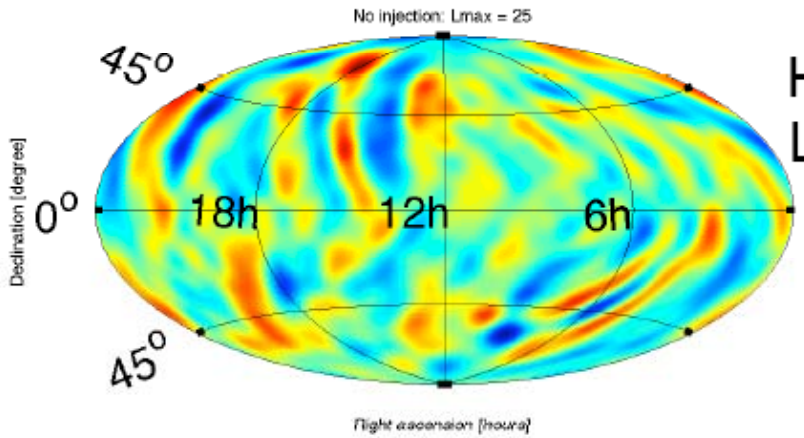


Spherical Harmonics Lmax = 20

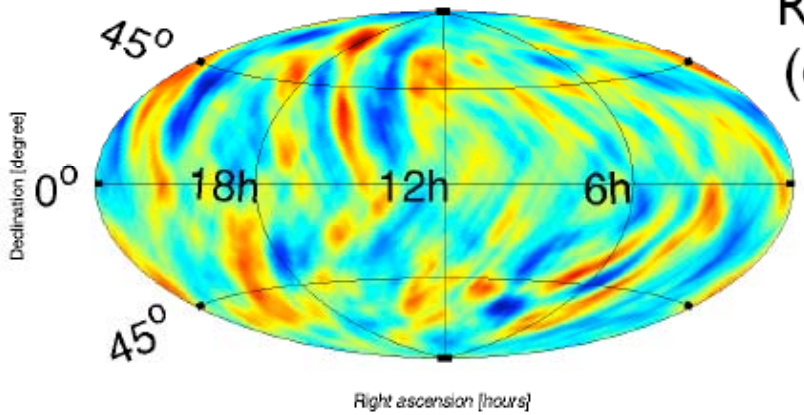


Radiometer (dirty map)

Special case

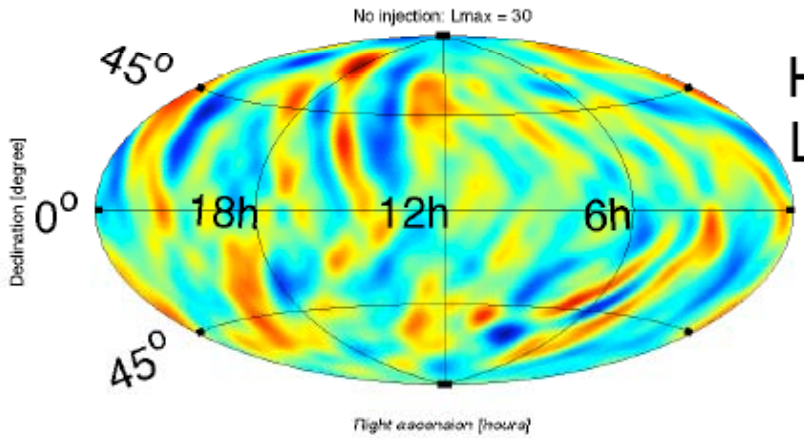


Spherical Harmonics Lmax = 25

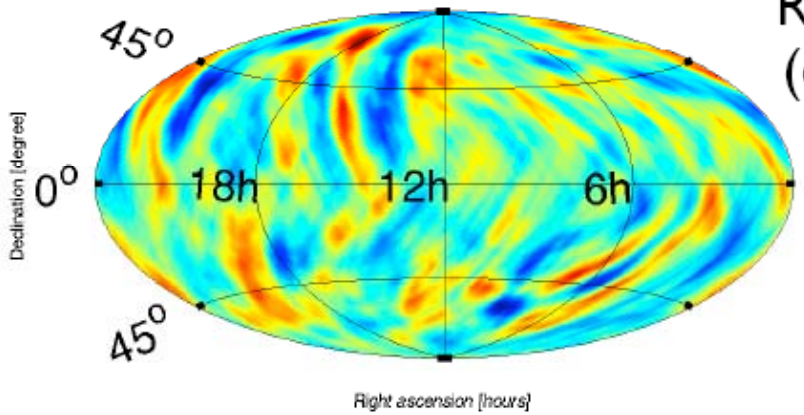


Radiometer (dirty map)

Special case



Spherical Harmonics Lmax = 30



Radiometer (dirty map)