# Controlling radiation pressure instabilities in Enhanced LIGO


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## The Sigg-Sidles instability[1] in cavities

The torques acting on a suspended mirror with light on it are the natural pendulum restoring torque and a torque due to radiation pressure:

$$\tau_{pend} = \alpha I \omega_{pend}^2$$

$$\tau_{rp} = \frac{2P}{c} x$$

- $\alpha$ - angle of mirror from nominal axis
- $I$ - mirror moment of inertia
- $\omega$ - pendulum resonant frequency
- $P$ - power of incident light
- $c$ - speed of light
- $x$ - distance of beam to center of mirror

The torsional spring constant of the pendulum is

$$k_{pend} = I\omega_{pend}^2$$

whereas the torsional spring constant due to radiation pressure in a cavity is given by the torsional stiffness matrix

$$k_{rp} = -\frac{2PL}{c(1-g_1 g_2)} \begin{bmatrix} g_2 & 1 \\ 1 & g_1 \end{bmatrix}$$

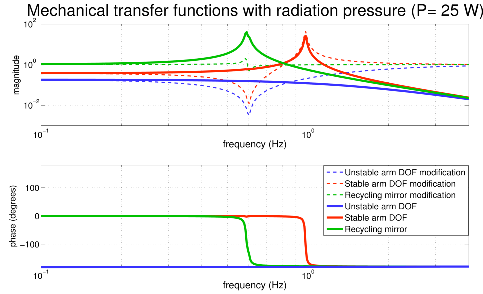
- $L$ - cavity length
- $g = 1 - L/R$ - cavity g-factor
- $R$ - mirror radius of curvature

The resonating light in a cavity acts as an optical spring. The eigenvalues of $k_{rp}$ are the spring constants of the two optical modes. When combined with the pendula, the two new modes, hard and soft, have resonant frequencies

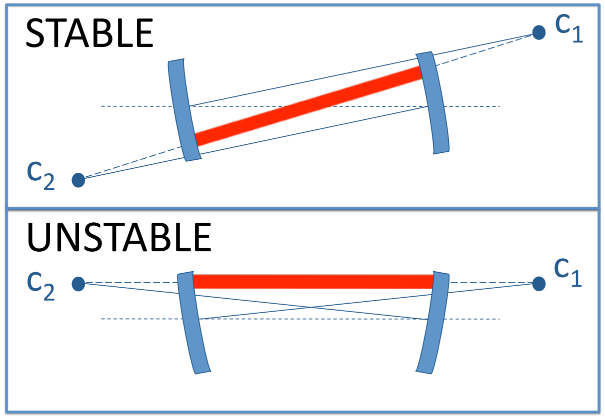
$$f_{hard} = \frac{1}{2\pi}\sqrt{\frac{k_{pend} + k_{hard}^{optical}}{I}} \qquad f_{soft} = \frac{1}{2\pi}\sqrt{\frac{k_{pend} + k_{soft}^{optical}}{I}}$$

When the torque due to radiation pressure exceeds the pendulum restoring torque *and* acts in the opposite direction, $f_{soft}$ becomes imaginary and the mode is now unstable. When $\tau_{rp}$ and $\tau_{pend}$ act in the same direction, $f_{hard}$ increases and the mode is stable.

Mechanical transfer functions with radiation pressure (P= 25 W)

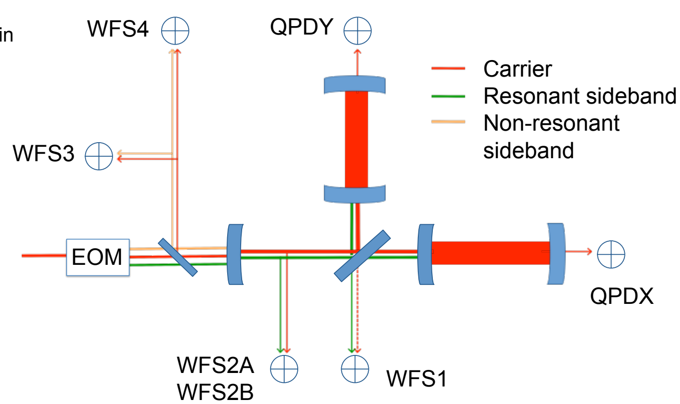



| PITCH | Intra-cavity power (kW) | Pendulum frequency (Hz) | f_hard (Hz) | f_soft (Hz) |
|---|---|---|---|---|
| Initial LIGO | 15 | 0.6 | 0.69 | 0.25 i |
| Enhanced LIGO | 87.5 | 0.6 | 1.0 | 1.46 i |




The four times increase in power from Initial LIGO to Enhanced LIGO requires an upgrade to the Angular Sensing and Control system. Increased radiation pressure effects on the mirrors must be compensated by changing the control basis to directly address the cavity instabilities.

## Angular sensing and control topology




## Angular sensing and control

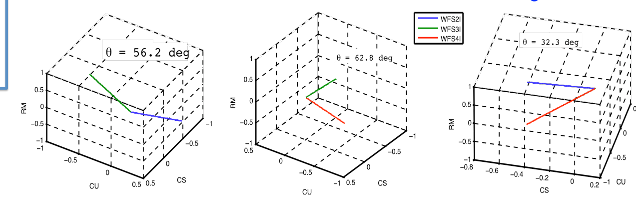
The eigenvectors of $k_{rp}$ describe how to construct the soft (stable) and hard (unstable) modes for one arm. The two arms' modes are combined to form four interferometer degrees of freedom:

DU - differential unstable
CU - common unstable
DS - differential stable
CS - common stable

The angular control matrix.

| | DU | CU | DS | CS | RM |
|---|---|---|---|---|---|
| ETMX | 1.0 | 1.0 | 0.87 | 0.87 | 0 |
| ETMY | -1.0 | 1.0 | -0.87 | 0.87 | 0 |
| ITMX | 0.87 | 0.87 | -1.0 | -1.0 | 0 |
| ITMY | -0.87 | 0.87 | 1.0 | -1.0 | 0 |
| RM | 0 | 0 | 0 | 0 | 1.0 |

Each of these degrees of freedom are excited and responses are measured in the wavefront sensors to construct a sensing matrix.

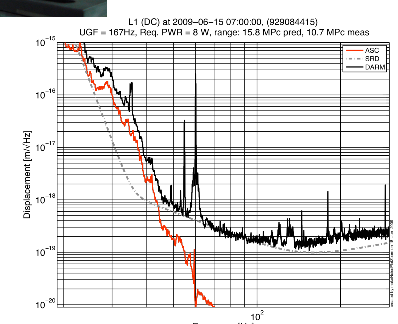



Projections of a normalized common DOF sensing matrix

Error signals are multiplied by the sensing matrix, sent through digital filters, multiplied by the control matrix and sent to the optics. The digital control filters must add phase to the unstable mode without taking too much away from the stable mode.

Wavefront sensors (WFS) are RF quadrant photodiodes used to detect the angular motions of the optics.

Angular sensing and control noise couples to the gravitational wave readout signal, limiting sensitivity at 40 Hz and below. Careful tuning of the control filters can reduce coupling.




[1] J.A.Sidles and D.Sigg. Optical torques in suspended fabry-perot interferometers. *Physics Letters A*, 354:167–172, 2006.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